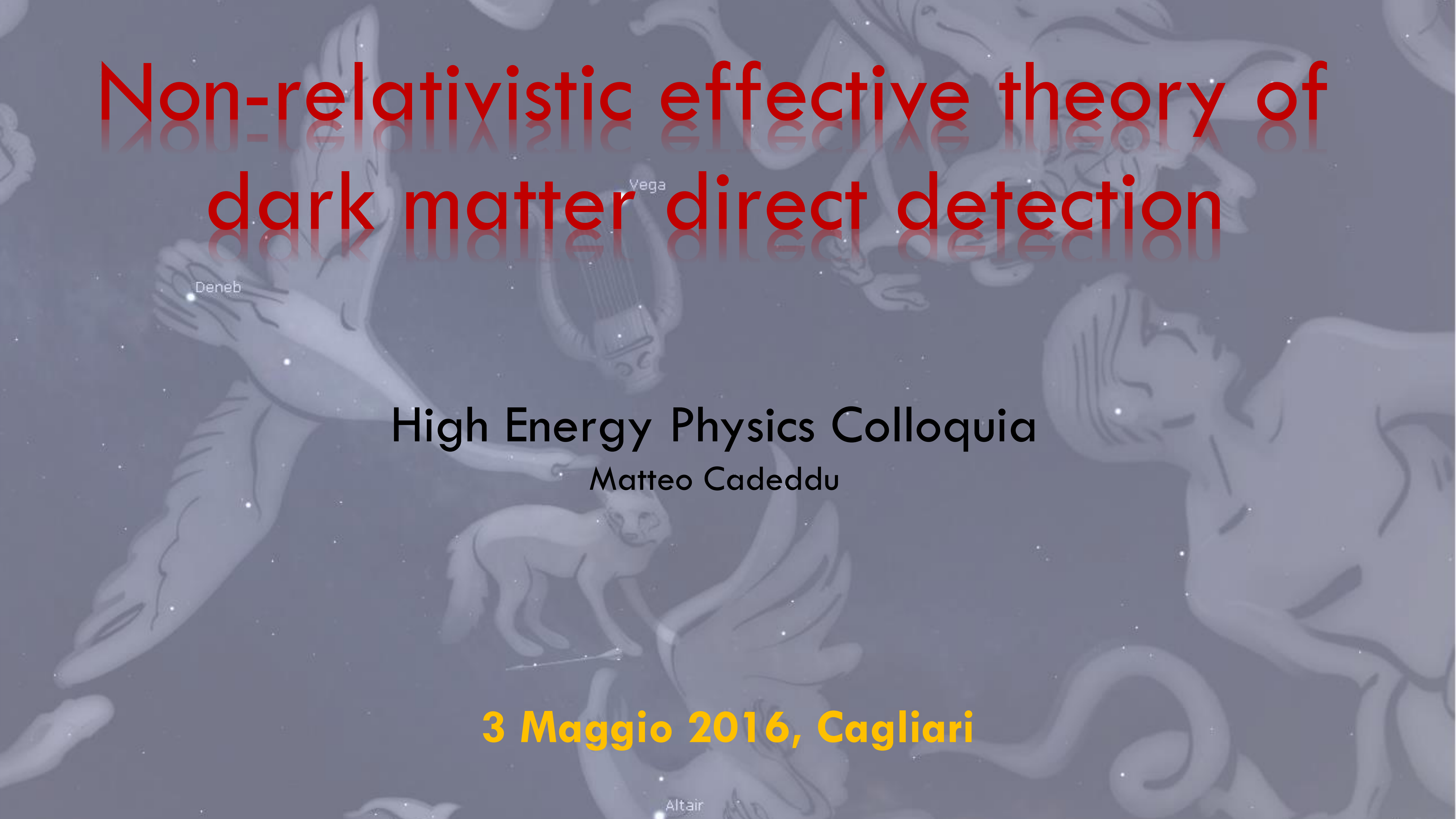
Non-relativistic effective theory of dark matter direct detection



High Energy Physics Colloquia

Matteo Cadeddu

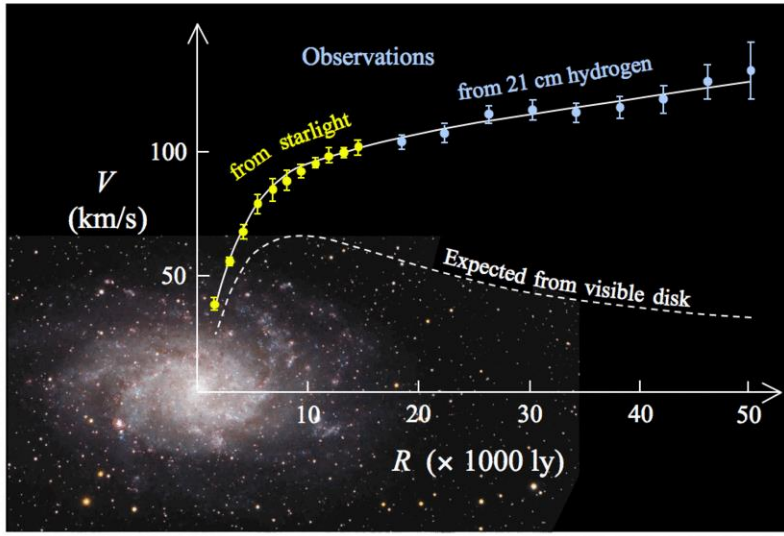
3 Maggio 2016, Cagliari

Observational evidence for Dark Matter

- The evidence for the existence of Dark Matter (DM) is overwhelming, and comes from a wide variety of astrophysical measurements.

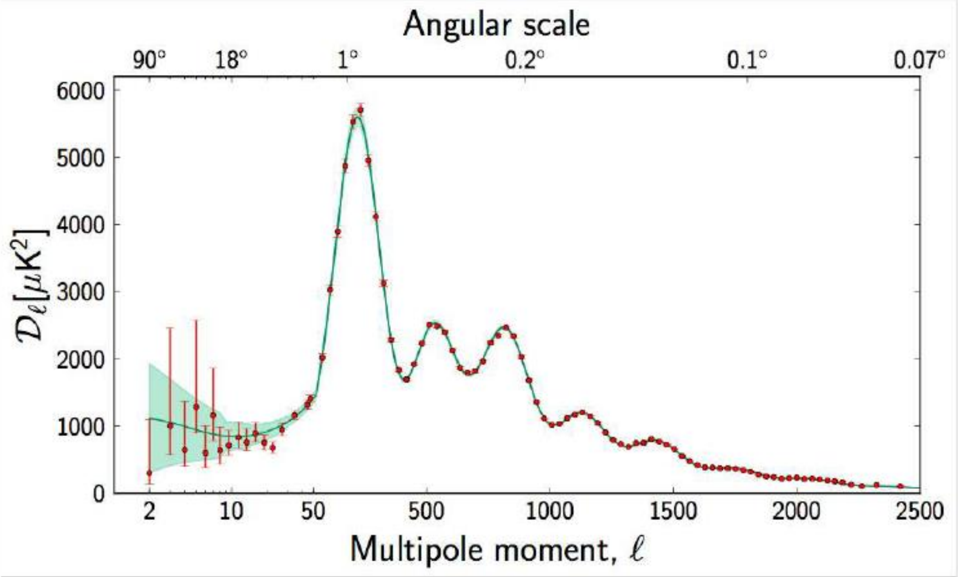
Galaxy rotation curves

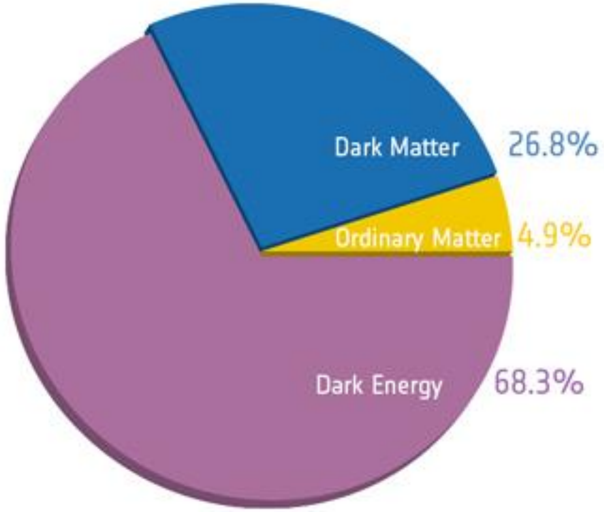
In the 1970s, Ford and Rubin discovered that rotation curves of galaxies are flat. The simplest explanation is that galaxies contain far more mass than can be explained by the bright stellar objects in the galactic disks.



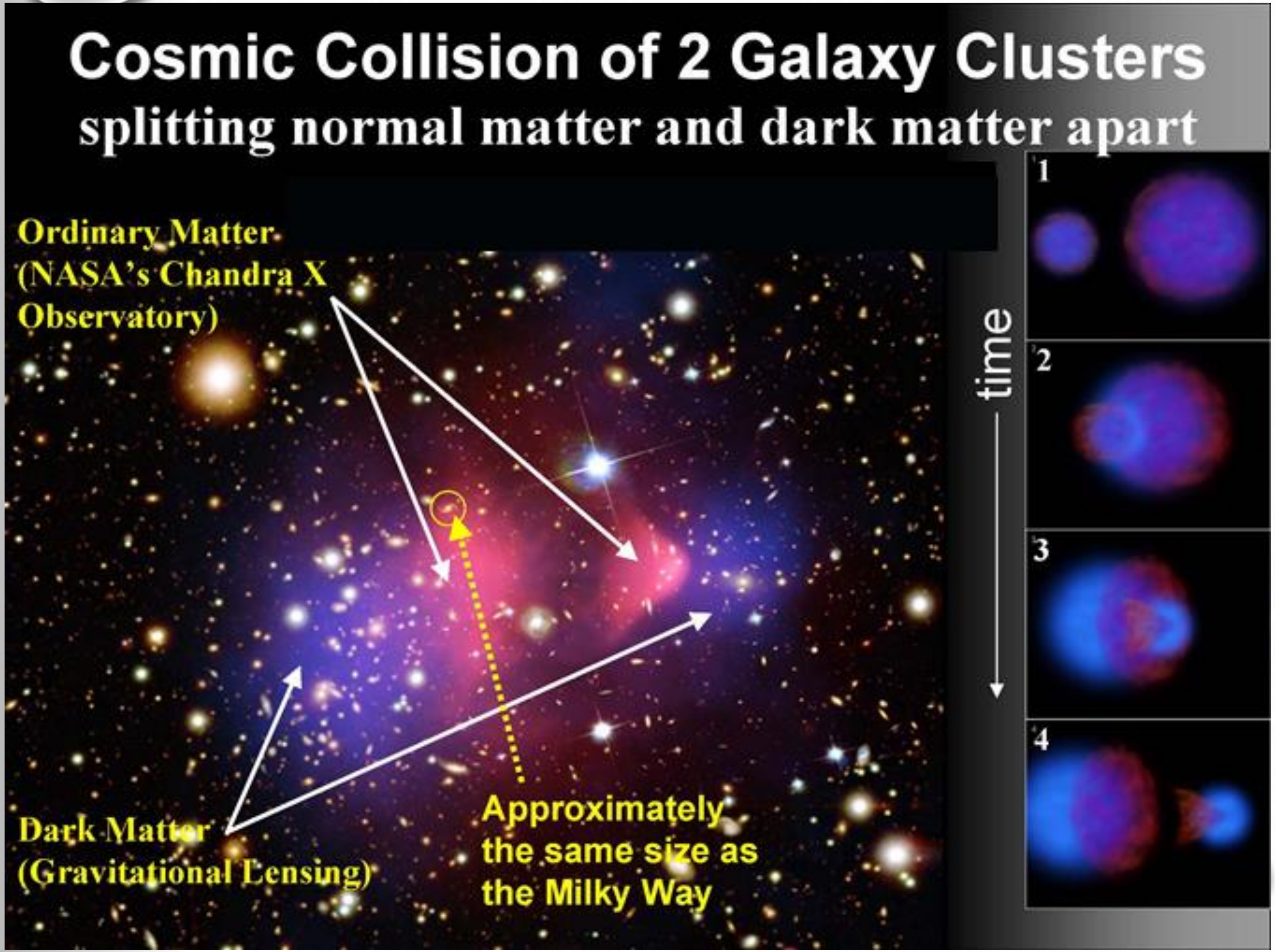
CMB

CMB temperature anisotropy angular power spectrum seen by Planck, with the predictions for the best fit of the standard cosmological model parameters

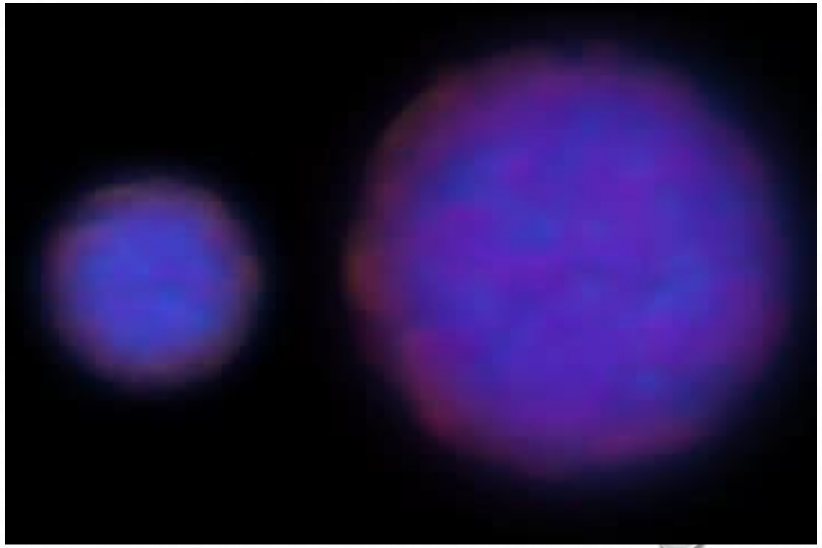




Observational evidence for Dark Matter



Bullet cluster: A collision of galactic clusters (the bullet cluster) shows baryonic matter (pink) as separate from dark matter (blue), whose distribution is deduced from gravitational lensing.

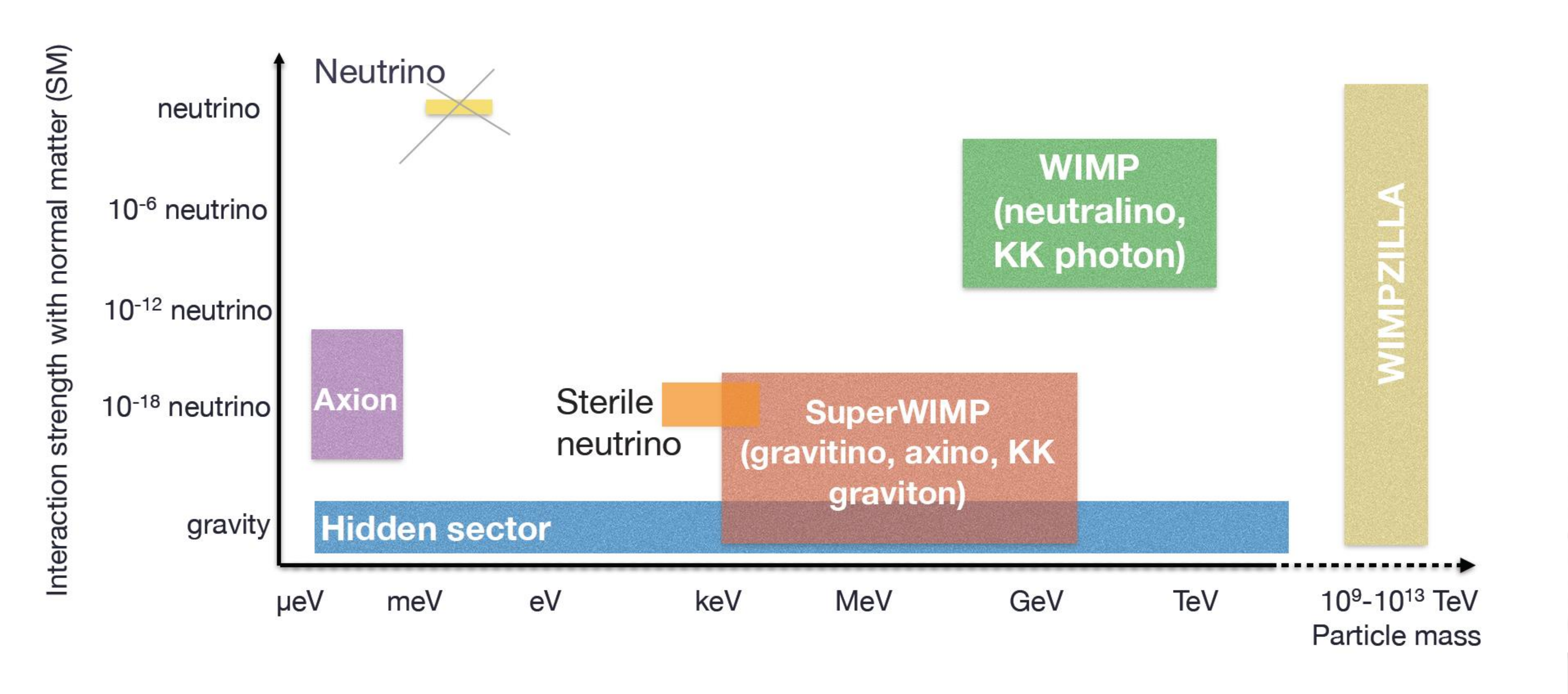


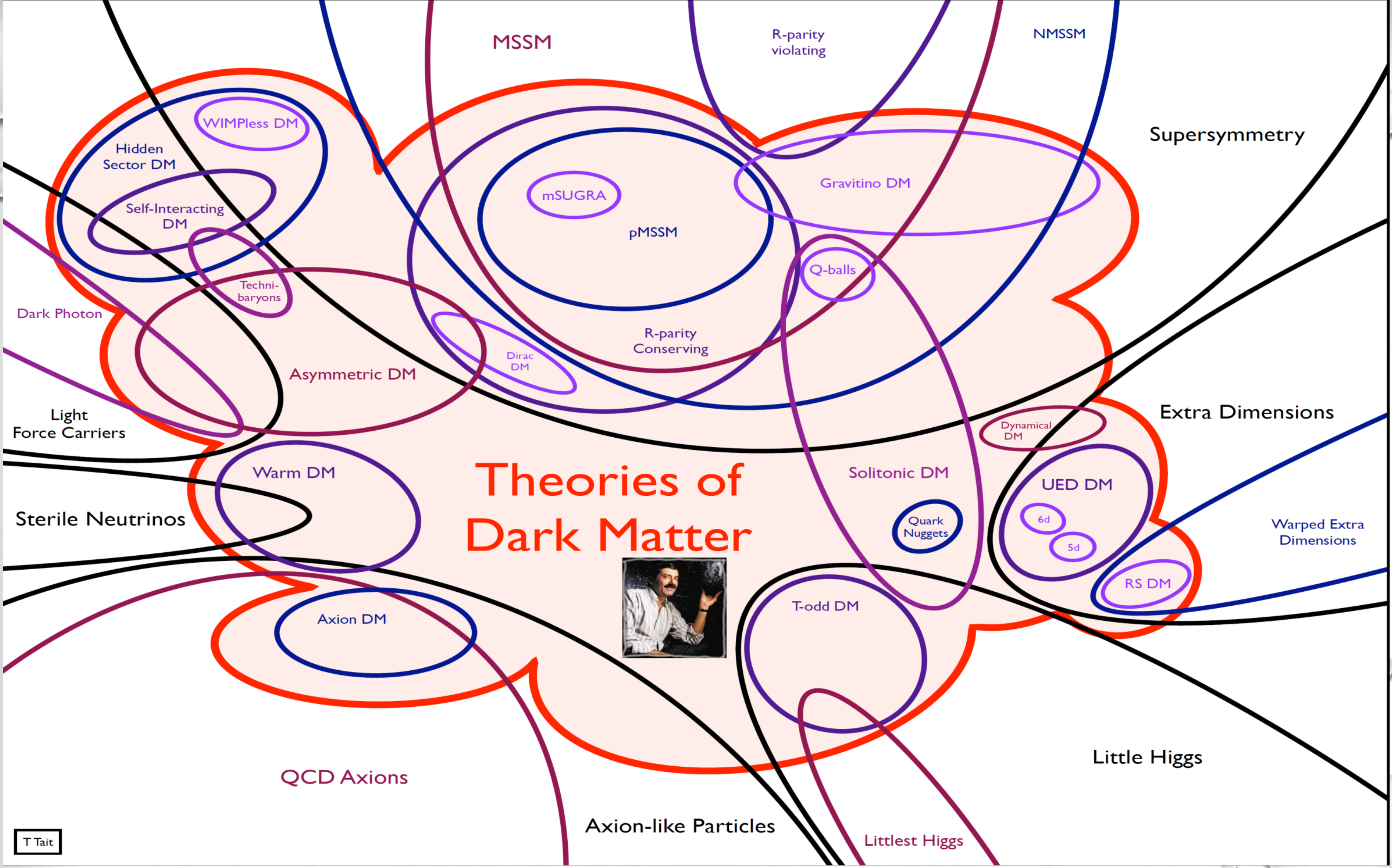
A plausible explanation of DM is that it is constituted by

Weakly Interacting Massive Particles (WIMPs)

DM candidates

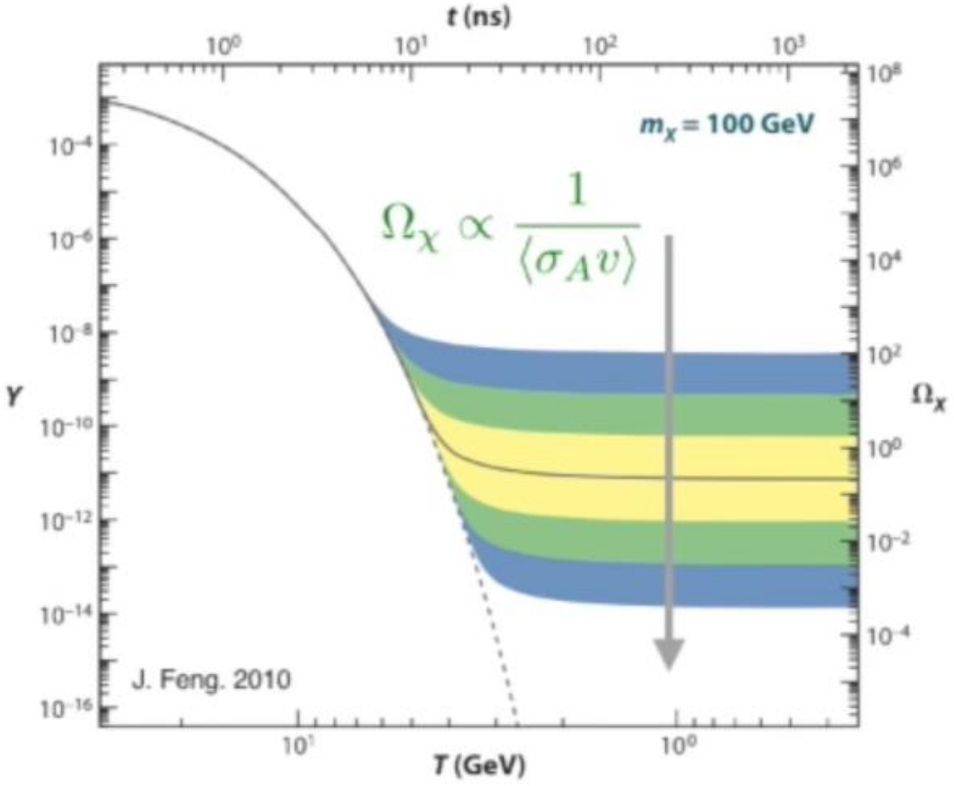
The mass and cross section span many orders of magnitude




Theories of Dark Matter
MSSM
R-parity violating
NMSSM
Supersymmetry
WIMPless DM
Hidden Sector DM
Self-Interacting DM
Techni-baryons
mSUGRA
pMSSM
Gravitino DM
Q-balls
Dark Photon
R-parity Conserving
Dirac DM
Asymmetric DM
Light Force Carriers
Dynamical DM
Extra Dimensions
Warm DM
Solitonic DM
UED DM
Sterile Neutrinos
Quark Nuggets
6d
5d
Warped Extra Dimensions
RS DM
T-odd DM
Axion DM
Little Higgs
QCD Axions
Axion-like Particles
Littlest Higgs
T Tait

WIMP Miracle

- **In thermal equilibrium in the early Universe**
- Freeze-out: *when annihilation rate drops below expansion rate and $M_{WIMP} > T$ ('cold')*
- Their relic density can account for the dark matter *if the annihilation cross section is weak (pb range)*



$$\Omega_\chi h^2 \simeq 3 \times 10^{-27} cm^3 s^{-1} \frac{1}{\langle \sigma_A v \rangle}$$

$$\Omega_\chi h^2 = \Omega_{cdm} h^2 \simeq 0.1141 \Rightarrow \langle \sigma_A v \rangle \simeq 3 \times 10^{-26} cm^3 s^{-1}$$

How to detect Weakly Interacting Massive Particles

Direct detection

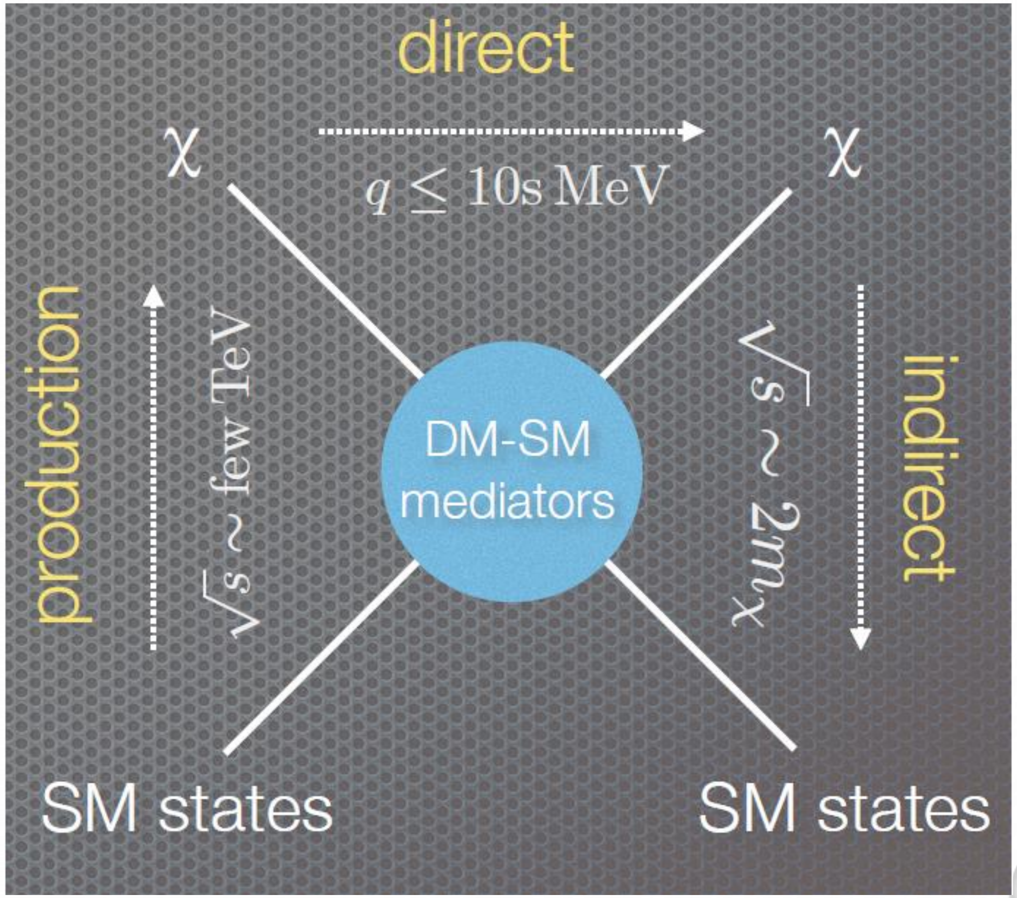
Nuclear recoils from elastic scattering
Dependence on A, J; annual modulation, directionality
Local densitiy and v-distribution

Indirect detection

high-energy neutrinos, gammas,
look at over-dense regions in the sky
astrophysics backgrounds difficult

Accelerator searches

missing ET, mono-‘objects’, etc
can it establish that the new particle is the DM?



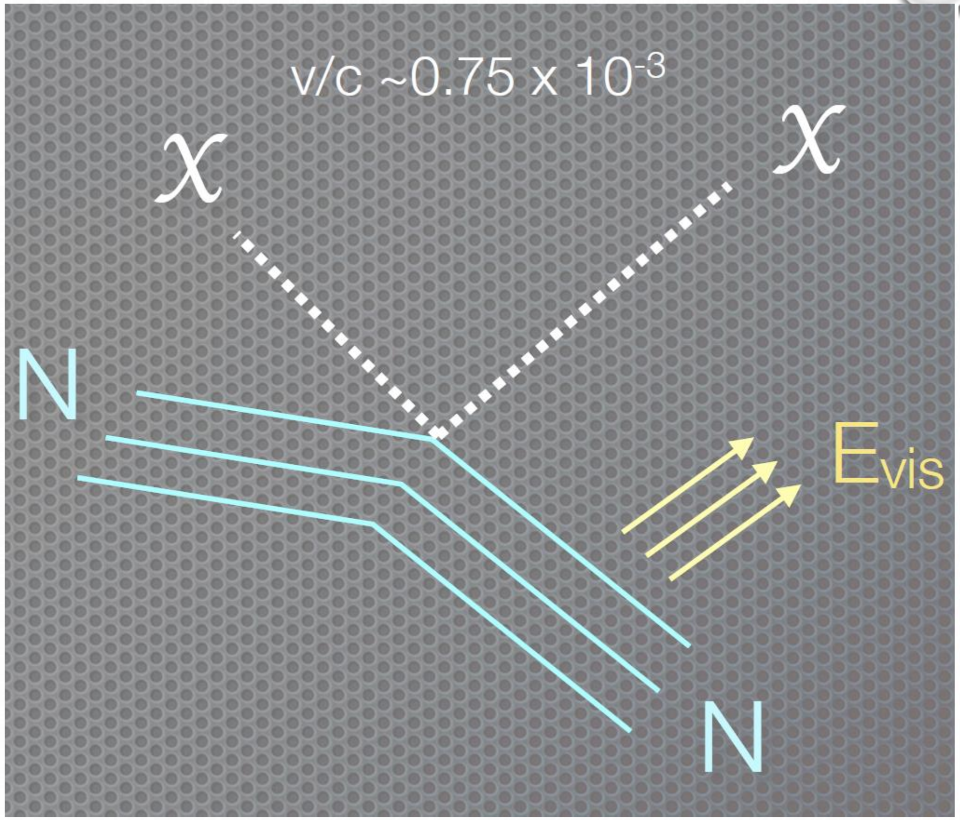
WIMP detection in the laboratory

REVIEW D VOLUME 31, NUMBER 12

Detectability of certain dark-matter candidates

Mark W. Goodman and Edward Witten
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544
(Received 7 January 1985)

We consider the possibility that the neutral-current neutrino detector recently proposed by Drukier and Stodolsky could be used to detect some possible candidates for the dark matter in galactic halos. This may be feasible if the galactic halos are made of particles with coherent weak interactions and masses $1-10^6$ GeV; particles with spin-dependent interactions of typical weak strength and masses $1-10^2$ GeV; or strongly interacting particles of masses $1-10^{13}$ GeV.



By searching for collisions of invisibles particles with atomic nuclei => Evis (q ~tens of MeV)

Need *very low energy thresholds*

Need *ultra-low backgrounds*, good background understanding and discrimination

Need *large detector masses*

$$10\ keV \lesssim E_R = \frac{q^2}{2m_N} \lesssim 100\ keV$$

$$15\ MeV \lesssim q \lesssim 150\ MeV$$

Direct detection WIMP flux on Earth

- For a typical WIMP mass of 100 GeV$/c^2$, the expected WIMP flux on Earth is

$$\phi_\chi = \frac{\rho}{M_\chi} \times \langle v \rangle \sim 10^5\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- The flux is sufficiently large that, even though WIMPs are weakly interacting, a small but potentially measurable fraction will elasticaly scatter off nuclei in a Earth-bound detector
- Direct Dark Matter detection experiments aim to detect WIMPs via nuclear recoils which are caused by WIMP-nucleus elastic scattering

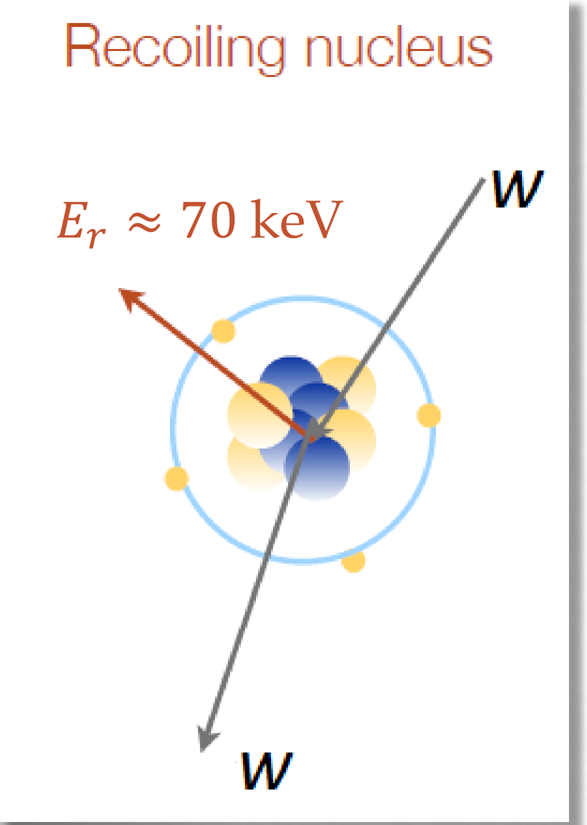
- Assuming a scattering cross section of $10^{-38} cm^2$, the expected rate (for a nucleus with atomic mass A=100) would be

$$R = \frac{N_A}{A} \times \phi_\chi \times \sigma \sim 0.13\ \mathrm{events\ kg^{-1}yr^{-1}}$$

The standard WIMP recoil spectrum

Standard **recoil spectrum**, i.e. differential event rate **per unit detector** mass:

$$\frac{dR}{dE_r} = \frac{1}{M_T}\frac{1}{M_w} F^2(E_r)\,\rho \int_{v_n=\sqrt{M_T E_{th}/(2\mu^2)}}^{v_{max}} dv\, v\, f(\boldsymbol{v}) \frac{d\sigma}{dE_r}$$



Physics

$\frac{d\sigma}{dE_r}$ → WIMP-nucleon differential cross section

M_w → WIMP mass

μ → WIMP-nucleus reduced mass

Detector

A → atomic mass of target material

M_T → Nucleus mass

$F(E_r)$ → The finite size of the nucleus is implemented with **Helm form Factor**

E_{th} → Energy threshold

Astrophysics (DM halo properties)

ρ → WIMP mass density

$f(\boldsymbol{v})$ → WIMP velocity distribution

v_n → minimum WIMP speed required to transfer an energy E_r to the nucleus of mass M_n in the detector.

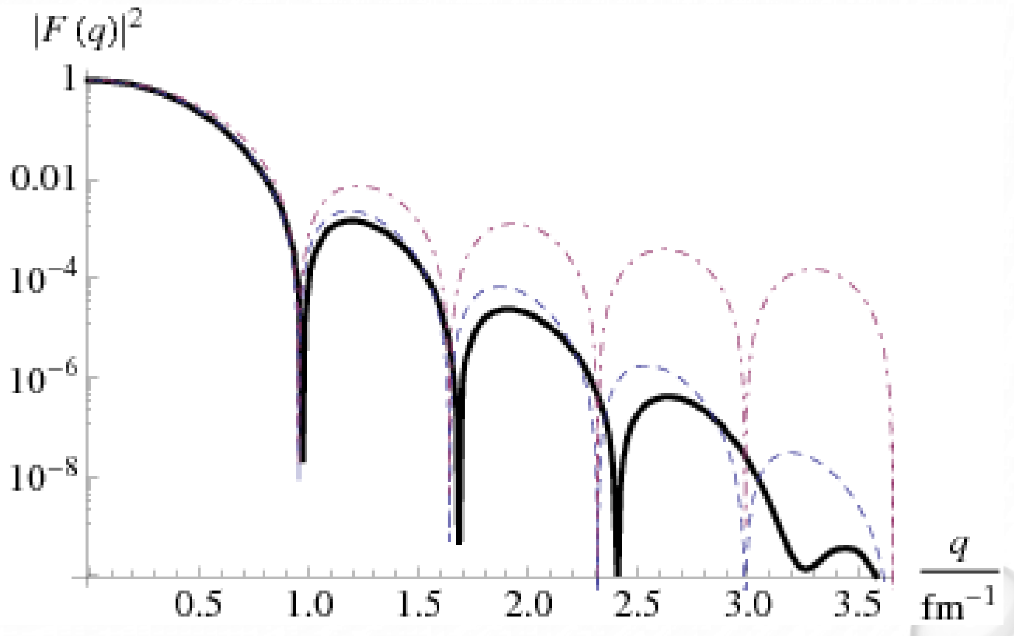
E_r → Recoiling nucleus energy

"Standard" scattering cross section

- **scalar interactions** (**spin-independent** neutralino-nucleus elastic cross section) with a pointlike nucleus of Z protons and N neutrons can be written as

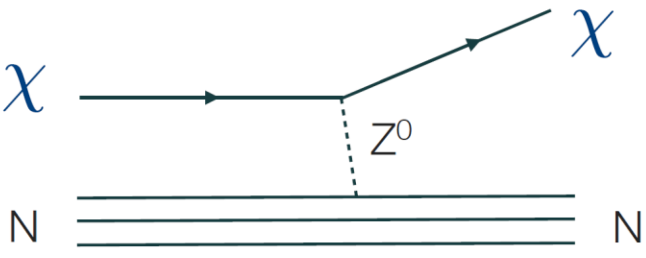
$$\sigma_i^{SI} = \frac{\mu_i^2}{\pi}\left[Z G_s^p + (A - Z) G_s^n\right]^2 \xrightarrow{G_s^p = G_s^n} \frac{\mu_i^2}{\pi}\left(G_s^p\right)^2 A^2$$

G_s^p, G_s^n: scalar four-fermion couplings to proton and neutron with point-like protons and neutrons

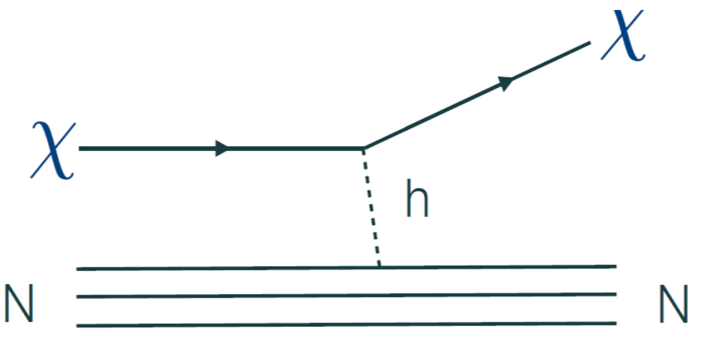


$$\sigma_i^{SI} = \sigma_p A^2 \left(\frac{\mu_i}{\mu_p}\right)^2$$

$$\frac{dR_i}{dE} = \frac{\rho \sigma_i^{SI} |F(q)|^2}{2 m \mu_i^2} \int_{v > q/2\mu} \frac{f(\vec{v}, t)}{v} d^3 v.$$



$\sigma_p \sim 10^{-39}\ cm^2$



$\sigma_p \sim 10^{-45} - 10^{-47}\ cm^2$

-> Nuclei with larger A favourable (but see nuclear form factor corrections)

The Helm Form factor

- The nuclear form factor, F(q), is taken to be the Fourier transform of a spherically symmetric ground state mass distribution normalized so that F(0) = 1:

$$F(q) = \frac{1}{M}\int \rho_{\text{mass}}(r)e^{-i\mathbf{q}\cdot\mathbf{r}}d^3r = \frac{1}{M}\int_0^\infty \rho_{\text{mass}}(r)\frac{\sin qr}{qr}4\pi r^2 dr.$$

Since the mass distribution in the nucleus is difficult to probe, it is generally assumed that mass and charge densities are proportional

$$\rho_{\text{mass}}(r) = \frac{M}{Ze}\rho_{\text{charge}}(r)$$

so that charge densities, determined through elastic electron scattering, can be utilized instead.

It is convenient to have an analytic expression. This expression has been provided by the **Helm form factor**, given by

$$|F^{SI}(q)|^2 = \left(\frac{3j_1(qR_1)}{qR_1}\right)^2 e^{-q^2s^2}$$

Spin dependent scattering cross section

- In general, interactions leading to WIMP-nucleus scattering are parameterized as:

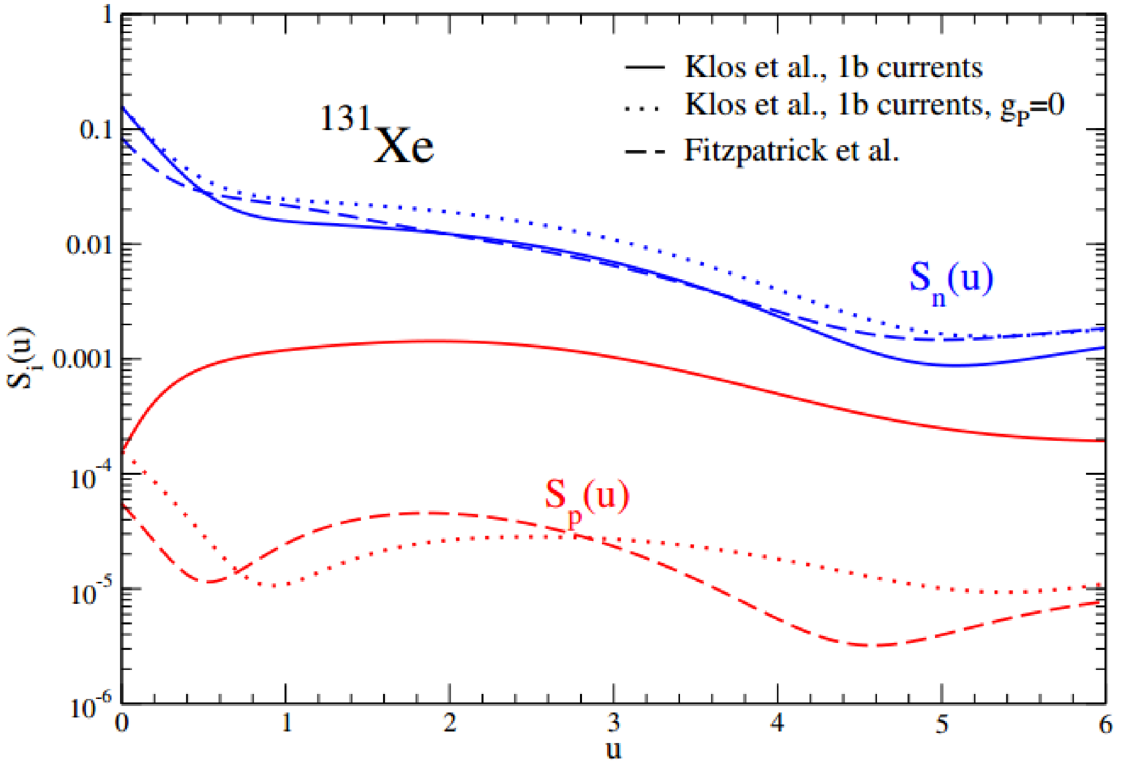
- **spin-spin interactions** (coupling to the **nuclear spin $\boldsymbol{J_N}$**, from axial-vector part of $\mathcal{L}$)

SPIN DEPENDENT

$$\sigma_{SD} \sim \mu^2 \frac{J_N + 1}{J_N} \left(a_p \langle S_p \rangle + a_n \langle S_n \rangle\right)^2$$

a_p, a_n: effective couplings to proton and neutron;

$\langle S_p \rangle$ and $\langle S_n \rangle$ expectation values of the proton and neutrons spins within the nucleus

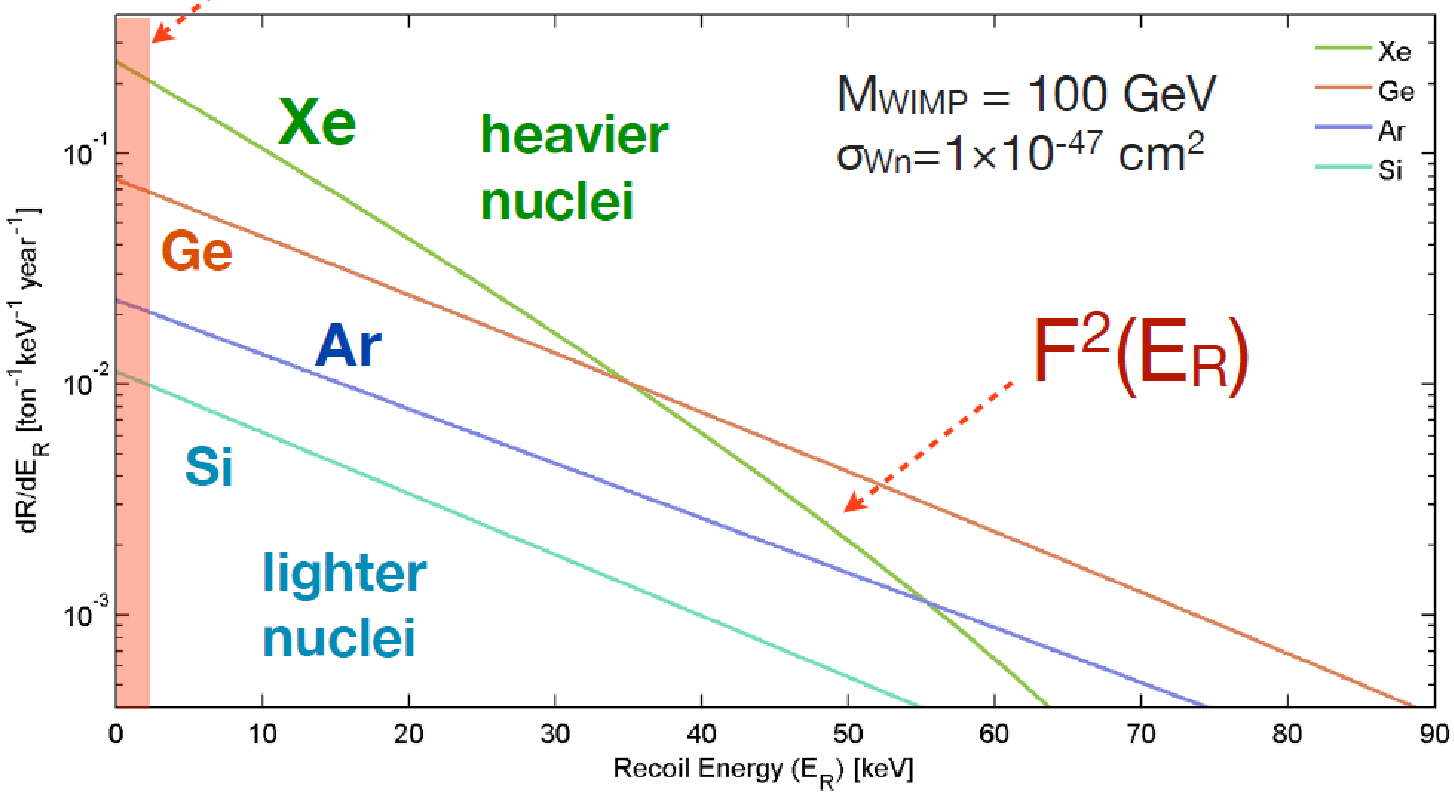


Expected interaction rates for SI interactions

$$\frac{dR_i}{dE} = \frac{\rho \sigma_i^{SI} |F(q)|^2}{2 m \mu_i^2} \int_{v>q/2\mu} \frac{f(\vec{v},t)}{v} d^3v$$

$$\sigma_i^{SI} = \sigma_p A^2 \left(\frac{\mu_i}{\mu_p}\right)^2$$

$$v_{min} = \sqrt{\frac{m_N E_{th}}{2\mu^2}}$$



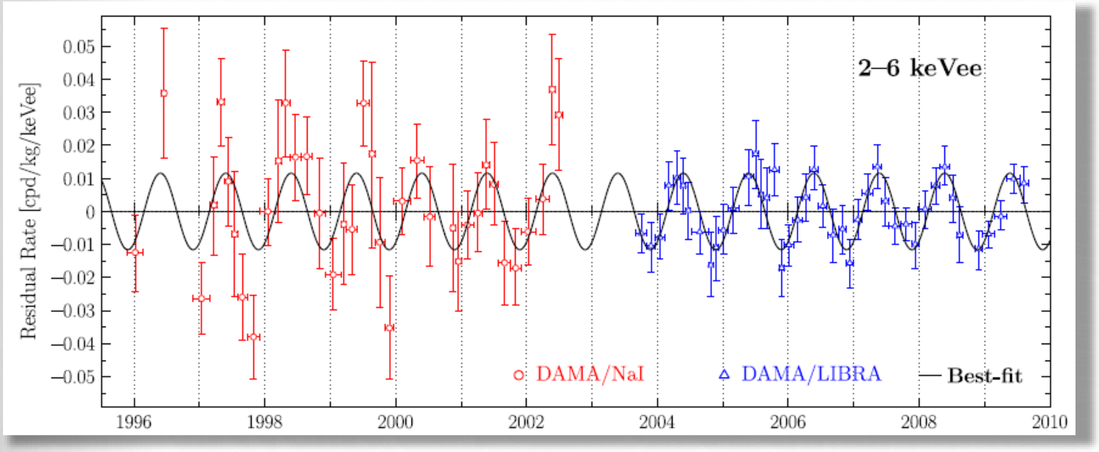
For a given detector, the expected number of recoil events with recoil energy in the range (E1, E2) is the sum over the nuclear species in the detector given by

$$N_{E_1-E_2} = \sum_i \int_{E_2}^{E_1} \frac{dR_i}{dE} \mathcal{E}_i(E) dE$$

$$\mathcal{E}_i = \mathcal{M}_i T_i \epsilon_i(E)$$

Current picture of WIMP detection

DAMA/LIBRA [1] the scientific case
Annual modulation: Signal or Background?
Origin is unclear!



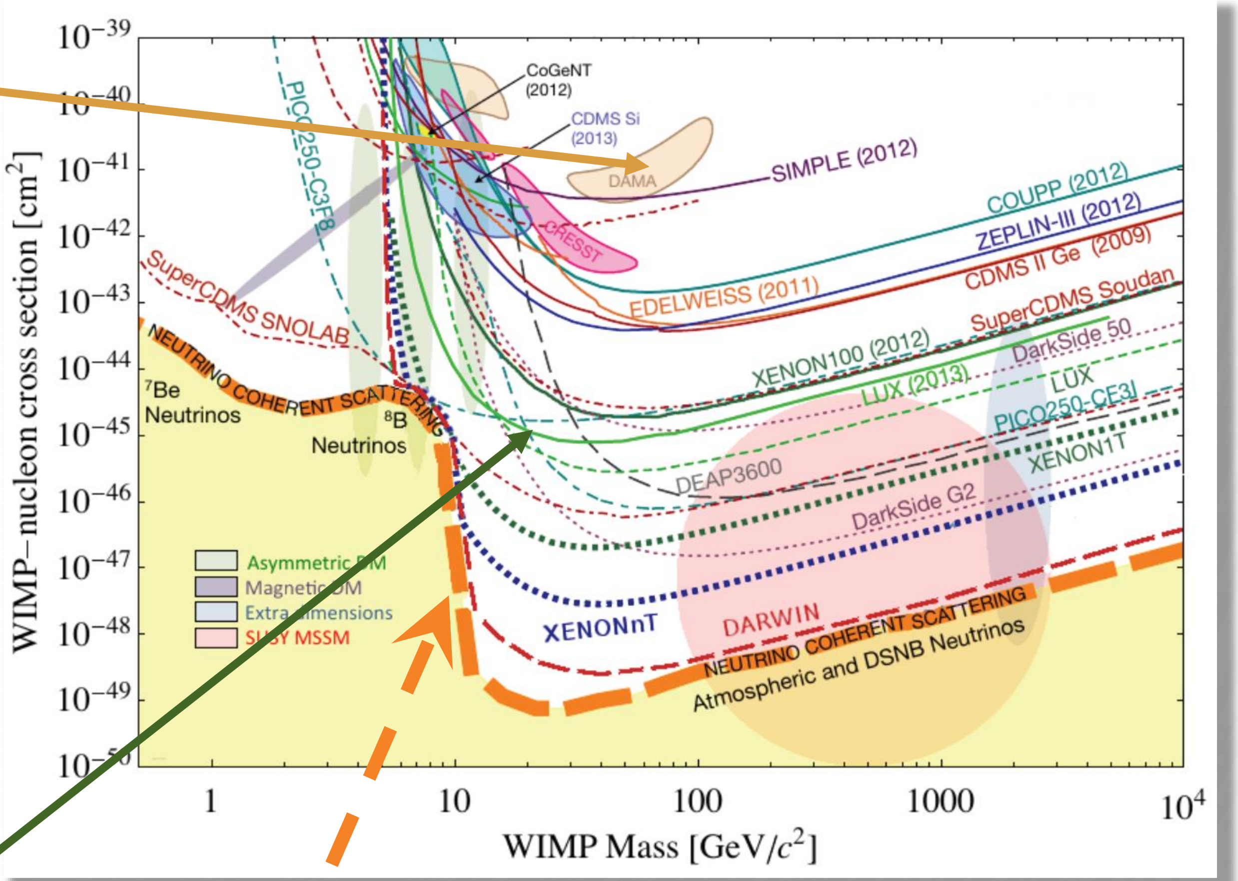
[1] R. Bernabei et al. The European Physical Journal C. 1434-6044, 2008

Abstract: "[…] The presence of Dark Matter particles in the galactic halo is supported at 8.2 σ C.L.."

Period: $T = 0.999 \pm 0.001$ yr,

Phase: $t_0 = 0.400 \pm 0.019$ (May 26±7 days)

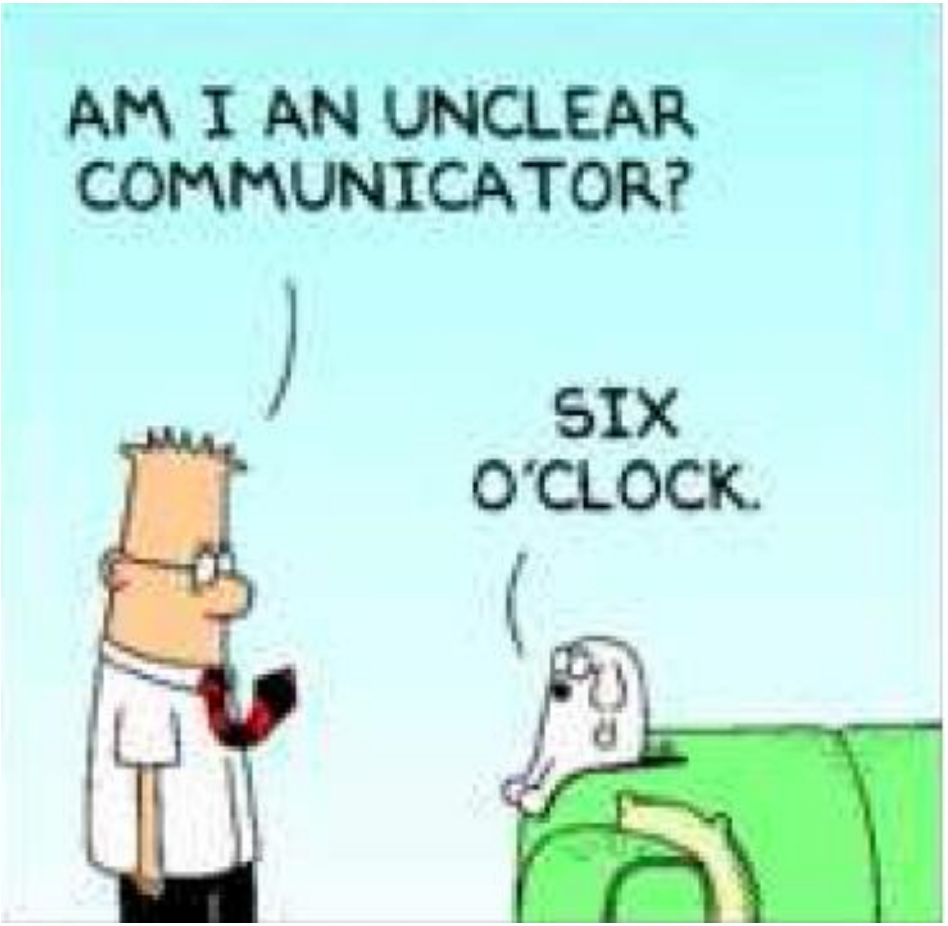
LUX (2013) : $\sigma_{W-n} \lesssim 10^{-45}\ cm^2$



HOWEVER
All the experiments must face the neutrino floor

Thanks for your attention

BACKUP



WIMP-nucleus differential cross section

- The WIMP-nucleus cross section encodes the particle physics inputs including the WIMP interaction properties

- It depends fundamentally on the **WIMP-quark** interaction strenght, which is calculated from the microscopic description of the model, in terms of an effective Lagragian describing the interaction of the WIMP candidate with quarks and gluons

- In a next step, the **WIMP-nucleon** cross section, using hadronic matrix elements that describe the nucleon contents in the quarks, is calculated

- In a third step, usign **nuclear wave functions**, the spin and the nuclear components of nucleons ore added to obtain the matrix element of **WIMP-nucleus** cross section as a function of momentum transfer. This step introduces a form factor supression which reduces the cross section for heavy WIMPs and/or nuclei (analogous to low-energy electromagnetic scattering of electrons from nuclei)

- **More recently: efforts to treat WIMP interactions more generally, using the tools of EFT**

- One writes down all WIMP-nucleon operators consistent with general symmetry arguments

- Then the interactions are imbedded in the nucleus -> nuclear operators -> response function that describe the WIMP-nucleus elastic scattering

Goal of NR-EFT approach

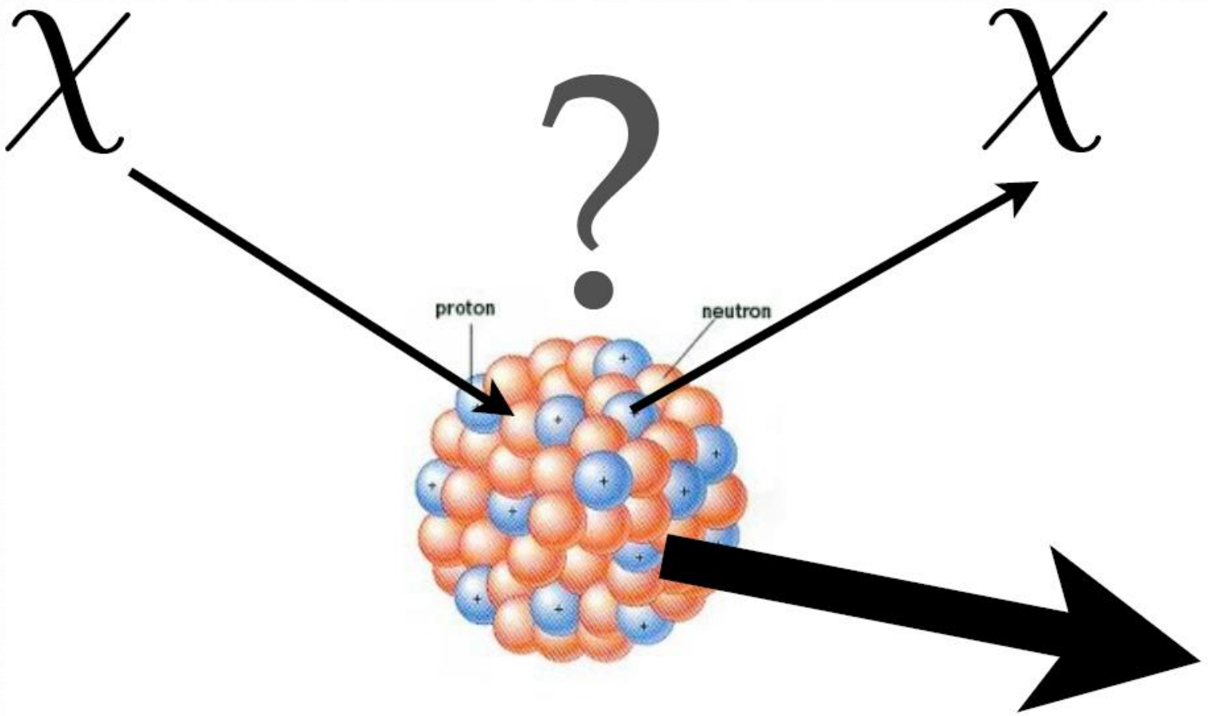
- Ignore UV model prejudice

- Parameterize theory in terms of IR quantities, with direct connection to experimental observables

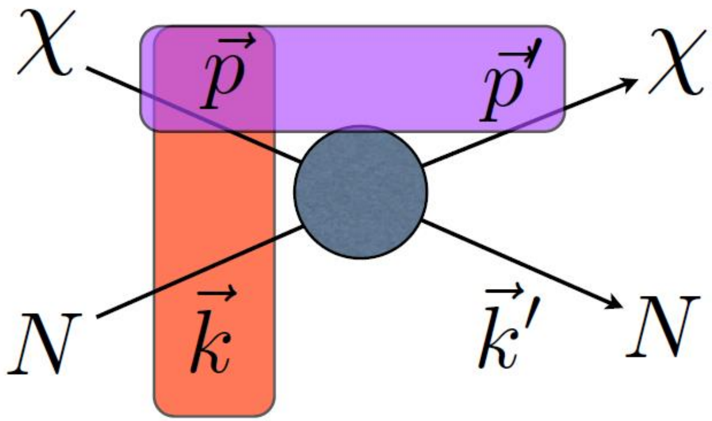
- Constrain these low-energy parameters directly

1) What are all possible WIMP-nucleon interactions?

2) What are all the ways different elements can respond?



Basics of the WIMP-nucleon Effective Theory



The kinetic action is just the usual, non-relativistic form

$$\mathcal{L}_{\text{kin}} = 2m_\phi \phi^+(y)\left(i\frac{\partial}{\partial t} - \frac{\vec{\nabla}^2}{2m_\phi}\right)\phi^-(y)$$

By momentum-conservation, the momentum transfer q is both $q = p' - p = k - k'$

There are several important symmetries that restrict the possible form of interactions.

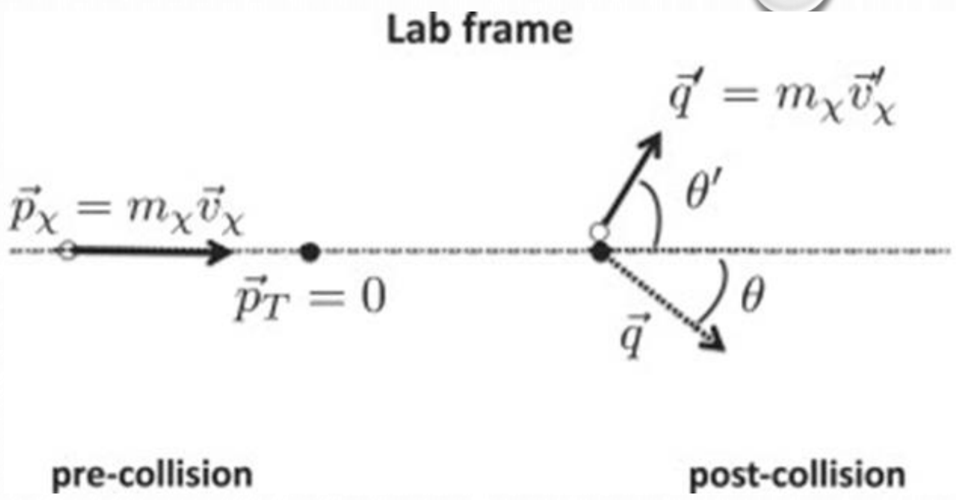
-**Galilean invariance**: it is just a constant shift in all velocities.
Between p, k and q, there are only two independent momenta that can arise in any interaction.

-momentum transfer $\vec{q}$ is Galilean invariant $\vec{q} = \vec{p} - \vec{p}'$

-the relative incoming velocity $\vec{v} = \vec{v}_{\chi,\text{in}} - \vec{v}_{N,\text{in}}$, which is just the velocity of the incoming dark matter particle in the nucleon rest frame.

Basics of the WIMP-nucleon Effective Theory

The final kinematic constraint is **energy conservation.**



This is easiest to impose by passing to the center-of-mass system, where the total kinetic energy is

$$E = \frac{1}{2}\mu_N v_{rel}^2$$

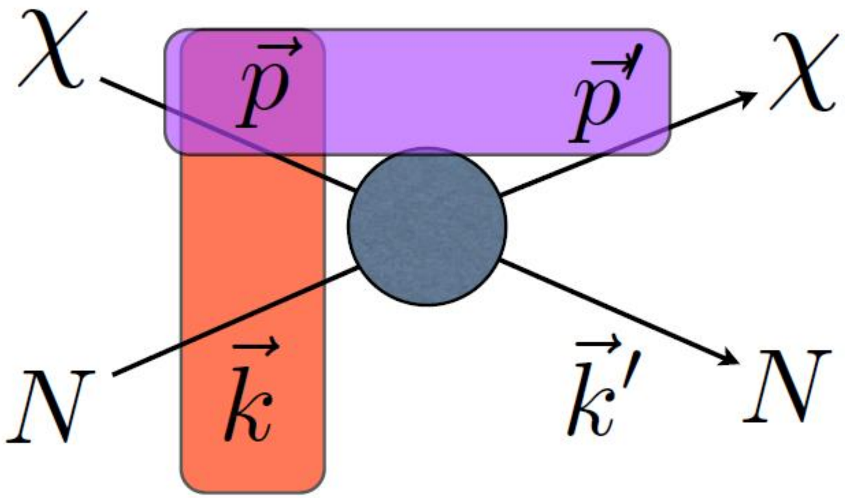
$\mu_N = \frac{m_N m_\chi}{m_N + m_\chi}$ is the dark-matter-nucleon reduced mass and $v_{rel} = v_\chi - v_N$

Energy conservation

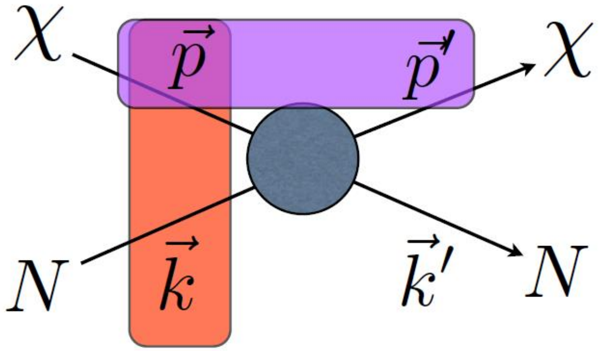
$$E_{in} = \frac{1}{2}\mu_N v^2 \, ; \quad E_{out} = \frac{1}{2}\mu_N \left(\vec{v} + \frac{\vec{q}}{\mu_N}\right)^2$$

E_{in}=E_{out} ⇨

$$\vec{v} \cdot \vec{q} = -\frac{q^2}{2\mu_N}$$



Basics of the WIMP-nucleon Effective Theory



The next major constraint is **Hermiticity** of the interaction.

This is essentially equivalent to crossing symmetry, because Hermitian conjugation exchanges incoming for outgoing particles i.e. $(\phi^-)^\dagger = \phi^\dagger$

Consequently, the momentum transfer $\vec{q}$ is effectively anti-Hermitian, and it will be more convenient to work with the Hermitian operator $i\vec{q}$. Under exchange of incoming and outgoing particles, $\vec{v}$ does not have defined parity

$$\vec{v} \xrightarrow{\dagger} \vec{v}_{\chi,out} - \vec{v}_{N,out} = \vec{v} + \frac{\vec{q}}{\mu_N}$$

However, we can easily construct a similar quantity that is Hermitian

$$\vec{v}^\perp \equiv \vec{v} + \frac{\vec{q}}{2\mu_N}$$

The reason for this notation is that, by the energy-conservation condition $\vec{v}^\perp \cdot \vec{q} = 0$

Basics of the WIMP-nucleon Effective Theory

Finally, we must include the particle **spins**. In the non-relativistic limit, we can write down the dark matter and nuclear spins $\vec{S}_\chi$ and $\vec{S}_N$ as operators directly.

If dark matter is a spin-1/2 particle, then these spins operators are simply $\frac{1}{2}\vec{\sigma}$ where σ^i are Pauli sigma matrices, acting on the χ and N spinors; for vector dark matter, they are spin-1 representations of the angular momentum generators J^i acting on the χ vector; and for scalars, they simply do not appear.

These are invariant under Hermitian conjugation.

So we have for our complete set of **Galilean, Hermitian invariants** the following:

$$i\vec{q}, \quad \vec{v}^\perp, \quad \vec{S}_\chi, \quad \vec{S}_N$$

Basics of the WIMP-nucleon Effective Theory

In addition to the above symmetries, there are strong constraints on **violations of CP symmetry**. Since ultimately our non-relativistic theory must be embedded in a Lorentz invariant quantum field theory, this is equivalent to **T symmetry**.

Spins behave like angular momentum, and thus change sign under **T**.

Velocities all change direction under **T**, so $\vec{v}^{\perp}$ and $\vec{q}$ change sign as well.

Finally, although we will not impose P as a symmetry, it will be helpful to classify all operators according to whether they are even or odd under P.

In this case, spins do not change sign, whereas $\vec{v}^{\perp}$ and $\vec{q}$ do.

	$\dagger$	T	P
$\vec{S}$	$+1$	-1	$+1$
$i\vec{q}$	$+1$	$+1$	-1
$\vec{v}^{\perp}$	$+1$	-1	-1

Basics of the WIMP-nucleon Effective Theory

Since we are interested in *elastic scattering direct detection*, all effective operators will be **four-field operators**, of the form

$$\mathcal{L}_{int} = \chi^+ \mathcal{O}_\chi \chi^- N^+ \mathcal{O}_N N^+ \equiv \mathcal{O} \chi^+ \chi^- N^+ N^-$$

Furthermore, the momentum-transfer-squared q^2 is a completely invariant scalar quantity that depends only on dark matter kinematic quantities, and thus if $\mathcal{O}$ is an operator allowed by all symmetries of the theory, then $q^{2n}\mathcal{O}$ is as well.

It is therefore natural to classify all such operators as a single one with a q^2-dependent coeffiecient, or

DM-form factor

$$c_0 \mathcal{O} + c_2 q^2 \mathcal{O} + c_4 q^4 \mathcal{O} + \ldots \equiv F_{\mathcal{O}}\left(\frac{q^2}{\Lambda^2}\right) \mathcal{O}$$

Basics of the WIMP-nucleon Effective Theory

We are now ready to present the possible non-relativistic interactions. The general Lagrangian is

$$\mathcal{L}_{\rm int} = \sum_{N=n,p} \sum_i c_i^{(N)} \mathcal{O}_i \chi^+ \chi^- N^+ N^-$$

With these, the above operators provide the most general effective theory at the dark matter nucleon level that can arise from exchange of a spin-0 or spin-1.

T-even operators

1. P-even, S_χ-independent

$$\mathcal{O}_1 = \mathbf{1}, \quad \mathcal{O}_2 = (v^\perp)^2, \quad \mathcal{O}_3 = i\vec{S}_N \cdot (\vec{q} \times \vec{v}^\perp)$$

2. P-even, S_χ-dependent

$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N, \quad \mathcal{O}_5 = i\vec{S}_\chi \cdot (\vec{q} \times \vec{v}^\perp), \quad \mathcal{O}_6 = (\vec{S}_\chi \cdot \vec{q})(\vec{S}_N \cdot \vec{q}),$$

3. P-odd, S_χ-independent

$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp,$$

4. P-odd, S_χ-dependent

$$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp, \quad \mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \vec{q})$$

T-violating operators

5. P-odd, S_χ-independent

$$\mathcal{O}_{10} = i\vec{S}_N \cdot \vec{q},$$

6. P-odd, S_χ-dependent

$$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \vec{q}.$$

In addition, there are four operators that are products of the ones above

$\mathcal{O}_{10}\mathcal{O}_5$, $\mathcal{O}_{10}\mathcal{O}_8$, $\mathcal{O}_{11}\mathcal{O}_3$ and $\mathcal{O}_{11}\mathcal{O}_7$

Velocity operators

Finally, **we need to evaluate matrix elements of the nucleon-level operators from the effective theory inside of a target nucleus.**
An atomic nucleus is a many-body bound state of nucleons.

Separate out $\vec{v}^\perp$ into a term $\vec{v}_T^\perp$ that acts on the coherent center-of-mass velocity of the atomic nucleus as a whole, and a term $\vec{v}_N^\perp$ that acts only on the relative distances of the nucleons within the nucleus. We can write

$$\vec{v}^\perp = \frac{1}{2}\left(\vec{v}_{\chi,\mathrm{in}} + \vec{v}_{\chi,\mathrm{out}} - \vec{v}_{N,\mathrm{in}} - \vec{v}_{N,\mathrm{out}}\right) = \vec{v}_T^\perp + \vec{v}_N^\perp$$

where

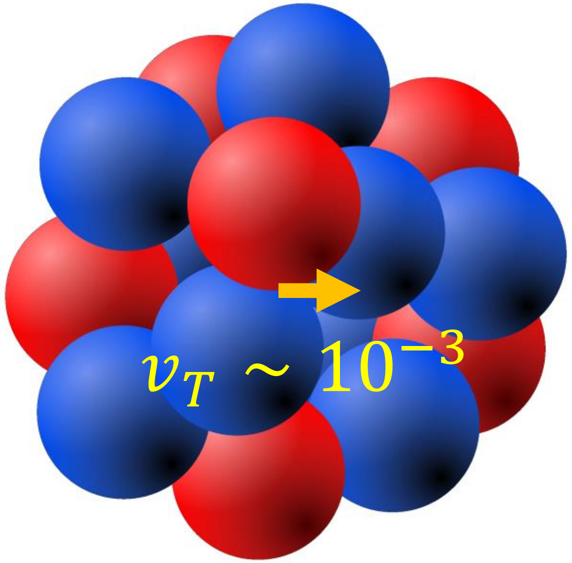
$$\vec{v}_T^\perp = \frac{1}{2}(\vec{v}_{\chi,\mathrm{in}} + \vec{v}_{\chi,\mathrm{out}} - \vec{v}_{T,\mathrm{in}} - \vec{v}_{T,\mathrm{out}}) = \vec{v}_T + \frac{\vec{q}}{2\mu_T}$$

acts only on the center of mass motion of the nucleus (here, $\vec{v}_T = \vec{v}_{\chi,\mathrm{in}} - \vec{v}_{T,\mathrm{in}}$ is the incoming dark matter velocity in the lab frame). Also, $\vec{v}_N^\perp$ is just

$$\vec{v}_N^\perp = -\frac{1}{2}(\vec{v}_{N,\mathrm{in}} + \vec{v}_{N,\mathrm{out}})$$

Velocity operators

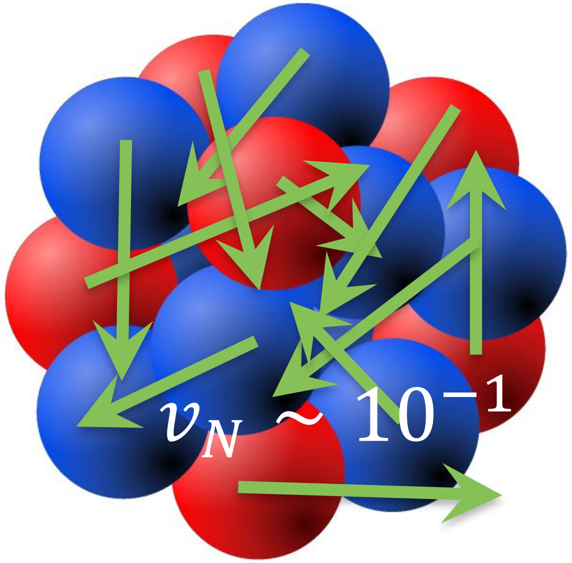
$\vec{v}_T^{\perp}$ acts on the coherent center-of-mass velocity of the atomic nucleus as a whole.



$\vec{v}_T^{\perp}$ is determined by the kinematics of the DM-nucleus scattering process -> not require any knowledge of the internal structure of the nucleus. Its magnitude is given by

$$m_T v_T \sim q$$

$\vec{v}_N^{\perp}$ that acts only on the relative distances of the nucleons within the nucleus.



This will lead to a **relative kinematic enhancement** of $\frac{m_T}{m_N} = A$ for $\vec{v}_N^{\perp}$ compared to $\vec{v}_T^{\perp}$.

$\vec{v}_N^{\perp}$ depends on the internal distribution of nucleons in the nucleons and thus is determined by

$$m_N \vec{v}_N^{\perp} \sim q$$

Form factors

- At low momentum-transfer, the internal structure of atomic nuclei can be summarized in just a finite number of macroscopic quantities.

- In the case of the standard **spin-independent** interaction $\mathcal{O}_1$ or **spin-dependent interaction** $\mathcal{O}_4$, these are the atomic number **A** and charge **Z** or nucleon spin expectation values $\langle S_n \rangle$, $\langle S_p \rangle$, respectively.

- However, there are many more possible macroscopic quantities that appear associated with our different type of interactions than just these usual ones.

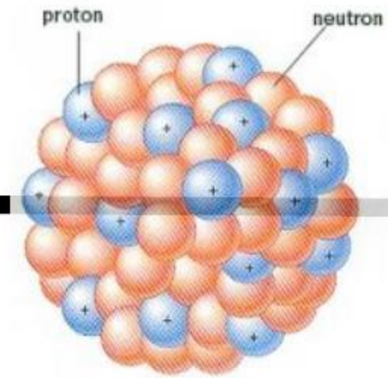
- Furthermore, at finite momentum-transfer, there are multiple possible **form factors** associated with the **nuclear responses** that are required for calculating event rates.

In order to obtain these nuclear responses, one needs detailed input from nuclear physics on the **wave functions** of nucleons inside the nucleus.

Nuclear responses

This is a concrete problem for nuclear physics - what are the form factors for all interactions?

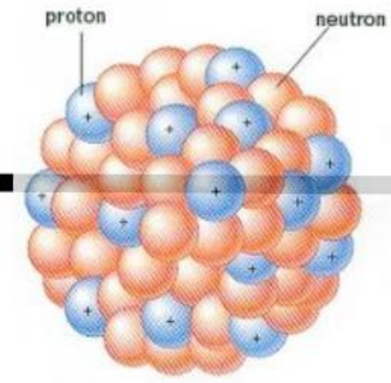
$\vec{S}_\chi \cdot \vec{S}_N$ →



Nuclear form factor

$F_N(q^2)$

$\vec{S}_\chi \cdot \vec{v}^\perp$ →



?

Additional form factors

- Input internal structure of the nucleus to calculate cross-sections for all operators in the effective theory

Additional form factors

All possible cross-sections can be worked out in terms of a few response function

$$M \sim 1$$

$$\Sigma \sim \langle \vec{S} \rangle \rightarrow \Sigma' \sim \langle \vec{S} \rangle|_{\mathrm{trans.}}, \quad \Sigma'' \sim \langle \vec{S} \rangle|_{\mathrm{long.}}$$

$$\Phi \sim \langle \vec{S} \cdot \vec{L} \rangle \rightarrow \tilde{\Phi}' \sim \langle \vec{S} \cdot \vec{L} \rangle, \quad \Phi'' \sim \vec{q}\langle \vec{S} \cdot \vec{L} \rangle - \langle \vec{L}(\vec{S} \cdot \vec{q}) \rangle$$

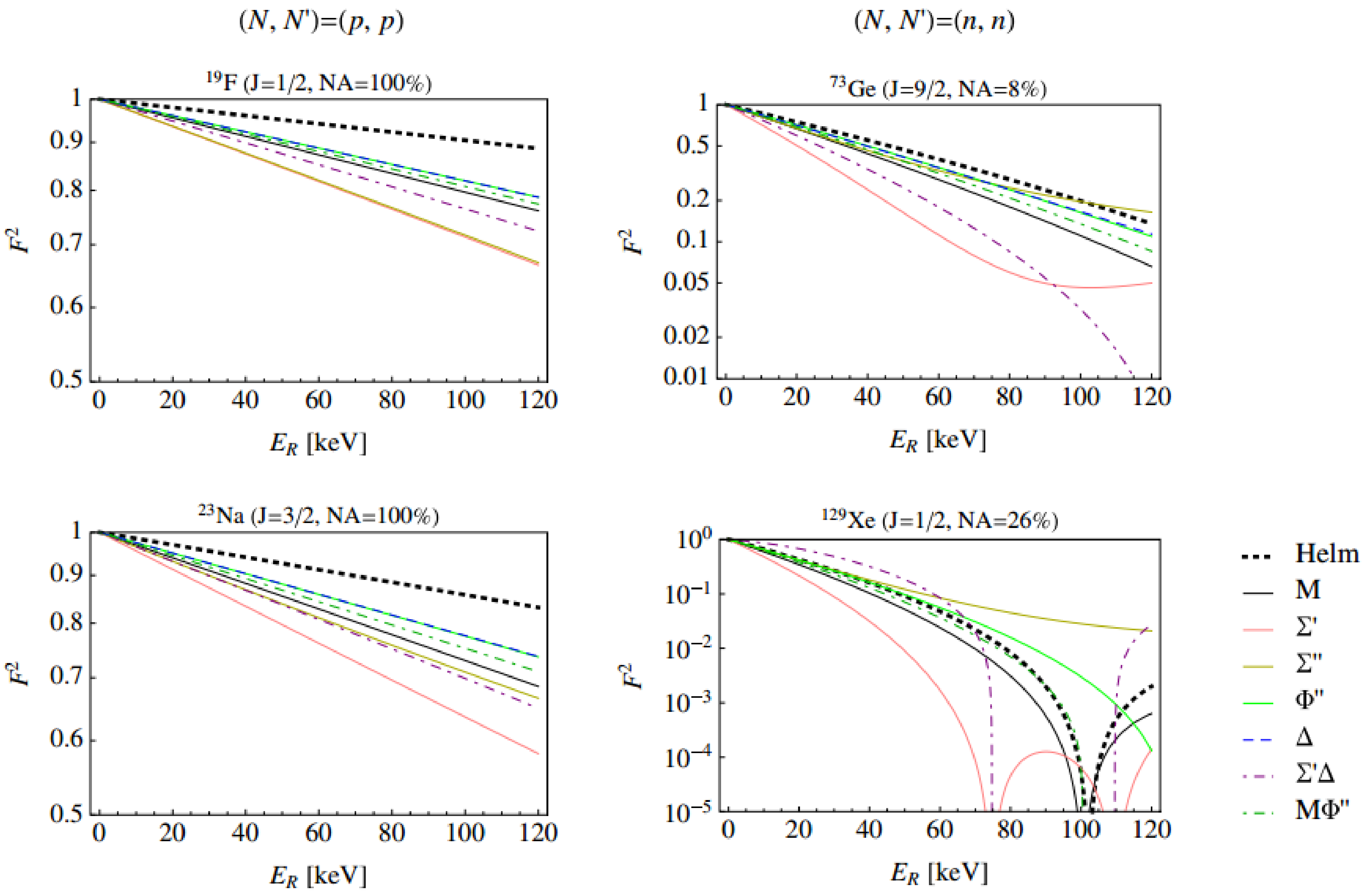
$$\tilde{\Delta} \sim \langle \vec{L} \rangle_{\mathrm{trans.}}$$

Much of this variation can be captured by two "generalized SI" and two "generalized SD" interactions

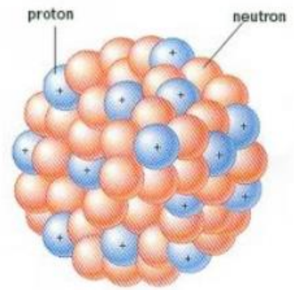
$$\left.\begin{array}{l} M \sim 1 \\ \Phi \sim \langle \vec{S} \cdot \vec{L} \rangle \end{array}\right\} \text{generalized SI}$$

$$\left.\begin{array}{l} \Sigma' \sim \langle \vec{S} \rangle|_{\mathrm{trans.}} \\ \tilde{\Delta} \sim \langle \vec{L} \rangle_{\mathrm{trans.}} \end{array}\right\} \text{generalized SD}$$

EFT Nuclear responses comparison



The EFT rate

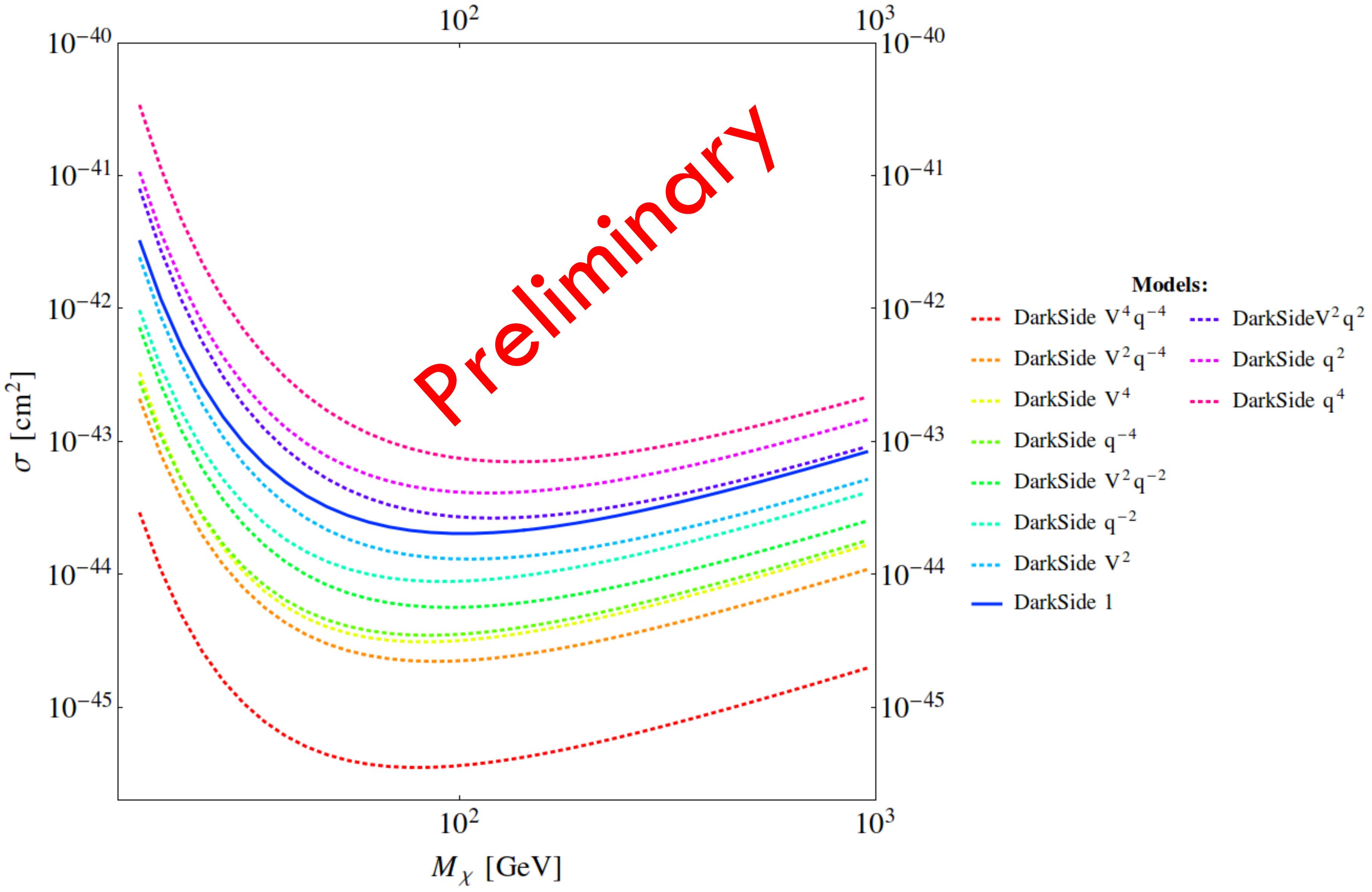


$$\frac{dR_D}{dE_R} = N_T \frac{\rho_\chi m_T}{32\pi m_\chi^3 m_N^2} \times \left\langle \frac{1}{v} \sum_{ij} \sum_{N,N'=p,n} c_i^{(N)} c_j^{(N')} F_{ij}^{(N,N')}(v^2, q^2) \right\rangle$$

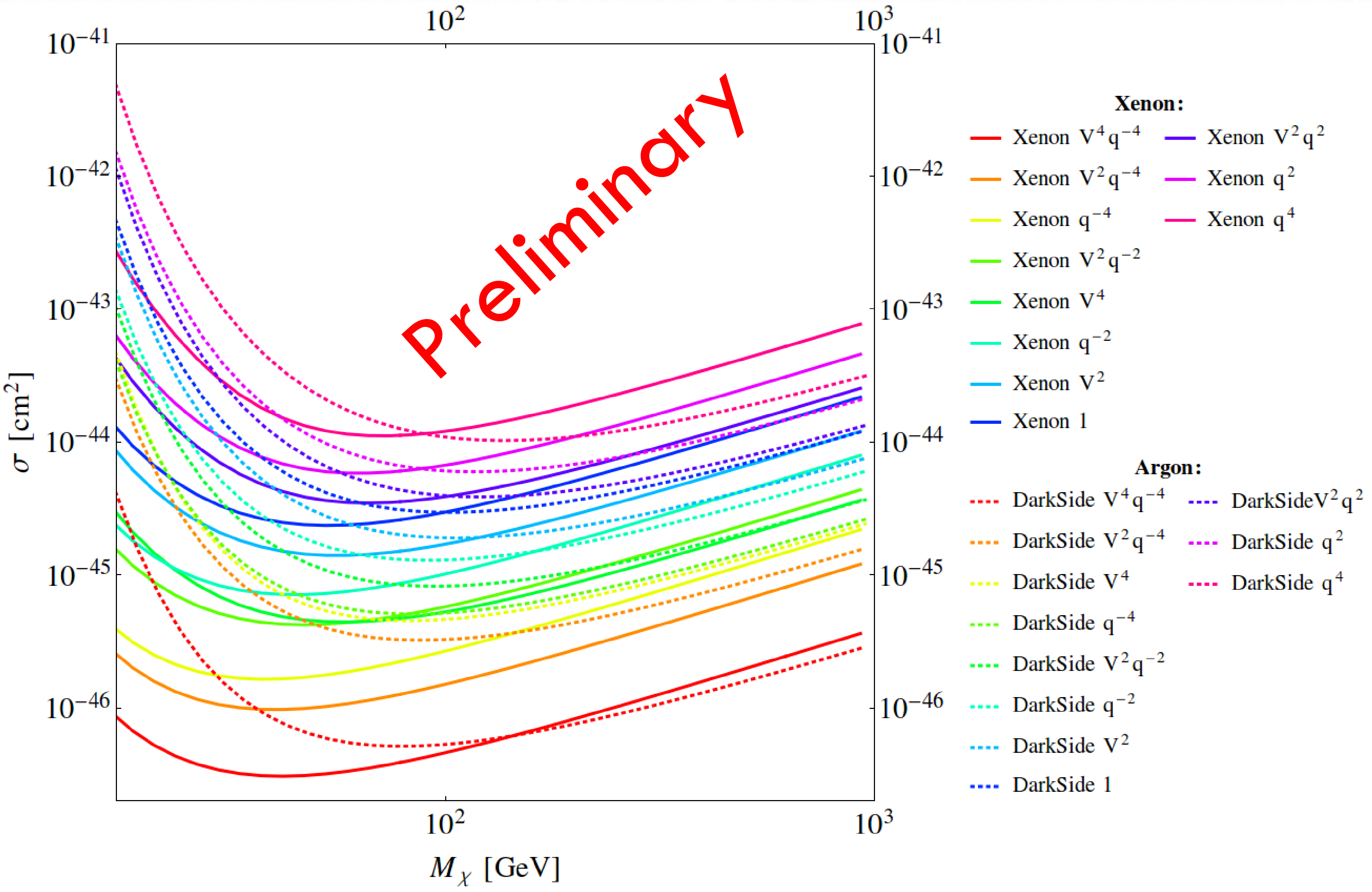
Non relativistic reduction

j	$\mathcal{L}^j_{\rm int}$	Nonrelativistic reduction	$\sum_i c_i \mathcal{O}_i$	P/T
1	$\bar\chi\chi\bar N N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
2	$i\bar\chi\chi\bar N\gamma^5 N$	$i\frac{\vec q}{m_N}\cdot\vec S_N$	$\mathcal{O}_{10}$	O/O
3	$i\bar\chi\gamma^5\chi\bar N N$	$-i\frac{\vec q}{m_\chi}\cdot\vec S_\chi$	$-\frac{m_N}{m_\chi}\mathcal{O}_{11}$	O/O
4	$\bar\chi\gamma^5\chi\bar N\gamma^5 N$	$-\frac{\vec q}{m_\chi}\cdot\vec S_\chi\frac{\vec q}{m_N}\cdot\vec S_N$	$-\frac{m_N}{m_\chi}\mathcal{O}_6$	E/E
5	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
6	$\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}N$	$\frac{\vec q^{\,2}}{2m_N m_{\rm M}}1_\chi 1_N + 2\big(\frac{\vec q}{m_\chi}\times\vec S_\chi + i\vec v^\perp\big)\cdot\big(\frac{\vec q}{m_{\rm M}}\times\vec S_N\big)$	$\frac{\vec q^{\,2}}{2m_N m_{\rm M}}\mathcal{O}_1 - 2\frac{m_N}{m_{\rm M}}\mathcal{O}_3 + 2\frac{m_N^2}{m_{\rm M}m_\chi}\left(\frac{q^2}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6\right)$	E/E
7	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu\gamma^5 N$	$-2\vec S_N\cdot\vec v^\perp + \frac{2}{m_\chi}i\vec S_\chi\cdot(\vec S_N\times\vec q)$	$-2\mathcal{O}_7 + 2\frac{m_N}{m_\chi}\mathcal{O}_9$	O/E
8	$i\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$2i\frac{\vec q}{m_{\rm M}}\cdot\vec S_N$	$2\frac{m_N}{m_{\rm M}}\mathcal{O}_{10}$	O/O
9	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\chi\bar N\gamma_\mu N$	$-\frac{\vec q^{\,2}}{2m_\chi m_{\rm M}}1_\chi 1_N - 2\big(\frac{\vec q}{m_N}\times\vec S_N + i\vec v^\perp\big)\cdot\big(\frac{\vec q}{m_{\rm M}}\times\vec S_\chi\big)$	$-\frac{\vec q^{\,2}}{2m_\chi m_{\rm M}}\mathcal{O}_1 + \frac{2m_N}{m_{\rm M}}\mathcal{O}_5 - 2\frac{m_N}{m_{\rm M}}\left(\frac{\vec q^{\,2}}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6\right)$	E/E
10	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}N$	$4\big(\frac{\vec q}{m_{\rm M}}\times\vec S_\chi\big)\cdot\big(\frac{\vec q}{m_{\rm M}}\times\vec S_N\big)$	$4\big(\frac{\vec q^{\,2}}{m_{\rm M}^2}\mathcal{O}_4 - \frac{m_N^2}{m_{\rm M}^2}\mathcal{O}_6\big)$	E/E
11	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\chi\bar N\gamma^\mu\gamma^5 N$	$4i\big(\frac{\vec q}{m_{\rm M}}\times\vec S_\chi\big)\cdot\vec S_N$	$4\frac{m_N}{m_{\rm M}}\mathcal{O}_9$	O/E
12	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$-\big[i\frac{\vec q^{\,2}}{m_\chi m_{\rm M}} - 4\vec v^\perp\cdot\big(\frac{\vec q}{m_{\rm M}}\times\vec S_\chi\big)\big]\frac{\vec q}{m_{\rm M}}\cdot\vec S_N$	$-\frac{m_N}{m_\chi}\frac{\vec q^{\,2}}{m_{\rm M}^2}\mathcal{O}_{10} - 4\frac{\vec q^{\,2}}{m_{\rm M}^2}\mathcal{O}_{12} - 4\frac{m_N^2}{m_{\rm M}^2}\mathcal{O}_{15}$	O/O
13	$\bar\chi\gamma^\mu\gamma^5\chi\bar N\gamma_\mu N$	$2\vec v^\perp\cdot\vec S_\chi + 2i\vec S_\chi\cdot\big(\vec S_N\times\frac{\vec q}{m_N}\big)$	$2\mathcal{O}_8 + 2\mathcal{O}_9$	O/E
14	$\bar\chi\gamma^\mu\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}N$	$4i\vec S_\chi\cdot\big(\frac{\vec q}{m_{\rm M}}\times\vec S_N\big)$	$-4\frac{m_N}{m_{\rm M}}\mathcal{O}_9$	O/E
15	$\bar\chi\gamma^\mu\gamma^5\chi\bar N\gamma^\mu\gamma^5 N$	$-4\vec S_\chi\cdot\vec S_N$	$-4\mathcal{O}_4$	E/E
16	$i\bar\chi\gamma^\mu\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}\gamma^5 N$	$4i\vec v^\perp\cdot\vec S_\chi\frac{\vec q}{m_{\rm M}}\cdot\vec S_N$	$4\frac{m_N}{m_{\rm M}}\mathcal{O}_{13}$	E/O
17	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\gamma^5\chi\bar N\gamma_\mu N$	$2i\frac{\vec q}{m_{\rm M}}\cdot\vec S_\chi$	$2\frac{m_N}{m_{\rm M}}\mathcal{O}_{11}$	O/O
18	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}N$	$\frac{\vec q}{m_{\rm M}}\cdot\vec S_\chi\big[i\frac{\vec q^{\,2}}{m_N m_{\rm M}} - 4\vec v^\perp\cdot\big(\frac{\vec q}{m_{\rm M}}\times\vec S_N\big)\big]$	$\frac{\vec q^{\,2}}{m_{\rm M}^2}\mathcal{O}_{11} + 4\frac{m_N^2}{m_{\rm M}^2}\mathcal{O}_{15}$	O/O
19	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\gamma^5\chi\bar N\gamma_\mu\gamma^5 N$	$-4i\frac{\vec q}{m_{\rm M}}\cdot\vec S_\chi\vec v_\perp\cdot\vec S_N$	$-4\frac{m_N}{m_{\rm M}}\mathcal{O}_{14}$	E/O
20	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}\gamma^5 N$	$4\frac{\vec q}{m_{\rm M}}\cdot\vec S_\chi\frac{\vec q}{m_{\rm M}}\cdot\vec S_N$	$4\frac{m_N^2}{m_{\rm M}^2}\mathcal{O}_6$	E/E

DarkSide preliminary result



Argon Vs Xenon



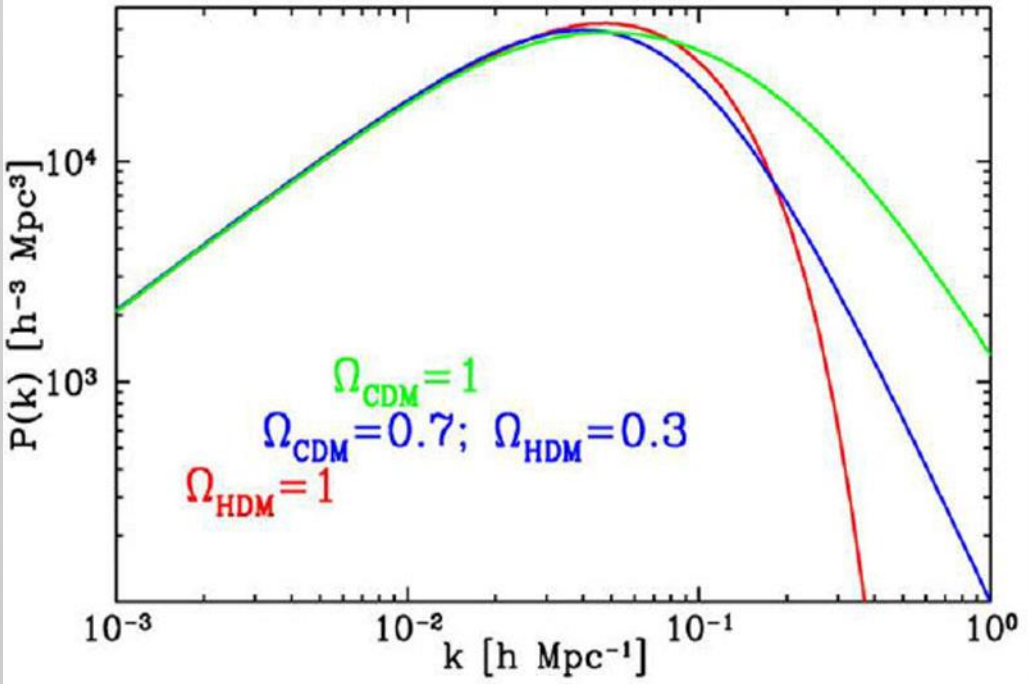
Non relativistic reduction

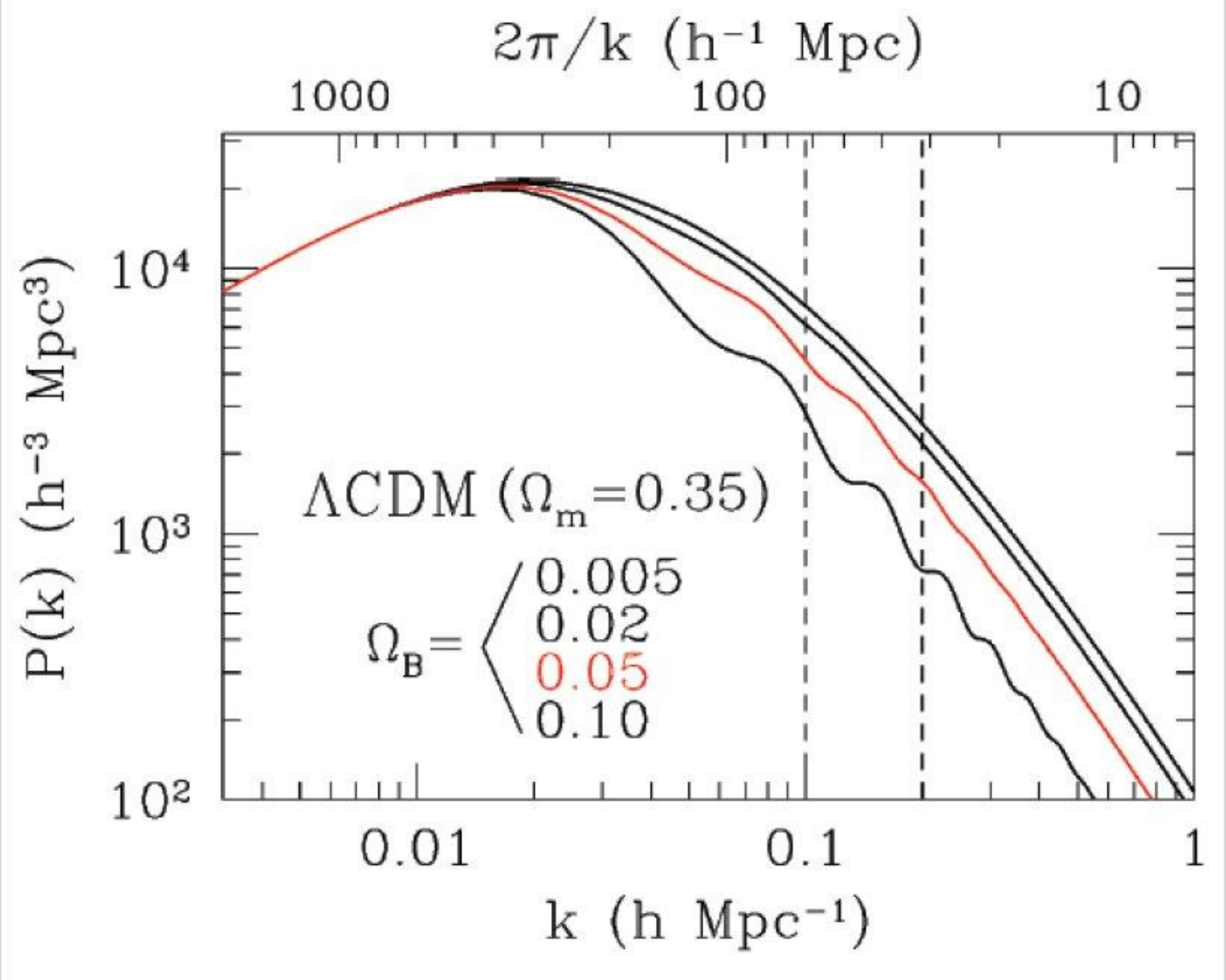
j	$\mathcal{L}^j_{\rm int}$	Nonrelativistic reduction	$\sum_i c_i \mathcal{O}_i$	P/T
1	$\bar{\chi}\chi\bar{N}N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
2	$i\bar{\chi}\chi\bar{N}\gamma^5 N$	$i\frac{\vec{q}}{m_N}\cdot\vec{S}_N$	$\mathcal{O}_{10}$	O/O
3	$i\bar{\chi}\gamma^5\chi\bar{N}N$	$-i\frac{\vec{q}}{m_\chi}\cdot\vec{S}_\chi$	$-\frac{m_N}{m_\chi}\mathcal{O}_{11}$	O/O
4	$\bar{\chi}\gamma^5\chi\bar{N}\gamma^5 N$	$-\frac{\vec{q}}{m_\chi}\cdot\vec{S}_\chi\frac{\vec{q}}{m_N}\cdot\vec{S}_N$	$-\frac{m_N}{m_\chi}\mathcal{O}_6$	E/E
5	$\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
6	$\bar{\chi}\gamma^\mu\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}N$	$\frac{\vec{q}^{\,2}}{2m_N m_{\rm M}}1_\chi 1_N + 2\left(\frac{\vec{q}}{m_\chi}\times\vec{S}_\chi + i\vec{v}^\perp\right)\cdot\left(\frac{\vec{q}}{m_{\rm M}}\times\vec{S}_N\right)$	$\frac{\vec{q}^{\,2}}{2m_N m_{\rm M}}\mathcal{O}_1 - 2\frac{m_N}{m_{\rm M}}\mathcal{O}_3 + 2\frac{m_N^2}{m_{\rm M}m_\chi}\left(\frac{q^2}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6\right)$	E/E
7	$\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu\gamma^5 N$	$-2\vec{S}_N\cdot\vec{v}^\perp + \frac{2}{m_\chi}i\vec{S}_\chi\cdot(\vec{S}_N\times\vec{q})$	$-2\mathcal{O}_7 + 2\frac{m_N}{m_\chi}\mathcal{O}_9$	O/E
8	$i\bar{\chi}\gamma^\mu\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$2i\frac{\vec{q}}{m_{\rm M}}\cdot\vec{S}_N$	$2\frac{m_N}{m_{\rm M}}\mathcal{O}_{10}$	O/O
9	$\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\chi\bar{N}\gamma_\mu N$	$-\frac{\vec{q}^{\,2}}{2m_\chi m_{\rm M}}1_\chi 1_N - 2\left(\frac{\vec{q}}{m_N}\times\vec{S}_N + i\vec{v}^\perp\right)\cdot\left(\frac{\vec{q}}{m_{\rm M}}\times\vec{S}_\chi\right)$	$-\frac{\vec{q}^{\,2}}{2m_\chi m_{\rm M}}\mathcal{O}_1 + \frac{2m_N}{m_{\rm M}}\mathcal{O}_5 - 2\frac{m_N}{m_{\rm M}}\left(\frac{\vec{q}^{\,2}}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6\right)$	E/E
10	$\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}N$	$4\left(\frac{\vec{q}}{m_{\rm M}}\times\vec{S}_\chi\right)\cdot\left(\frac{\vec{q}}{m_{\rm M}}\times\vec{S}_N\right)$	$4\left(\frac{\vec{q}^{\,2}}{m_{\rm M}^2}\mathcal{O}_4 - \frac{m_N^2}{m_{\rm M}^2}\mathcal{O}_6\right)$	E/E
11	$\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\chi\bar{N}\gamma^\mu\gamma^5 N$	$4i\left(\frac{\vec{q}}{m_{\rm M}}\times\vec{S}_\chi\right)\cdot\vec{S}_N$	$4\frac{m_N}{m_{\rm M}}\mathcal{O}_9$	O/E
12	$i\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$-\left[i\frac{\vec{q}^{\,2}}{m_\chi m_{\rm M}} - 4\vec{v}^\perp\cdot\left(\frac{\vec{q}}{m_{\rm M}}\times\vec{S}_\chi\right)\right]\frac{\vec{q}}{m_{\rm M}}\cdot\vec{S}_N$	$-\frac{m_N}{m_\chi}\frac{\vec{q}^{\,2}}{m_{\rm M}^2}\mathcal{O}_{10} - 4\frac{\vec{q}^{\,2}}{m_{\rm M}^2}\mathcal{O}_{12} - 4\frac{m_N^2}{m_{\rm M}^2}\mathcal{O}_{15}$	O/O
13	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma_\mu N$	$2\vec{v}^\perp\cdot\vec{S}_\chi + 2i\vec{S}_\chi\cdot\left(\vec{S}_N\times\frac{\vec{q}}{m_N}\right)$	$2\mathcal{O}_8 + 2\mathcal{O}_9$	O/E
14	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}N$	$4i\vec{S}_\chi\cdot\left(\frac{\vec{q}}{m_{\rm M}}\times\vec{S}_N\right)$	$-4\frac{m_N}{m_{\rm M}}\mathcal{O}_9$	O/E
15	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma^\mu\gamma^5 N$	$-4\vec{S}_\chi\cdot\vec{S}_N$	$-4\mathcal{O}_4$	E/E
16	$i\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}\gamma^5 N$	$4i\vec{v}^\perp\cdot\vec{S}_\chi\frac{\vec{q}}{m_{\rm M}}\cdot\vec{S}_N$	$4\frac{m_N}{m_{\rm M}}\mathcal{O}_{13}$	E/O
17	$i\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\gamma^5\chi\bar{N}\gamma_\mu N$	$2i\frac{\vec{q}}{m_{\rm M}}\cdot\vec{S}_\chi$	$2\frac{m_N}{m_{\rm M}}\mathcal{O}_{11}$	O/O
18	$i\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}N$	$\frac{\vec{q}}{m_{\rm M}}\cdot\vec{S}_\chi\left[i\frac{\vec{q}^{\,2}}{m_N m_{\rm M}} - 4\vec{v}^\perp\cdot\left(\frac{\vec{q}}{m_{\rm M}}\times\vec{S}_N\right)\right]$	$\frac{\vec{q}^{\,2}}{m_{\rm M}^2}\mathcal{O}_{11} + 4\frac{m_N^2}{m_{\rm M}^2}\mathcal{O}_{15}$	O/O
19	$i\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\gamma^5\chi\bar{N}\gamma_\mu\gamma^5 N$	$-4i\frac{\vec{q}}{m_{\rm M}}\cdot\vec{S}_\chi\vec{v}_\perp\cdot\vec{S}_N$	$-4\frac{m_N}{m_{\rm M}}\mathcal{O}_{14}$	E/O
20	$i\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_{\rm M}}\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_{\rm M}}\gamma^5 N$	$4\frac{\vec{q}}{m_{\rm M}}\cdot\vec{S}_\chi\frac{\vec{q}}{m_{\rm M}}\cdot\vec{S}_N$	$4\frac{m_N^2}{m_{\rm M}^2}\mathcal{O}_6$	E/E

Relativistic operators Vs NR operators and corresponding response functions

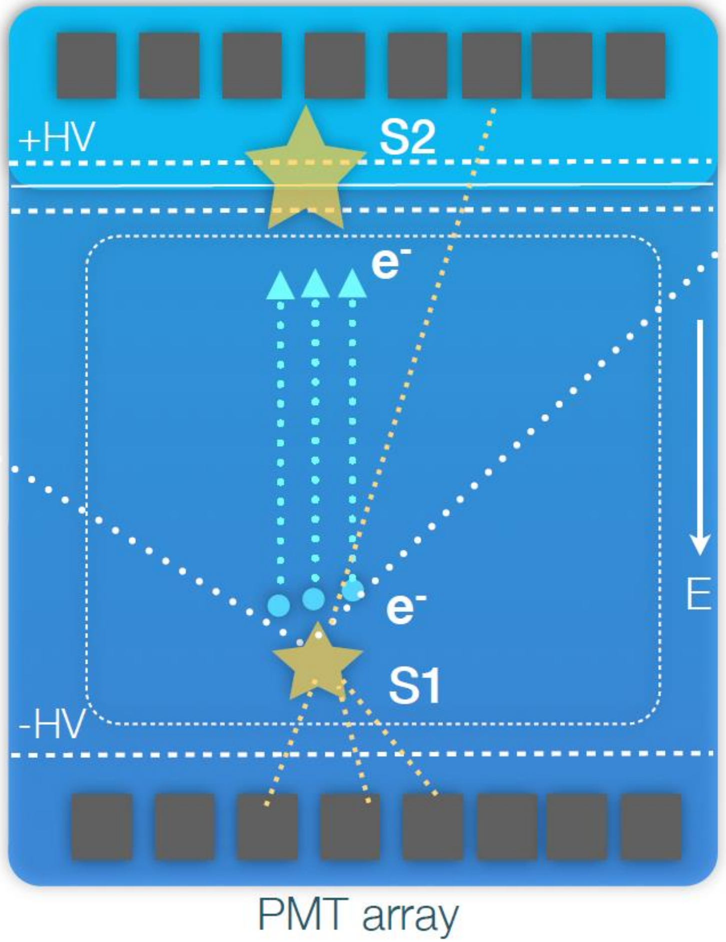
Model	Relativistic operators	Nonrelativistic operators	Response
Pseudo-mediated	$\mathcal{O}_2^{\rm rel} = i\bar{\chi}\chi\bar{N}\gamma^5 N$	$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}$	Σ''
	$\mathcal{O}_3^{\rm rel} = i\bar{\chi}\gamma^5\chi\bar{N}N$	$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}$	M
	$\mathcal{O}_4^{\rm rel} = \bar{\chi}\gamma^5\chi\bar{N}\gamma^5 N$	$\mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$	Σ''
Magnetic dipole	$\mathcal{O}_9^{\rm rel} = \bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\frac{K_\mu}{m_M}\bar{N}N$	$\mathcal{O}_1 = \mathbf{1}_\chi\mathbf{1}_N, \mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	M, Δ
	$\mathcal{O}_{10}^{\rm rel} = \bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar{N} i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N, \mathcal{O}_6$	Σ'', Σ'
Anapole	$\mathcal{O}_{13}^{\rm rel} = \bar{\chi}\gamma^\mu\gamma^5\chi\frac{K_\mu}{m_M}\bar{N}N$	$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp$	M, Δ
	$\mathcal{O}_{14}^{\rm rel} = \bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\frac{i\sigma_{\mu\nu}q^\nu}{m_M}N$	$\mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$	Σ'
Electric dipole	$\mathcal{O}_{17}^{\rm rel} = i\frac{P^\mu}{m_M}\bar{\chi}\gamma^\mu\gamma^5\chi\frac{K_\mu}{m_M}\bar{N}N$	$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}$	M
	$\mathcal{O}_{18}^{\rm rel} = i\frac{P^\mu}{m_M}\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\frac{i\sigma_{\mu\nu}q^\nu}{m_M}N$	$\mathcal{O}_{11}, \mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N})$	M, Φ'', Σ'
$\vec{L} \cdot \vec{S}$-generating	$\mathcal{O}_5^{\rm rel} = \frac{P^\mu}{m_M}\bar{\chi}\chi\frac{K_\mu}{m_M}\bar{N}N$	$\mathcal{O}_1$	M
	$\mathcal{O}_6^{\rm rel} = \frac{P^\mu}{m_M}\bar{\chi}\chi\bar{N}\frac{i\sigma_{\mu\nu}q^\nu}{m_M}N$	$\mathcal{O}_1, \mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	M, Φ'', Σ'
	and $\mathcal{O}_{10}^{\rm rel}$ (see above)		

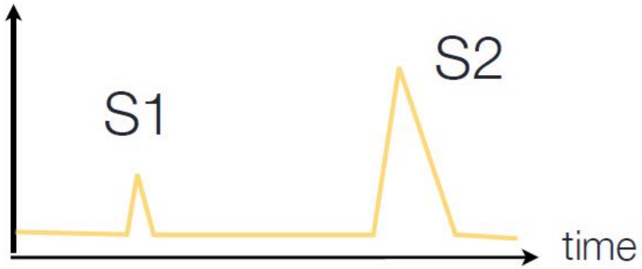
Matter Power Spectrum





Noble gases dark matter detectors





LXe: XENON100

LXe: LUX

LAr: DarkSide

XENON100 (LXe) and DarkSide (LAr) at LNGS

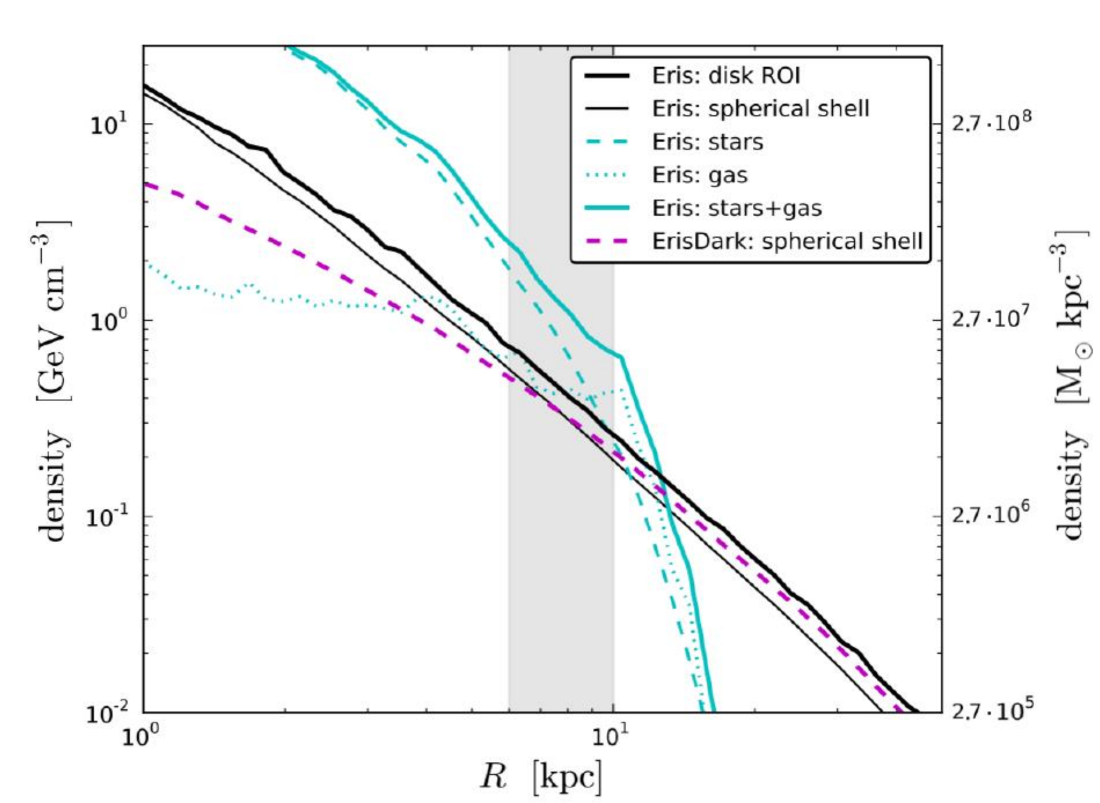
LUX (LXe) at SURF, PandaX (LXe) at CJPL

ArDM (LAr) at Canfranc

Target masses between ~ 50 kg - 1 ton

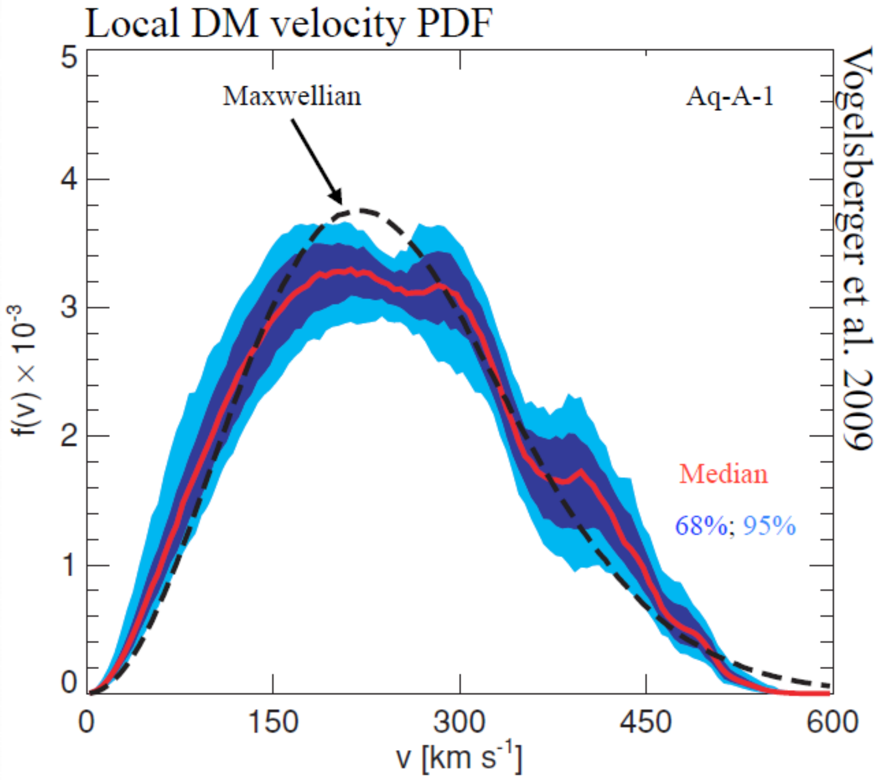
Astrophysics

Dark matter density profile in cosmological MW-like galaxy simulations with baryons (Eris) and DM only (ErisDark)



$$\rho(R_0) = 0.2 - 0.56\ \text{GeV}\ c^{-2} cm^{-3}$$
$$= 0.005 - 0.015\ M_\odot pc^{-3}$$

Velocity distribution of WIMPs in the galaxy



From cosmological simulations of (DM only) galaxy formation: departures from the simplest case of a Maxwell Boltzmann distribution.

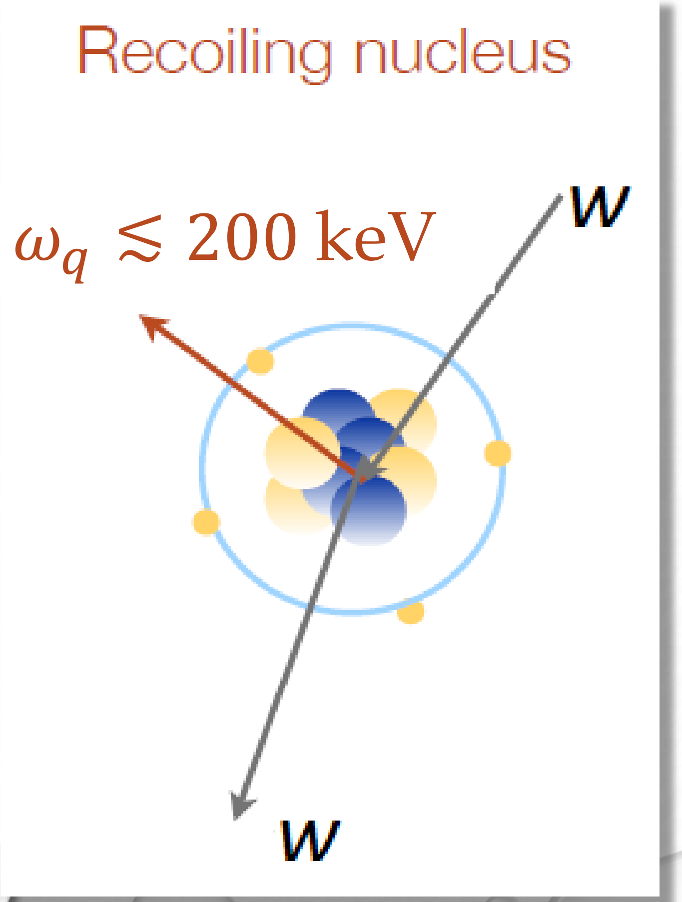
WIMP flux on Earth: ~$10^5\ cm^{-2}\ s^{-1}$ (M_W=100 GeV, for 0.3 GeV/cm^3)

Momentum scales VS energy scalse

So far, we have mainly discussed momentum scales. In addition, there is an energy scale associated with the scattering process, of size $\omega_q = q^2/2m_T \lesssim 200$ keV.

This is usually negligible, as the binding energy ω of nucleons is about 10 MeV per nucleon for most elements, and inelastic transitions are kinematically suppressed.

However, for nuclei with small splittings $\sim \omega_q$ between the ground state and an excited state, it could a affect direct detection rates.



Constructing the non relativistic operators

Because dark-matter–ordinary-matter interactions are more commonly described in relativistic notation, we begin by considering the nonrelativistic reduction of two familiar relativistic interactions. The **SI contact interaction** between a spin-1/2 WIMP and nucleon,

$$\mathcal{L}^{\mathrm{SI}}_{\mathrm{int}}(\vec{x}) = c_1 \bar{\Psi}_\chi(\vec{x})\Psi_\chi(\vec{x})\bar{\Psi}_N(\vec{x})\Psi_N(\vec{x})$$

can be reduced by replacing the spinors within the fields by their low-momentum forms

$$U(p) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \xi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m_\chi}\xi \end{pmatrix} \sim \begin{pmatrix} \xi \\ \frac{\vec{\sigma}\cdot\vec{p}}{2m}\xi \end{pmatrix}$$

To leading order in p/m_χ and p/m_N, we obtain the nonrelativistic operator

$$c_1 1_\chi 1_N \equiv c_1 \mathcal{O}_1$$

The nonrelativistic analog of the invariant amplitude is obtained by taking the matrix element of this operator between Pauli spinors ξ_N, ξ_χ

In the nonrelativistic reduction of the SD interaction

$$\mathcal{L}^{\mathrm{SD}}_{\mathrm{int}} = c_4 \bar{\chi}\gamma^\mu\gamma^5\chi \bar{N}\gamma_\mu\gamma^5 N$$

the leading term comes from the spatial components, with $\bar{\chi}\gamma^i\gamma^5\chi \sim \xi^\dagger_\chi \sigma^i \xi_\chi$ With $\sigma^i = 2S^i$

we obtain the **nonrelativistic operator**

$$-4c_4 \vec{S}_\chi \cdot \vec{S}_N \equiv -4c_4\mathcal{O}_4$$

“Standard” Spin Independent scattering cross section

The SI interaction of WIMPs with nuclei, assuming spin 1/2 WIMP (neutralinos), is described by the low-momentum-transfer Lagrangian

$$(1)\ \mathcal{L}_\chi^{SI} = \frac{G_F}{\sqrt{2}} \int d^3\boldsymbol{r}\ j(\boldsymbol{r})\, S(\boldsymbol{r})$$

scalar hadronic current

scalar leptonic current

- The leptonic current is given by kinematics of the WIMP field,

$$j(\boldsymbol{r}) = \bar{\chi}\chi = \delta_{S_f,S_i} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}$$

Where S_f, S_i = 1/2 are the final and initial spin projections of the WIMP and **q** is the momentum transfer from nucleons to WIMPs.

If one takes the hadronic current of the nucleons to be purely isoscalar with coupling c_0 and takes into account only the leading one-body currents, so that the **scalar nuclear current is a sum over single nucleons**

$$S(\boldsymbol{r}) = c_0 \sum_{I=1}^{A} \delta^{(3)}(r - r_i)$$

SI scattering cross section

The differential cross section for SI WIMP scattering off a nucleus with initial state $|i\rangle$ and final state $|f\rangle$ is obtained from the Lagrangian density

$$\frac{d\sigma}{dq^2} = \frac{2}{(2J_i+1)\pi v^2} \sum_{S_f,S_i} \sum_{M_f,M_i} \left| \langle f | \mathcal{L}_\chi^{SI} | i \rangle \right|^2 = \frac{8\, G_f^2}{(2J_i+1)v^2}\, F_S(q)$$

scalar structure factor.

As the target is unpolarized, one averages over initial projections and sums over the final ones.

The structure factor can be decomposed as a sum over multipoles (L) of the reduced matrix elements of the Coulomb projection CL of the scalar current.

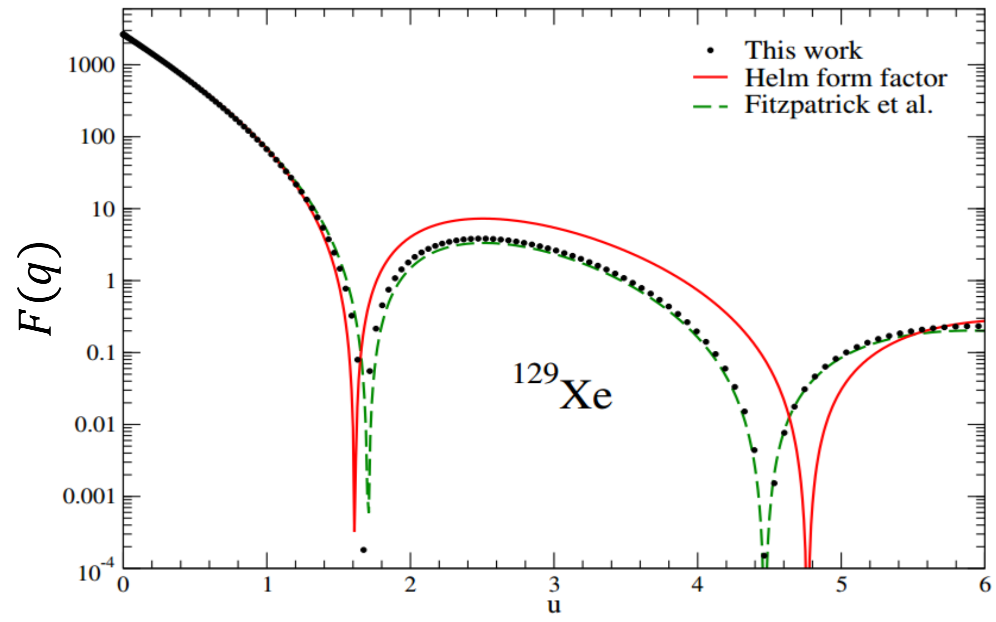
Spin Independent Form Factors

The **structure factors** $\boldsymbol{F_S(q)}$ are plotted as a function of the dimensionless variable $u = q^2b^2/2$, where **q** is the momentum transfer and b is the harmonic-oscillator length, defined as $b = \sqrt{\hbar/m\omega}$, with m the nucleon mass and ω the oscillator frequency, taken as

$$\hbar\omega = \left(45A^{-1/2} - 25A^{-2/3}\right).$$

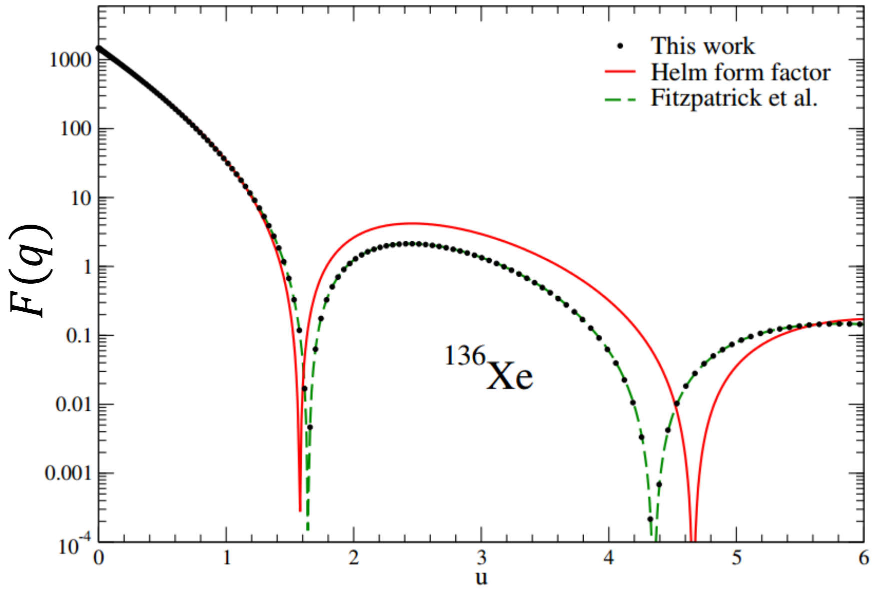
At **zero momentum transfer,** $F_0(q)$ receives contributions only from the L=0 multipole and is model-independent:

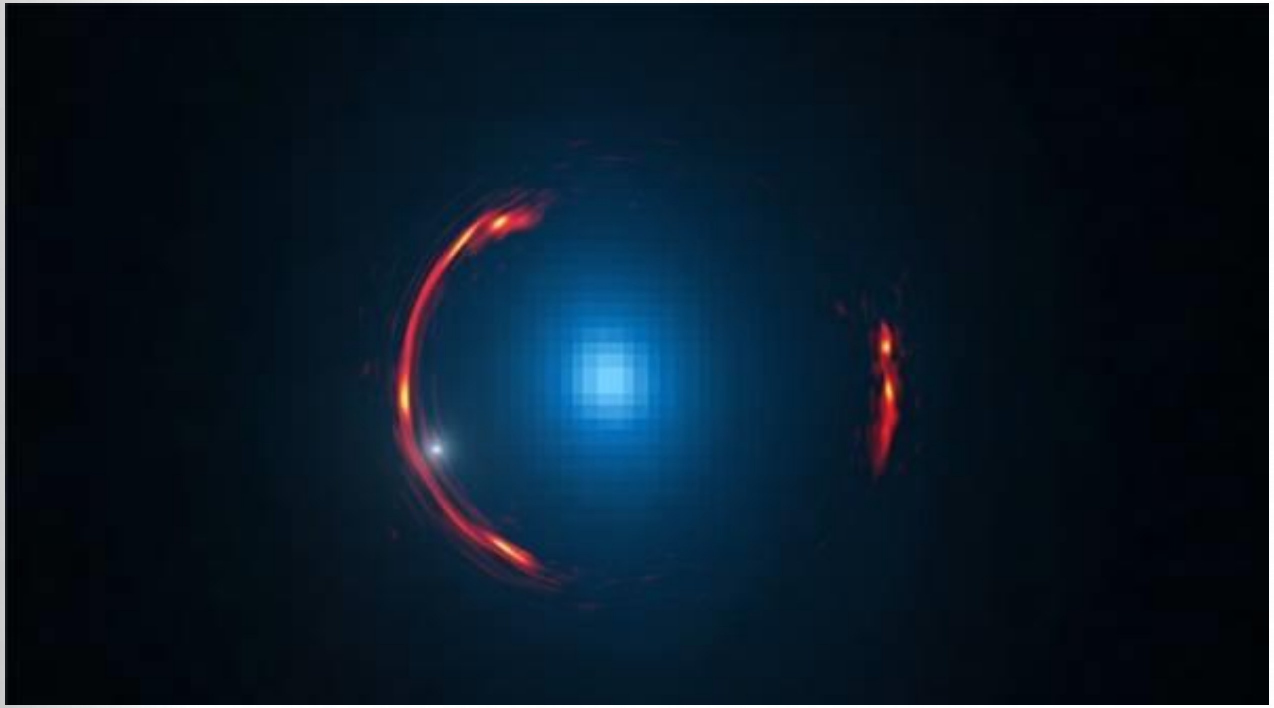
$$F_S(0) = c_0^2\, A^2\, \frac{2J+1}{4\pi}$$

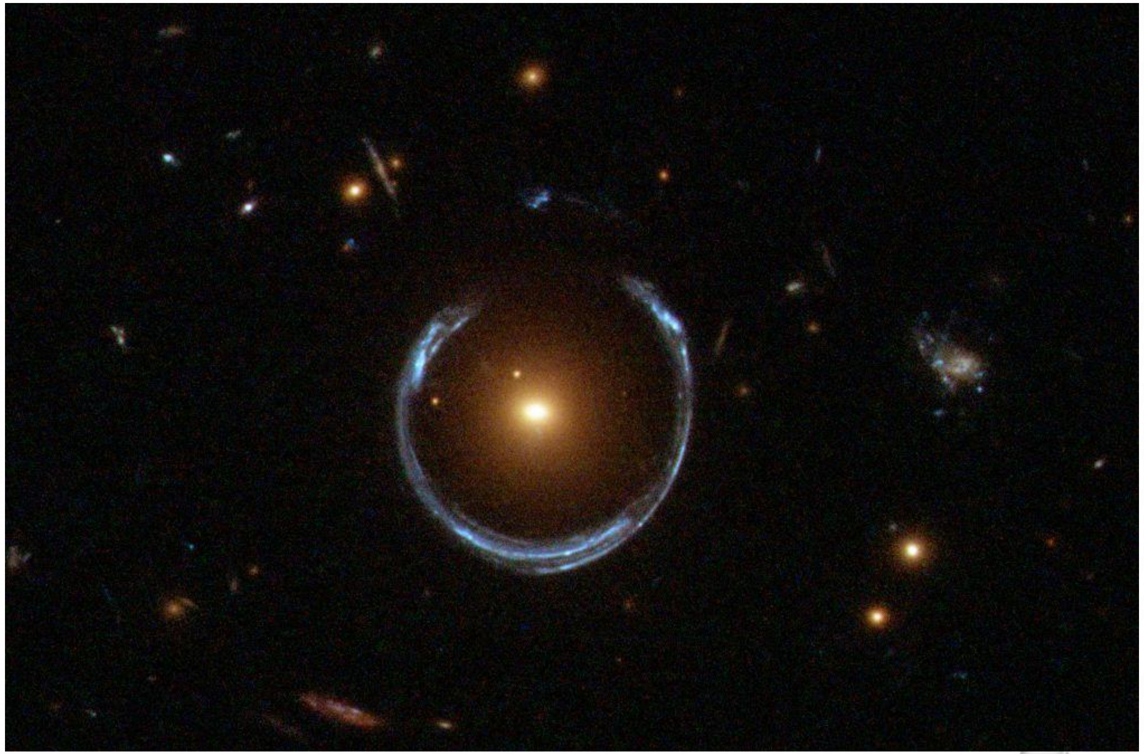


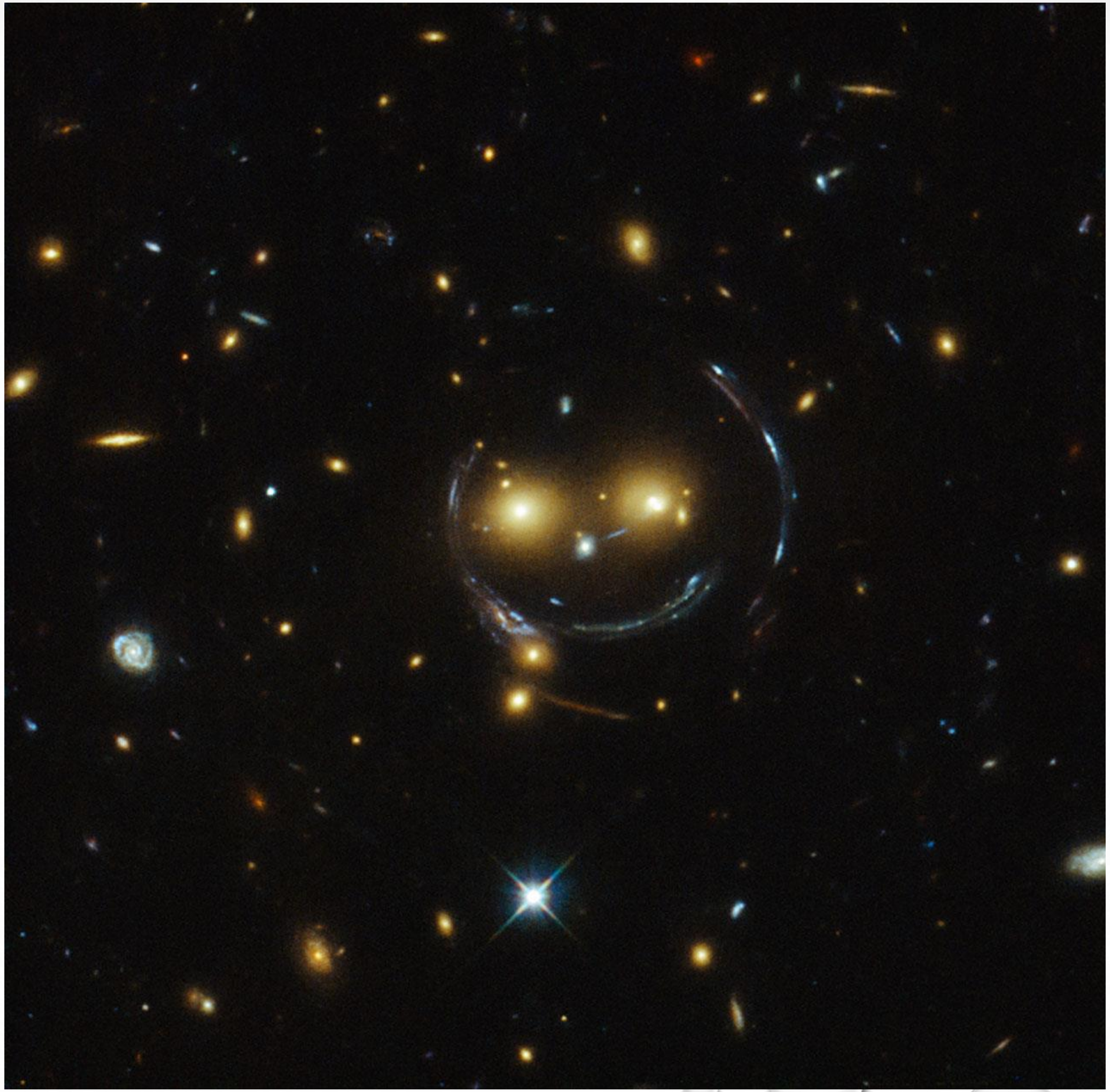
$$\frac{d\sigma_{SI}}{dq^2} = \sigma_{0,SI} \times F(q)$$

Scattering cross section in the limit of zero momentum transfer









Some formulas

$$\frac{d\sigma(v,E_R)}{dE_R} = 2m_T \frac{d\sigma(v,\vec{q}^2)}{d\vec{q}^2} = 2m_T \frac{1}{4\pi v^2}\left[\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\sum_{\text{spins}} |\mathcal{M}^{Nuc}|^2\right]$$

$$\sigma_T \equiv \frac{2\mu_T^2 v^2}{m_T}\frac{d\sigma_T}{dE_R} = \frac{\mu_T^2}{\pi}\langle|\mathcal{M}|^2\rangle^{\text{Nuc}}_{\text{nonrel}}$$

$$\langle|\mathcal{M}|^2\rangle^{\text{Nuc}}_{\text{nonrel}} = \sum_{N,N'=p,n}\left[\sum_{k=M,\Sigma',\Sigma''} R_k\left(v_T^{\perp 2},\frac{\vec{q}^2}{m_N^2},c_i^{(N)},c_j^{(N')}\right)\tilde{W}_k^{(N,N')}(y) + \frac{\vec{q}^2}{m_N^2}\sum_{k=\Phi'',\Delta,M\Phi'',\Delta\Sigma'} R_k\left(v_T^{\perp 2},\frac{\vec{q}^2}{m_N^2},c_i^{(N)},c_j^{(N')}\right)\tilde{W}_k^{(N,N')}(y)\right]$$

$$\begin{aligned}\frac{dR_D}{dE_R} = N_T\frac{dR_T}{dE_R} = N_T\int \frac{d\sigma(v,E_R)}{dE_R} v\,dn_\chi &= N_T n_\chi \int_{v>v_{\min}} \frac{d\sigma(v,E_R)}{dE_R} v f_E(\vec{v}) d^3v \\ &\equiv N_T n_\chi \left\langle v\frac{d\sigma(v,E_R)}{dE_R}\right\rangle_{v>v_{\min}}\end{aligned}$$

Conclusion

- Attention has been focused on a very small piece of all possible WIMP scattering

$$\mathbf{1} \quad \vec{S}_\chi \cdot \vec{S}_N \quad \text{vs.}$$

$$\mathbf{1} \qquad (\vec{S}_\chi \cdot \vec{q})(\vec{S}_N \cdot \vec{q})$$

$$\vec{S}_\chi \cdot \vec{S}_N \qquad i\vec{S}_\chi \cdot (\vec{S}_N \times \vec{q})$$

$$i\vec{S}_N \cdot (\vec{q} \times \vec{v}) \qquad \vec{S}_N \cdot \vec{v}^\perp$$

$$i\vec{S}_\chi \cdot (\vec{q} \times \vec{v}) \qquad \vec{S}_\chi \cdot \vec{v}^\perp$$

$$\ldots$$

Write theory in terms of IR quantities- this makes it much clearer what all possible interactions are.

Gives a concrete set of physical quantities that we need nuclear physics input to calculate

NR-EFT operators list

$$i\frac{\vec{q}}{m_N}, \quad \vec{v}^{\perp}, \quad \vec{S}_\chi, \quad \vec{S}_N$$

$$\mathcal{O}_1 = 1_\chi 1_N,$$
$$\mathcal{O}_2 = (v^{\perp})^2,$$
$$\mathcal{O}_3 = i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^{\perp}\right),$$
$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N,$$
$$\mathcal{O}_5 = i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^{\perp}\right),$$
$$\mathcal{O}_6 = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right),$$
$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^{\perp},$$
$$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^{\perp},$$
$$\mathcal{O}_9 = i\vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N}\right),$$
$$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N},$$
$$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}.$$

$$\mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^{\perp}),$$
$$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^{\perp})\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right),$$
$$\mathcal{O}_{14} = i\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)(\vec{S}_N \cdot \vec{v}^{\perp}),$$
$$\mathcal{O}_{15} = -\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left[(\vec{S}_N \times \vec{v}^{\perp}) \cdot \frac{\vec{q}}{m_N}\right],$$
$$\mathcal{O}_{16} = -\left[(\vec{S}_\chi \times \vec{v}^{\perp}) \cdot \frac{\vec{q}}{m_N}\right]\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right).$$

$$\mathcal{O}_{16} = \mathcal{O}_{15} + \frac{\vec{q}^{\,2}}{m_N^2}\mathcal{O}_{12}$$

Dark Side result

Results from the first use of low radioactivity argon in a dark matter search

P. Agnes *et al.* (DarkSide Collaboration)

Phys. Rev. D **93**, 081101(R) – Published 8 April 2016

ABSTRACT

Liquid argon is a bright scintillator with potent particle identification properties, making it an attractive target for direct-detection dark matter searches. The DarkSide-50 dark matter search here reports the first WIMP search results obtained using a target of low-radioactivity argon. DarkSide-50 is a dark matter detector, using a two-phase liquid argon time projection chamber, located at the Laboratori Nazionali del Gran Sasso. The underground argon is shown to contain ^{39}Ar at a level reduced by a factor $(1.4 \pm 0.2) \times 10^3$ relative to atmospheric argon. We report a background-free null result from (2616 ± 43) kg d of data, accumulated over 70.9 live days. When combined with our previous search using an atmospheric argon, the 90% C.L. upper limit on the WIMP-nucleon spin-independent cross section, based on zero events found in the WIMP search regions, is 2.0×10^{-44} cm^2 (8.6×10^{-44} cm^2, 8.0×10^{-43} cm^2) for a WIMP mass of 100 GeV$/c^2$ (1 TeV$/c^2$, 10 TeV$/c^2$).

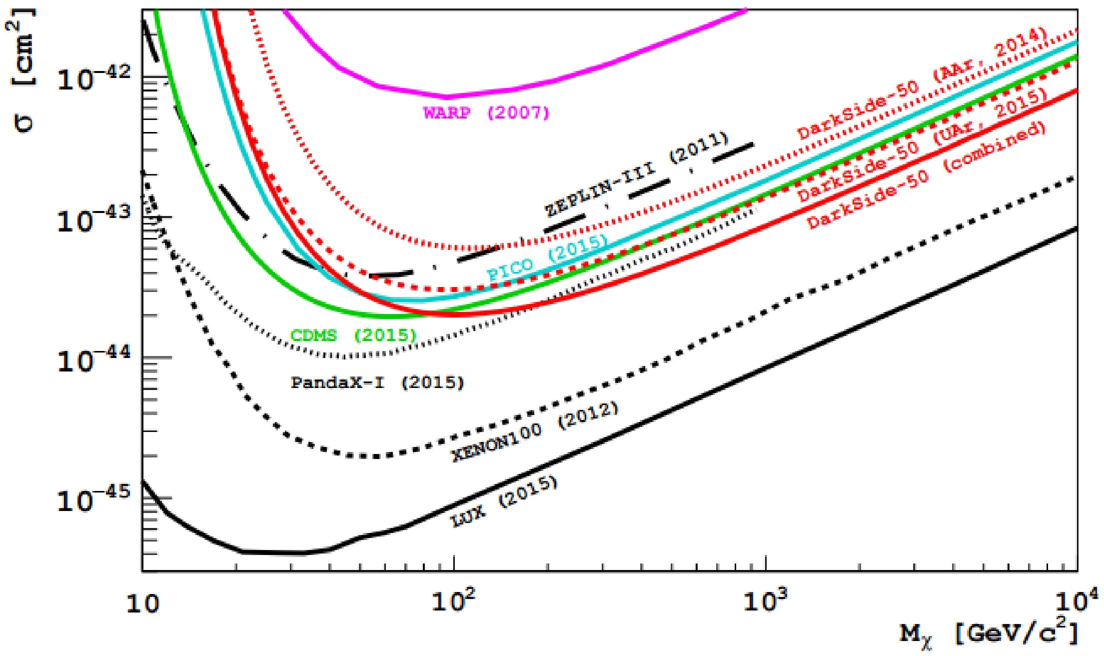


FIG. 5. Spin-independent WIMP-nucleon cross section 90 % C.L. exclusion plots for the DarkSide-50 AAr (dotted red) and UAr campaigns (dashed red), and combination of the UAr and AAr [38] campaigns (solid red). Also shown are results from LUX [58] (solid black), XENON100 [59] (dashed black), PandaX-I [60] (dotted black), CDMS [61] (solid green), PICO [62] (solid cyan), ZEPLIN-III [63] (dash dotted black) and WARP [64] (magenta).

Spin independent

The standard spin-independent (SI), WIMP-nucleus interaction can result from the effective Lagrangian,

$$\mathcal{L}_{\text{int}}^{\text{SI}} = \sum_{N=n,p} \frac{f_{\text{SI}}^N}{\Lambda^2} \bar{\chi}\chi\bar{N}N \rightarrow \sum_{N=n,p} c_1^N \mathcal{O}_1 \quad \text{with } c_1^N = \frac{f_{\text{SI}}^N}{\Lambda^2}$$

leading to the cross section,

$$\sigma_T^{\text{SI}} = \frac{\mu_T^2}{\pi}\frac{1}{\Lambda^4}\left({f_{\text{SI}}^p}^2 \tilde{W}_M^{(p,p)} + 2 f_{\text{SI}}^p f_{\text{SI}}^n \tilde{W}_M^{(p,n)} + {f_{\text{SI}}^n}^2 \tilde{W}_M^{(n,n)}\right)$$

which is often expressed as

$$\sigma_T^{\text{SI}} = \frac{\mu_T^2}{\mu_p^2}\sigma_p^{\text{SI}}(Z + (A - Z) f_{\text{SI}}^n / f_{\text{SI}}^p)^2 F^2$$

where the form factor F^2 is defined to be 1 at zero momentum transfer

$$F^2(y) \equiv \frac{{f_{\text{SI}}^p}^2 \tilde{W}_M^{(p,p)}(y) + 2 f_{\text{SI}}^p f_{\text{SI}}^n \tilde{W}_M^{(p,n)}(y) + {f_{\text{SI}}^n}^2 \tilde{W}_M^{(n,n)}(y)}{{f_{\text{SI}}^p}^2 \tilde{W}_M^{(p,p)}(0) + 2 f_{\text{SI}}^p f_{\text{SI}}^n \tilde{W}_M^{(p,n)}(0) + {f_{\text{SI}}^n}^2 \tilde{W}_M^{(n,n)}(0)}$$

and where σ_p^{SI} is the zero-momentum-transfer cross section off of protons

$$\sigma_p^{\text{SI}} = \frac{\mu_p^2}{\pi}\left(\frac{f_{\text{SI}}^p}{\Lambda^2}\right)^2.$$

Anapole Dark Matter

Majorana fermion DM scattering off of nucleons via a spin-1 mediator that kinetically mixes with the photon proceeds via the following effective interaction

$$\mathcal{L}_{\text{int}}^{\text{anapole}} = \frac{f_a}{M^2} \bar{\chi}\gamma^\mu\gamma^5\chi \mathcal{J}_\mu^{\text{EM}}$$

where,

$$\mathcal{J}_\mu^{\text{EM}} \equiv \sum_{N=n,p} \bar{N}\left(Q_N \frac{K_\mu}{2m_N} - \tilde{\mu}_N \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}\right)N \qquad \tilde{\mu} = \frac{\text{magnetic moment}}{\text{nuclear magneton}}$$

is the electromagnetic current restricted to nucleons.

In the non relativistic limit,

$$\mathcal{L}_{\text{int}}^{\text{anapole}} \to \frac{2f_a}{M^2} \sum_{N=n,p} (Q_N \mathcal{O}_8 + \tilde{\mu}_N \mathcal{O}_9)$$

Evaluating $\langle |\mathcal{M}|^2 \rangle_{\text{nonrel}}^{\text{Nuc}} = \sum_{N,N'=p,n} \left[\sum_{k=M,\Sigma',\Sigma''} R_k\left(v_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}, c_i^{(N)}, c_j^{(N')}\right) \tilde{W}_k^{(N,N')}(y) + \frac{\vec{q}^2}{m_N^2} \sum_{k=\Phi'',\Delta,M\Phi'',\Delta\Sigma'} R_k\left(v_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}, c_i^{(N)}, c_j^{(N')}\right) \tilde{W}_k^{(N,N')}(y) \right]$

and substituting the “WIMP form factors” Rk.

$$\sigma_T^{\text{anapole}} = \frac{\mu_T^2}{\pi}\left(\frac{f_a}{M^2}\right)^2 C_\chi \left\{ \vec{v}_T^{\perp 2} \tilde{W}_M^{(p,p)} + \frac{\vec{q}^2}{m_N^2}\left[\tilde{W}_\Delta^{(p,p)} - \tilde{\mu}_n \tilde{W}_{\Delta\Sigma'}^{(p,n)} - \tilde{\mu}_p \tilde{W}_{\Delta\Sigma'}^{(p,p)} + \frac{1}{4}\left(\tilde{\mu}_p^2 \tilde{W}_{\Sigma'}^{(p,p)} + 2\tilde{\mu}_n\tilde{\mu}_p \tilde{W}_{\Sigma'}^{(p,n)} + \tilde{\mu}_n^2 \tilde{W}_{\Sigma'}^{(n,n)}\right)\right]\right\}$$

$$\Rightarrow \quad \sigma_T^{\text{anapole}} = \frac{\mu_T^2}{\pi}\left(\frac{f_a}{M^2}\right)^2 \times \left(\left(\vec{v}^2 - \frac{\vec{q}^2}{4\mu_T^2}\right) Z^2 F(E_R)^2 + \vec{q}^2 \frac{J+1}{6J} \frac{\tilde{\mu}_T^2}{m_N^2}\right)$$