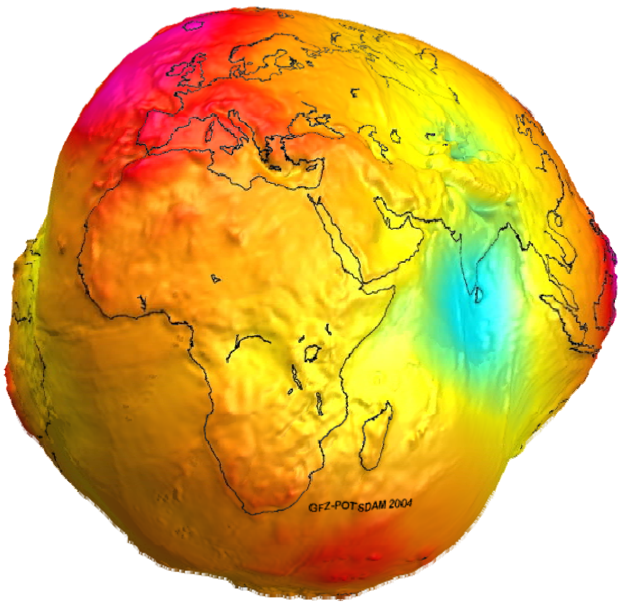
Testing the no-hair hypothesis



Vítor Cardoso

(CENTRA/Técnico & Perimeter)

...

Cagliari 2016

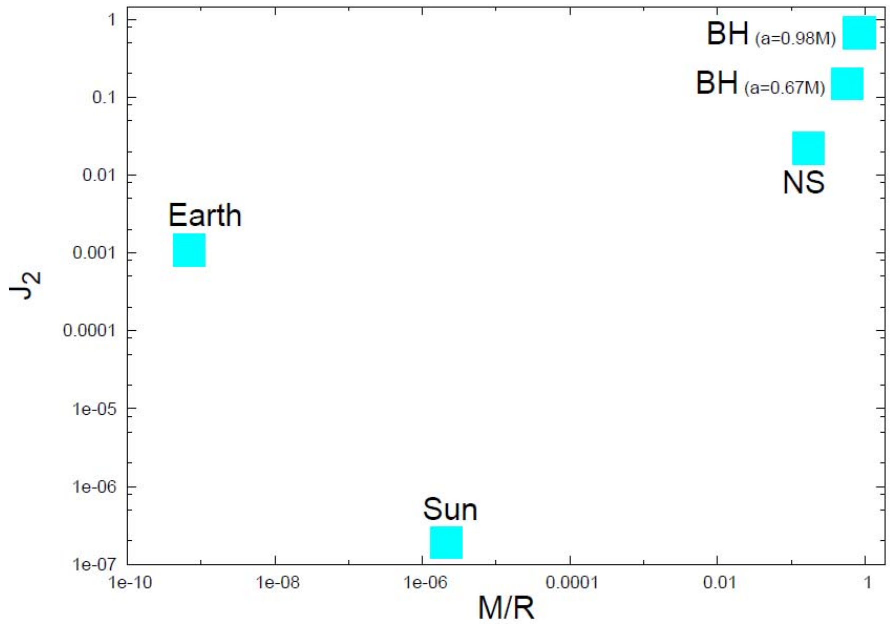
blackholes.ist.utl.pt

erc supports this project

$$\Phi(\boldsymbol{x}) = -G\sum_{l=0}^{\infty}\frac{M_l}{r^{l+1}}P_l(\cos\theta)$$

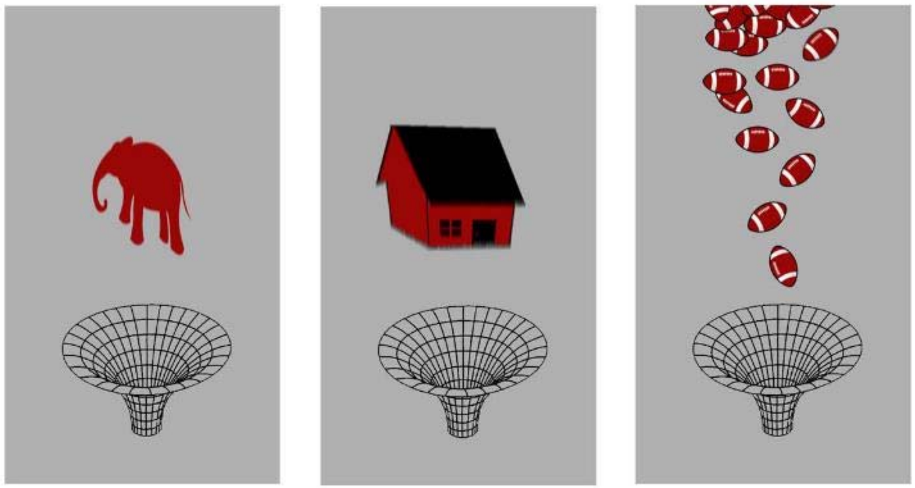
$$M_l = \int \rho(\boldsymbol{x})r^l P_l(\cos\theta)d^3x$$

$$J_l = -M_l/(MR^l)$$



“Black holes have no hair”

$$M_{2l} = (-1)^l M a^{2l}$$
$$S_{2l+1} = (-1)^l M a^{2l+1}$$



Incidentally, the first mention of the theorem was refused by PRD Editor Pasternak, on the grounds of being obscene (in Kip Thorne’s *Black Holes and Time Warps*)

Plan

The no-hair hypothesis

What is it

Why it's dead

Why we still care

Testing the no-hair hypothesis

Gravitational wave ringdown

Motion of stars and pulsars

Accretion disks

Black hole shadows

Conclusions

Dynamics of BHs & compact objects

Brito, Fujita, Hopper, Nerozzi (Franzin, Okawa, Pani, Rocha, Witek, Zilhão)

Barausse, Berti, Gualtieri, Herdeiro, Pretorius, Sperhake

✷ ✷ ✷

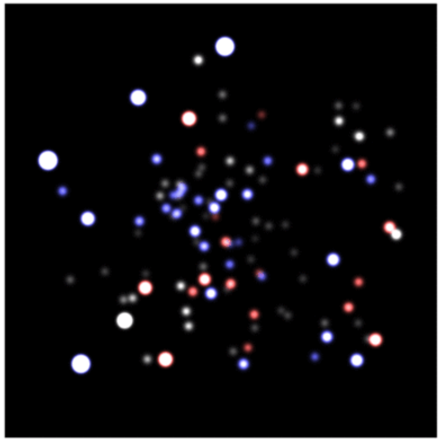
Brito, Cardoso, Pani, *Superradiance*, Lect. Notes Phys. (Springer-Verlag, 2015)

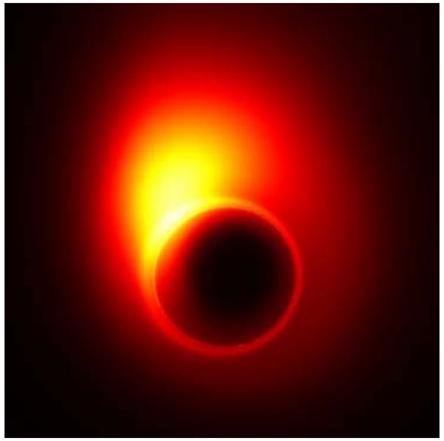
Cardoso, Franzin, Pani, Phys.Rev.Lett.116(2016)171101

Berti, Sesana, Barausse, Cardoso, Belczynski (2106, submitted)

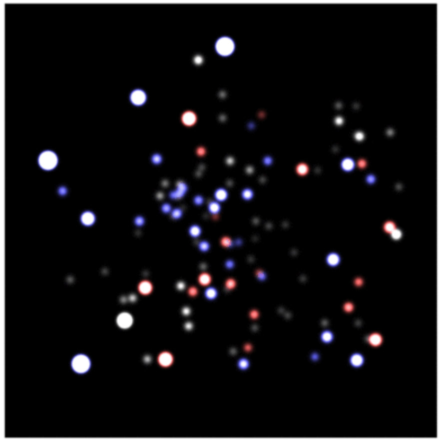
Cardoso, Gualtieri, *Testing the no-hair hypothesis*, to appear (2016)

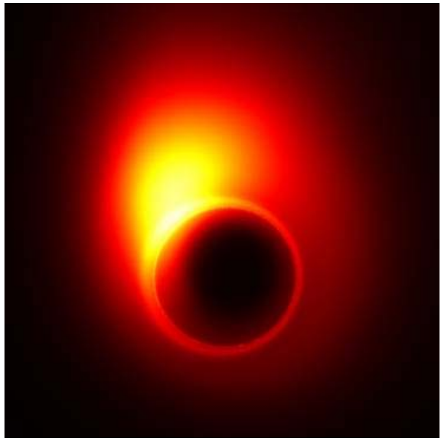
Massive, compact objects exist!



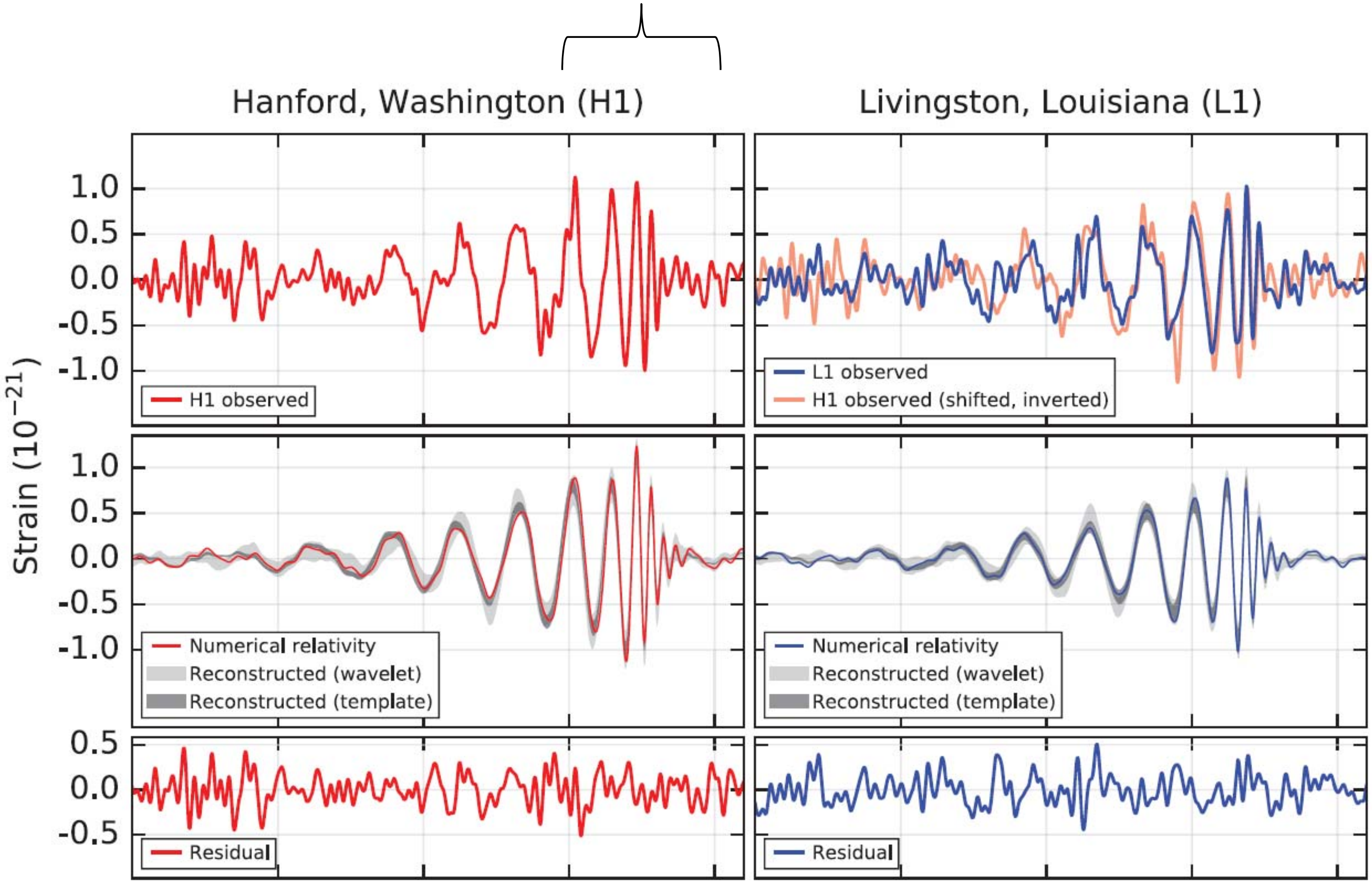


Massive, compact objects exist!





0.05 secs (~80 Schwarzschild radius)

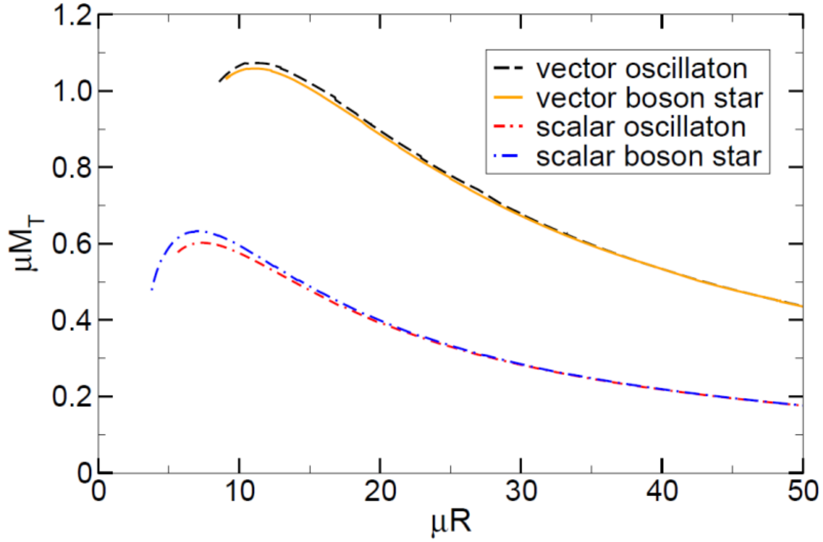


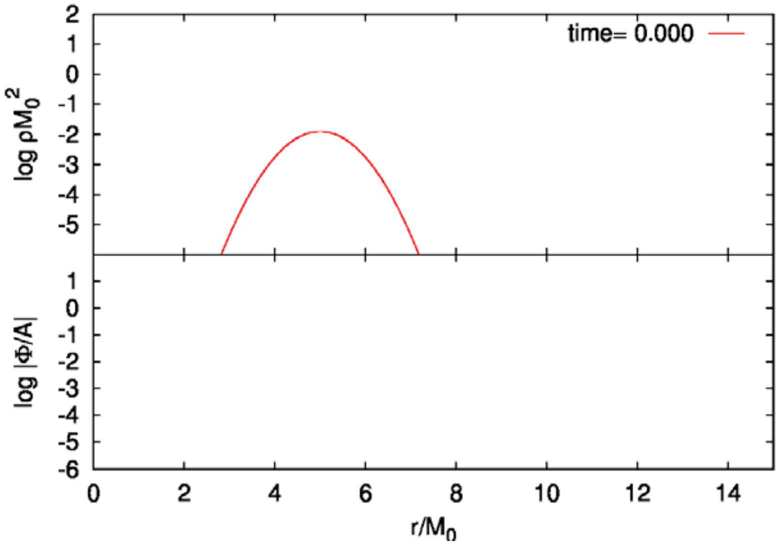
Abbott et al, Phys.Rev.Lett.116:061102 (2016)

Exotic Compact Objects (ECOs)

Boson Stars, Fermion-boson stars

(Kaup 1968; Ruffini, Bonazzolla 1969; Colpi et al 1986; Okawa et al 2014; Brito et al 2015)





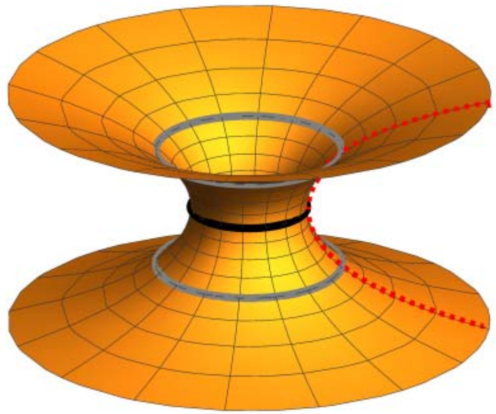
$$\frac{M_{\max}}{M_{\odot}} = 8 \times 10^{-11} \frac{\text{eV}}{m_B c^2}$$

Boson Stars, Fermion-boson stars

(Kaup 1968; Ruffini, Bonazzolla 1969, Colpi et al 1986, Brito et al 2015)

Wormholes

(Morris, Thorne 1988; Visser 1996)



Gravastars

(Mazur, Mottola 2001)

Superspinars (super-extremal Kerr singularity cut-off)

(Gimon, Horava 2009)

The Schwarzschild solution

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$\left(\left(1 - \frac{2M}{r}\right) \Psi'\right)' - \left(\frac{l(l+1)}{r^2} + \frac{2M(1-s^2)}{r^3}\right) \Psi = 0$$

$$\int_{2M}^{+\infty} dr \left(1 - \frac{2M}{r}\right) |\Psi'|^2 + \frac{l(l+1)}{r^3} \left(r + \frac{2M(1-s^2)}{l(l+1)}\right) |\Psi|^2 = 0$$

(i) Impossible to “anchor” massless scalars (or fermions) onto Schwarzschild BHs

(ii) Impossible to anchor massless multipoles l> s: no “protuberance” (hair)

(iii) Possible generalization that includes electric charge and rotation

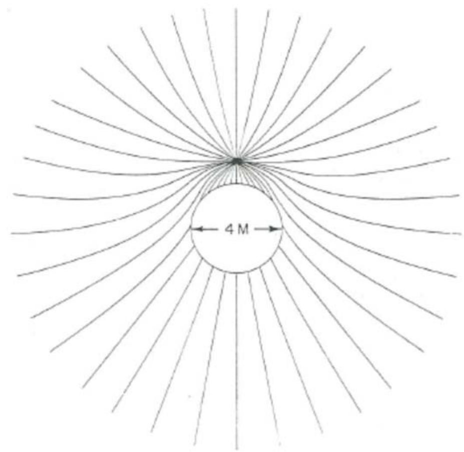


Fig. 4. Lines of force with the test charge momentarily at rest a $r = 3M$.

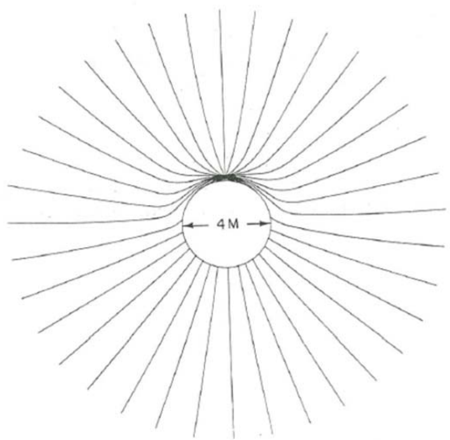
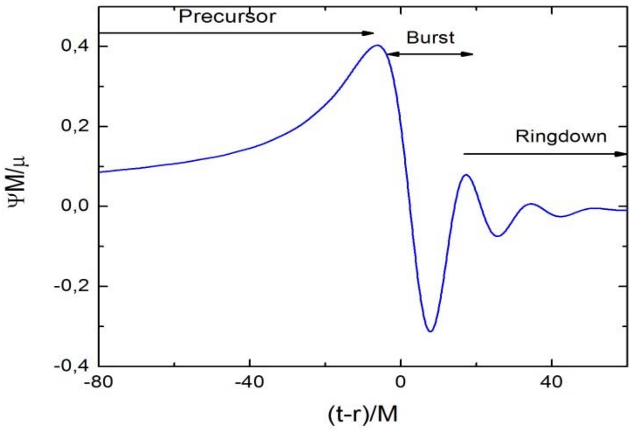
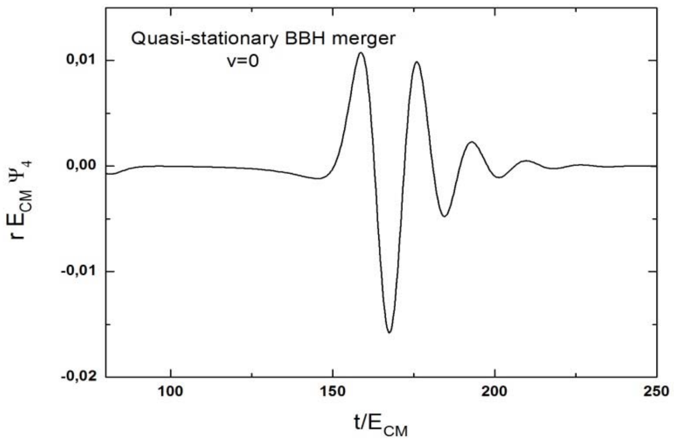


Fig. 5. Lines of force with the test charge momentarily at rest at $r = 2.2M$.

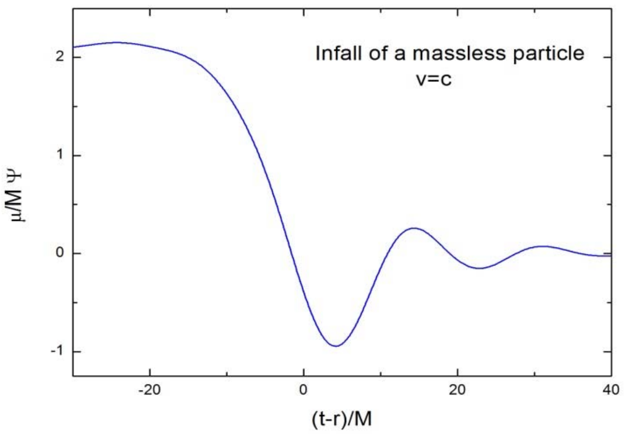
(Ginzburg, Ozernoy 1964; Cohen, Wald 1971; Ruffini 1973)



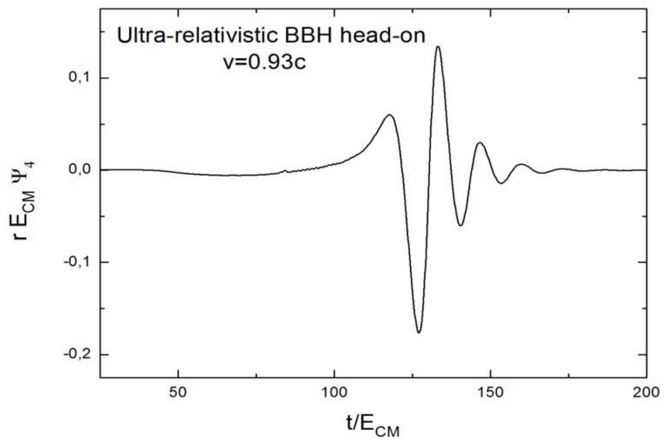
Davis et al, 1971



Anninos et al, 1993



Cardoso, Lemos, 2002



Sperhake et al, 2008

$$e^{-0.0898\, t/M_{BH}} \sin\left(0.374\, t/M_{BH}\right)$$

Followed by power-law decay

Uniqueness: the Kerr solution

(Kerr 1963)

Theorem 1 (Carter 1971; Robinson 1975):
A stationary, asymptotically flat, vacuum solution must be Kerr

$$ds^2 = \frac{\Delta - a^2 \sin^2\theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta)\sin^2\theta}{\Sigma} dt d\phi - \frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma} \sin^2\theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2$$

$$\Sigma = r^2 + a^2\cos^2\theta\,, \quad \Delta = r^2 + a^2 - 2Mr$$

Describes a rotating BH with mass M and angular momentum J=aM

Theorem 2 (Bekenstein 1972; Graham, Jha 2014):
Isolated, stationary BHs in the Einstein-Klein-Gordon or Einstein-Proca theory with a *time-independent boson* are described by Kerr family
(impossible to hold the hair)

Theorem 3 (Bekenstein 1972; Graham, Jha 2014):
Isolated, stationary BHs in the Einstein-Klein-Gordon theory with *one real scalar* are described by the Kerr family
(impossible not to radiate GWs)

The no-hair hypothesis

The Kerr geometry describes all black holes in our Universe

The Kerr geometry describes all massive, compact objects

The no-hair hypothesis *must* be wrong

Black holes surrounded by thin shells (stability for r>3M...)

(Frauendiener, Hoenselaers, Konrad 1990; Brady, Louko, Poisson 1991)

Anisotropic fluid hair

(Brown, Hussain 1997)

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

$$f = 1 - \frac{2M}{r} + \frac{Q_m^{2k}}{r^{2k}}$$

$$\rho = \frac{Q_m^{2k}(2k-1)}{8\pi r^{2k+2}}, \qquad P = k\rho$$

The no-hair hypothesis *must* be wrong

Black holes in EYM theory (with SU(2) gauge group, “colored BHs”)
(Bizon 1990)

Einstein-dilaton-Gauss-Bonnet
(Mignemi, Stewart 1993; Kanti et al 1995; Kleihaus, Kunz, Radu 2011)

$$S = \int d^4x\sqrt{-g}\left(R - \frac{1}{2}\partial_a\phi\partial^a\phi + \frac{\alpha}{4}e^{\phi}\mathcal{R}^2\right)$$

$$\mathcal{R} = {R_{abcd}}^2 - 4{R_{ab}}^2 + R^2$$

Dynamical-Chern-Simons
(Alexander, Yunes 2009, Pani et al 2011)

$$S = \int d^4x\sqrt{-g}\,\kappa R + \frac{\alpha}{4}\vartheta\,{}^*RR - \frac{\beta}{2}g^{ab}\nabla_a\vartheta\nabla_b\vartheta$$

$$*\,RR = \frac{1}{2}R_{abcd}\epsilon^{baef}R^{cd}_{ef}$$

The no-hair hypothesis *must* be wrong

Models of mini-charged DM predict heavy, fractional "electrons"
(Rujula, Glashow, Sarid 1990; Perl, Lee 1997; Holdom 1986; Sigurdson et al 2004)

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{16\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + 4\pi e j^{\mu}_{\rm em}A_{\mu} + 4\pi e_h j^{\mu}_{h}B_{\mu} + 4\pi\epsilon e j^{\mu}_{h}A_{\mu}\right)$$

BH solutions are Reissner-Nordstrom

Discharge mechanisms (mechanical, Schwinger, Hawking) suppressed
(Cardoso, Macedo, Pani, Ferrari 2016)

The no-hair hypothesis *must* be wrong

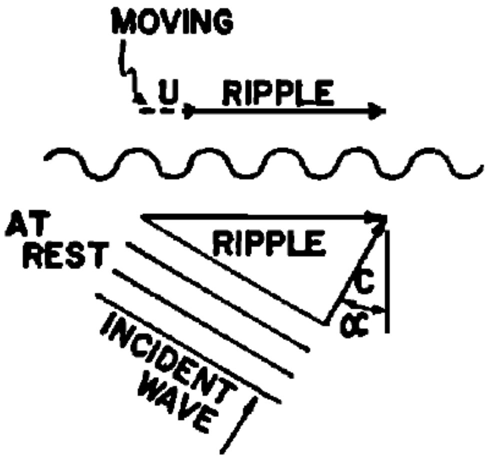
Hairy Kerr in minimally coupled KG theory (BS with BH at center)
(Herdeiro, Radu 2014)

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{\kappa} - \frac{1}{2}g^{\mu\nu}\bar{\Psi}_{,\mu}\Psi_{,\nu} - \frac{\mu_S^2\bar{\Psi}\Psi}{2}\right)$$

Evades theorems 1-3 with complex, time-dependent scalars but time-independent stress-tensor (prevents hair from falling *out*)

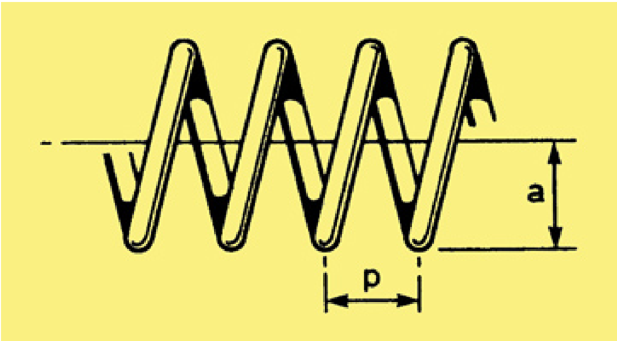
Superradiance prevents hair from falling *in* (Brito, Cardoso, Pani 2015)

Intermezzo: Friction & superradiance

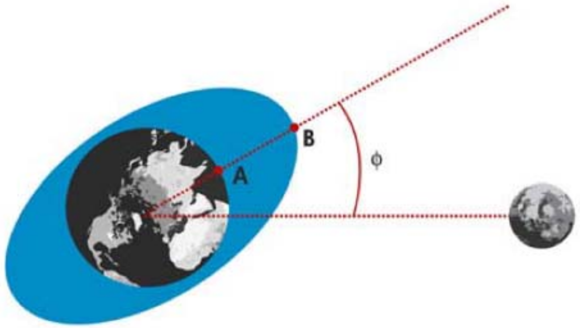


Ribner, J. Acous. Soc. Amer.29 (1957)

Tamm, Frank, Doklady AN SSSR 14 (1937)

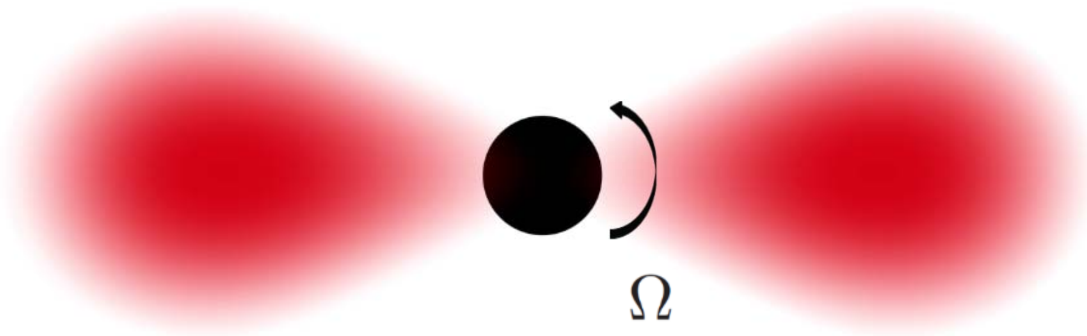


Pierce (& Kompfner), Bell Lab Series (1947)
Ginzburg, anomalous Doppler year



G. H. Darwin, Philos. Trans. R. Soc. London 171 (1880)

$$\Phi \sim e^{-i\omega t + im\phi} \;\rightarrow\; (Angular)\, phase\, velocity = \frac{\omega}{m}$$



$$\omega < m\Omega$$

Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. 14 (1971)

...and yet...

With exception of boson stars, no formation mechanism (yet) of ECOS

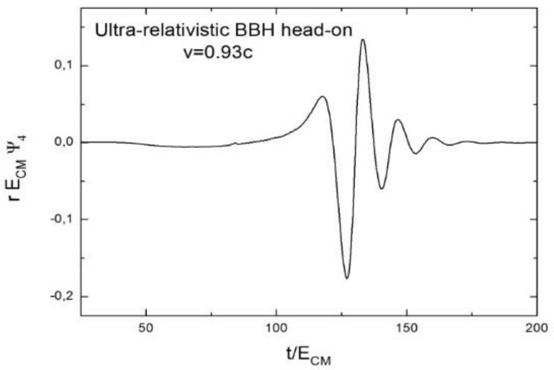
Compact objects plagued by linear and nonlinear instabilities
(Friedman 1978; Cardoso et al 2008; Brito et al 2015; Keir 2016)

If object too compact, distinction irrelevant
(Cardoso, Franzin, Pani 2016)

Large class of theories Kerr still solution
(Psaltis et al 2008; Barausse, Sotiriou 2008)

Tests of the no-hair hypothesis

Gravitational waves and ringdown modes



Multipolar structure: motion of stars and pulsars

Accretion disks

Black hole shadows

A linearized approach

A wave analysis

$$dr/dr_* \equiv f\,, \qquad f \equiv 1 - 2M/r$$

$$\frac{d^2\Psi}{dr_*^2} + \left[\omega^2 - V\right]\Psi = 0$$

$$V = f\left(\frac{\ell(\ell+1)}{r^2} + 2M\frac{1-s^2}{r^3}\right)$$

$$\Psi \sim e^{-i\omega(t \mp r_*)}\,, \quad r_* \to \pm\infty$$

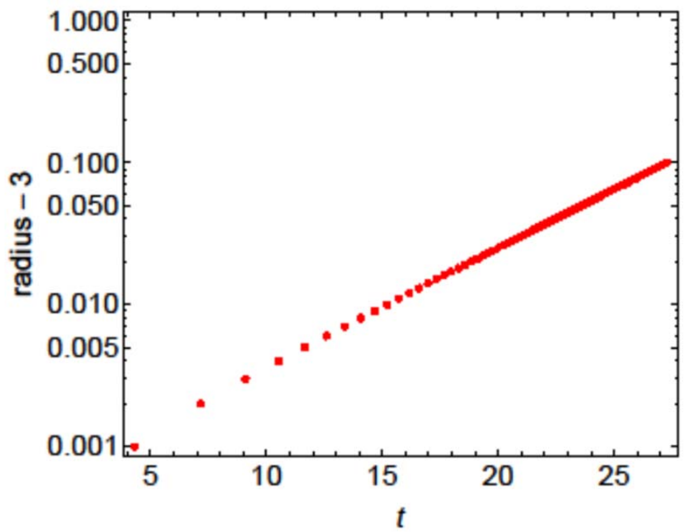
$$M\omega = 0.374 - 0.089i$$

The geodesic connection

$$\dot{\varphi} = L/r^2\,, \qquad \dot{t} = E/(1-2M/r)$$

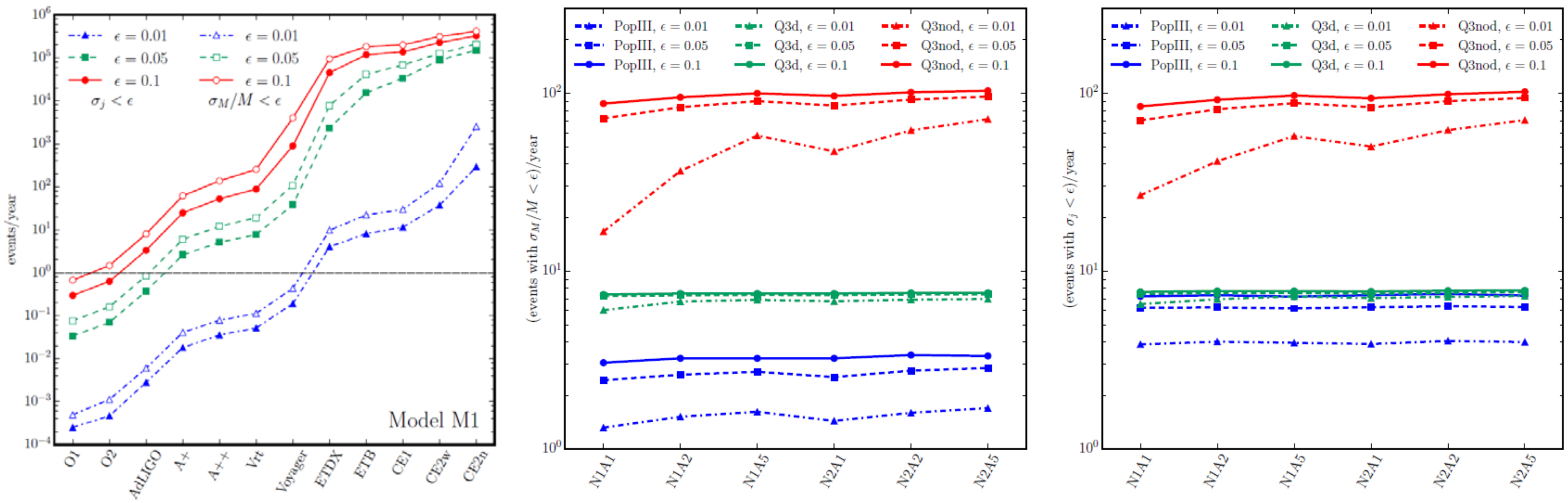
$$\dot{r}^2 = V = V_0 + V_0'(r - r_c) + \frac{1}{2}V_0''(r - r_c)^2$$

$$r - r_c \sim e^{\lambda t}\,, \qquad \lambda^2 = \frac{(1-2M/r_c)^2 V_0''}{2E^2}$$



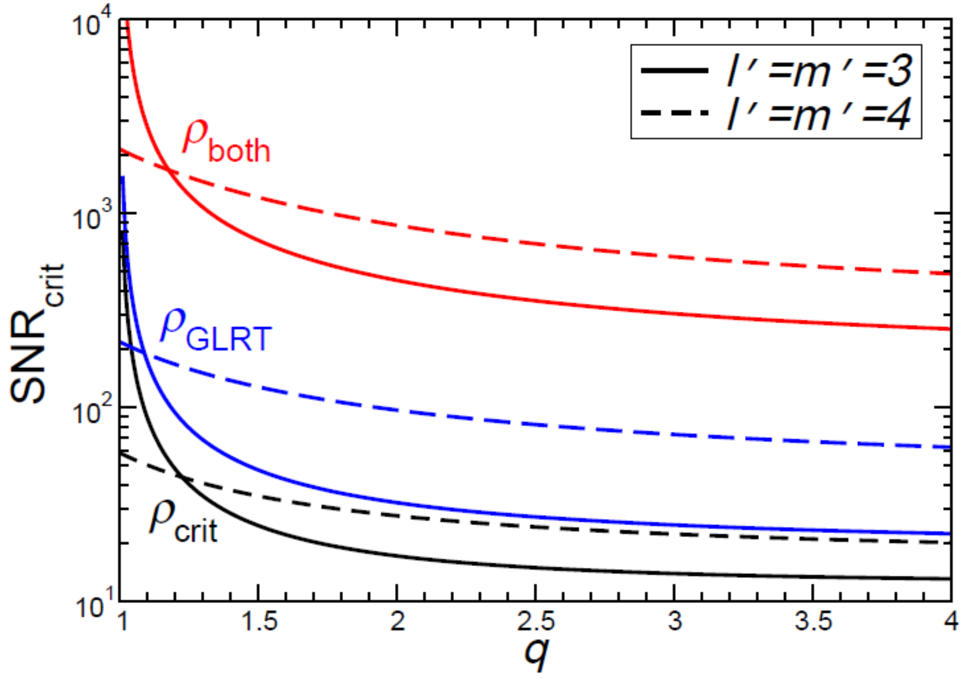
$$M\omega = 0.380\,, \quad M\lambda = 0.019$$

Can one hear the shape of a BH?



Berti, Sesana, Barausse, Cardoso, Belczynski (2016)

Can one hear the shape of gravity?



Berti, Cardoso, Cardoso, Cavaglia,
PRD76,104044(2007)

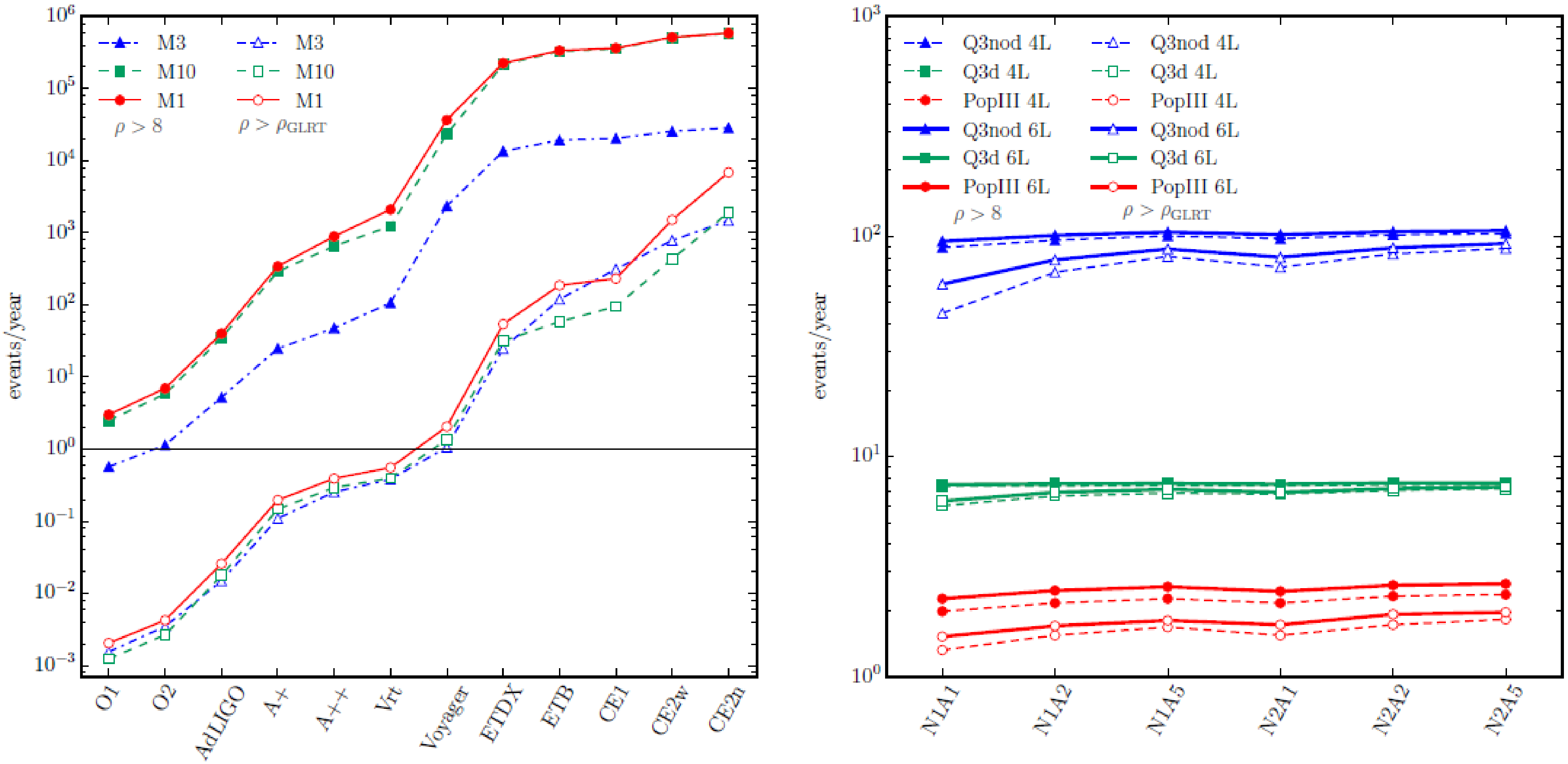
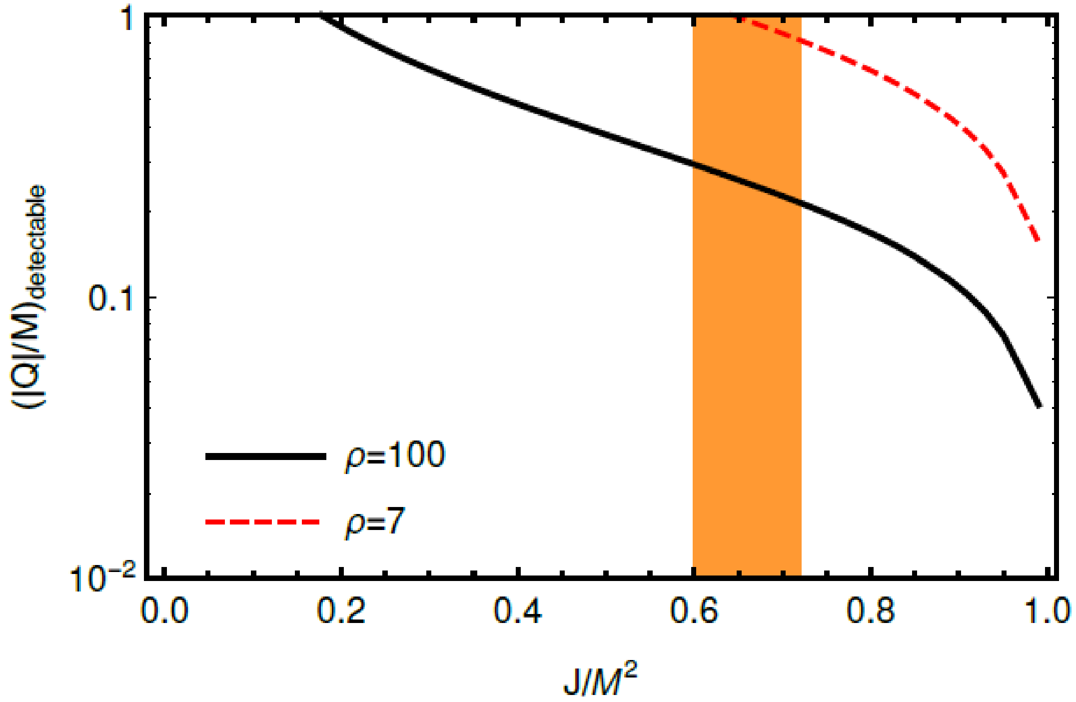


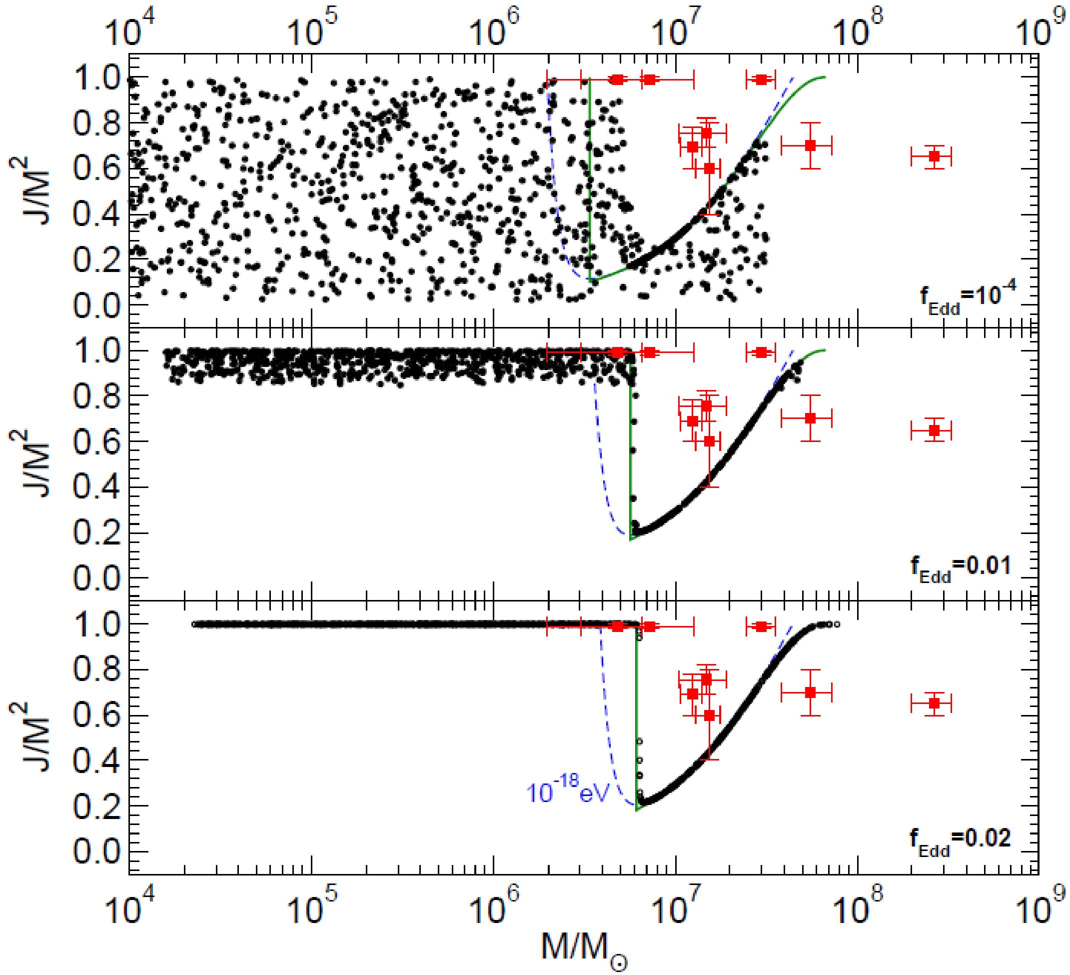
FIG. 2. Rates of binary BH mergers that yield detectable ringdown signals (filled symbols) and allow for no-hair theorem tests (hollow symbols). Left panel: rates per year for Earth-based detectors of increasing sensitivity in different stellar-mass BH formation models. Right panel: rates per year for 6-link (solid) and 4-link (dashed) eLISA configurations with varying armlength and acceleration noise in different supermassive BH assembly scenarios.

Berti, Sesana, Barausse, Cardoso, Belczynski (2016)



$$\frac{|Q|}{M} \lesssim 0.1\sqrt{\frac{100}{\rho}}$$

Cardoso, Macedo, Pani, Ferrari
JCAP 1605: 054 (2016)



$\mu = 10^{-18}\text{eV}$

Random distributions 1000 BHs, with initial mass between $\log_{10} M_0 \in [4, 7.5]$ and $J_0/M_0^2 \in [0.001, 0.99]$ extracted at $t = t_F$, with t_F distributed on a Gaussian centered at $\bar{t}_F \sim 2 \times 10^9$yr with width $\sigma = 0.1\bar{t}_F$.

Brito, Cardoso, Pani, CQG32 (2015) 13, 134001; Arvanitaki et al (2016)

Dirty effects: environment

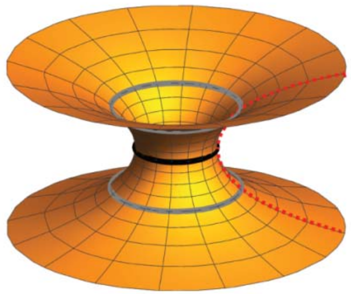
However, inasmuch as the goal of the gravitational wave observatories is to obtain astrophysical information of our universe (...), there is no doubt that we will eventually have to face this problem of the QNM spectra of dirty black holes.

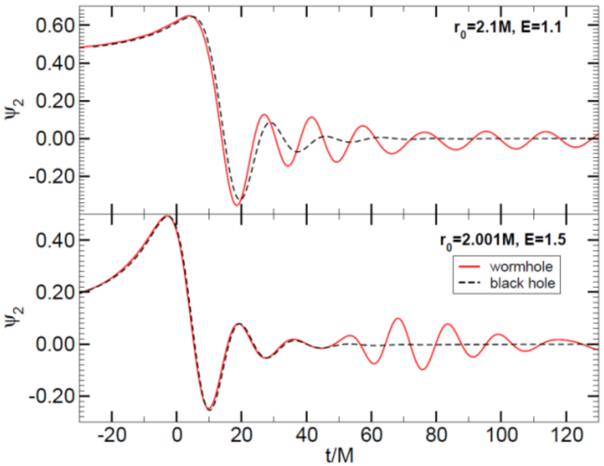
- Leung et al 1999

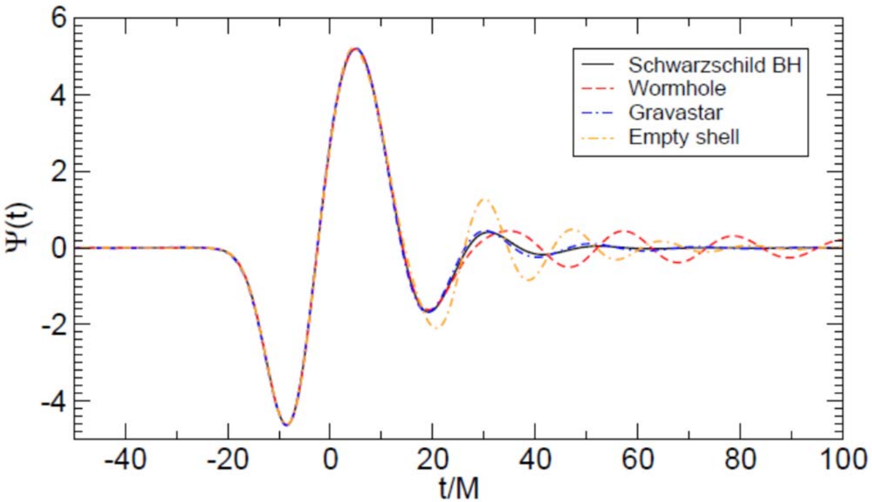
Correction	$\vert\delta_R\vert[\%]$	$\vert\delta_I\vert[\%]$
spherical near-horizon distribution	0.05	0.03
ring at ISCO	0.01	0.01
electric charge	10^{-5}	10^{-6}
magnetic field	10^{-8}	10^{-7}
gas accretion	10^{-11}	10^{-11}
DM halos	$10^{-21}\rho_3^{\rm DM}$	$10^{-21}\rho_3^{\rm DM}$
cosmological effects	10^{-32}	10^{-32}

Barausse, Cardoso, Pani 2014

Are we *really* observing black holes?







Cardoso, Franzin, Pani
PRL116 (2016), 171101

Conclusions

Exciting times for gravitational-wave physics!

Advances in theory and numerical methods. Advanced LIGO, network of detectors soon. Rates are under control (...)

Birth and interaction of massive objects, specially BHs; central engine of violent phenomena (GRBs, etc)

Demographics of very compact objects
(GWs determine mass and spin better than 1%!)

Gravitational wave astronomy *can* become a precision discipline, mapping compact objects throughout the entire visible universe.

Hundreds of ringdown observations, tests of GR and Kerr hypothesis will be done routinely.

"*After the advent of gravitational wave astronomy, the observation of these resonant frequencies might finally provide direct evidence of BHs with the same certainty as, say, the 21 cm line identifies interstellar hydrogen*" (S.Detweiler)

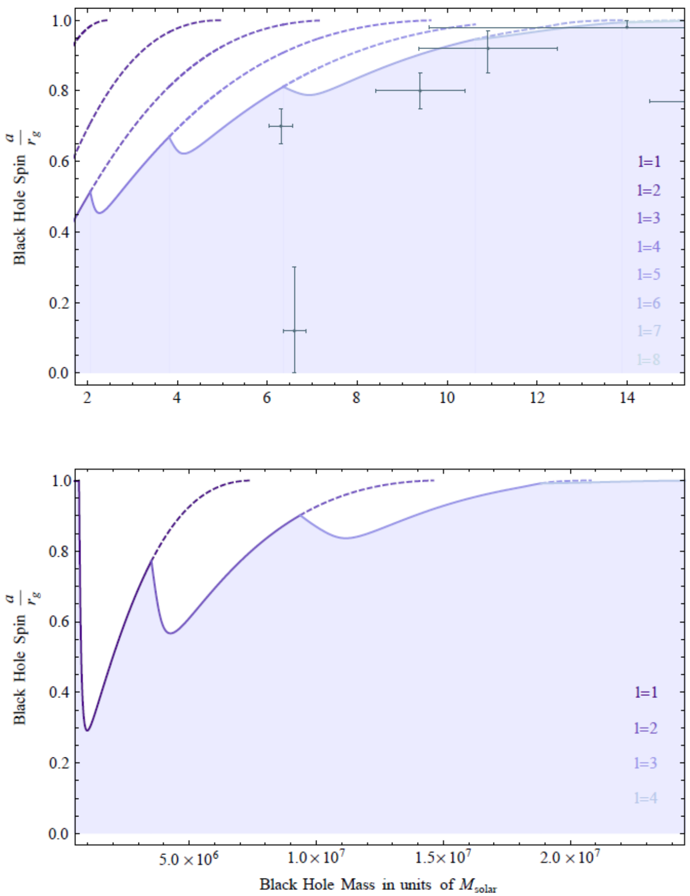
Time-response of BH is dominated by light-ring ringdown at early times, and shared by all horizonless compact objects. These vibrations modes do *not* show up as poles of the corresponding Green function

Can we discriminate competing gravity theories from ringdown observations?

...?

Thank you

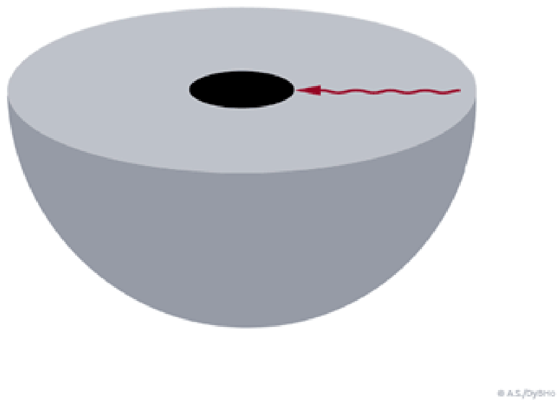
Strong field gravity and fundamental physics



Arvanitaki et al (2016)
Brito et al, CQG (2014)

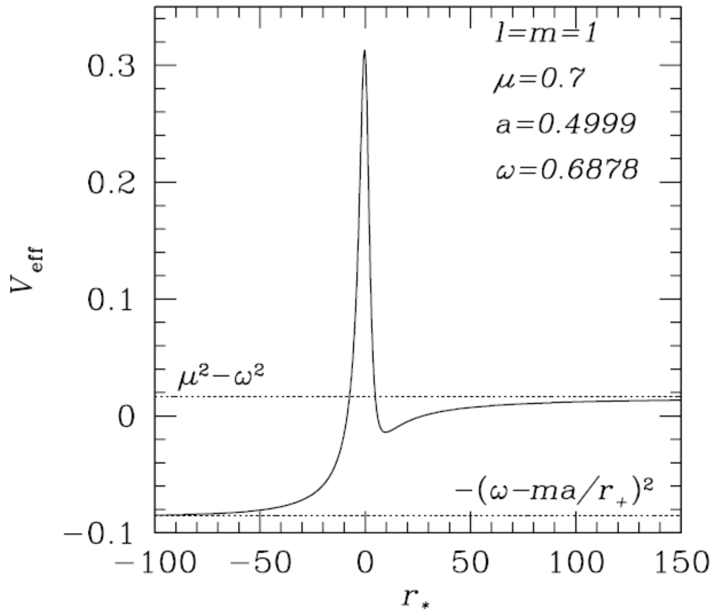
Superradiant instabilities

Can construct unstable states by forcing wave to bounce back



Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. 14 (1971); Detweiler PRD22:2323 (1980)

Cardoso, Dias, PRD70 (2004) ; Brito, Cardoso, Pani, arXiv:1501.06570

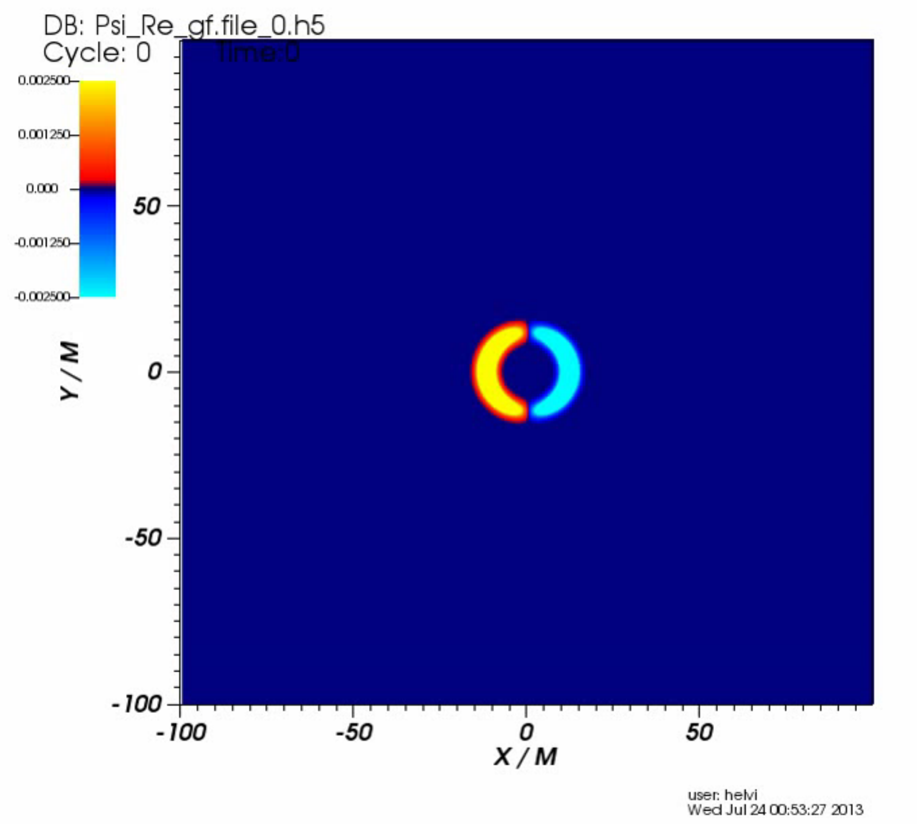


$$\tau \sim 100 \left(\frac{10^6 M_\odot}{M}\right)^8 \left(\frac{10^{-16}\text{eV}}{\mu}\right)^9 \text{ seconds}$$

Massive "states" around Kerr are linearly unstable

Damour et al '76; Detweiler PRD22:2323 (1980); see review Brito et al arXiv:1501.06570

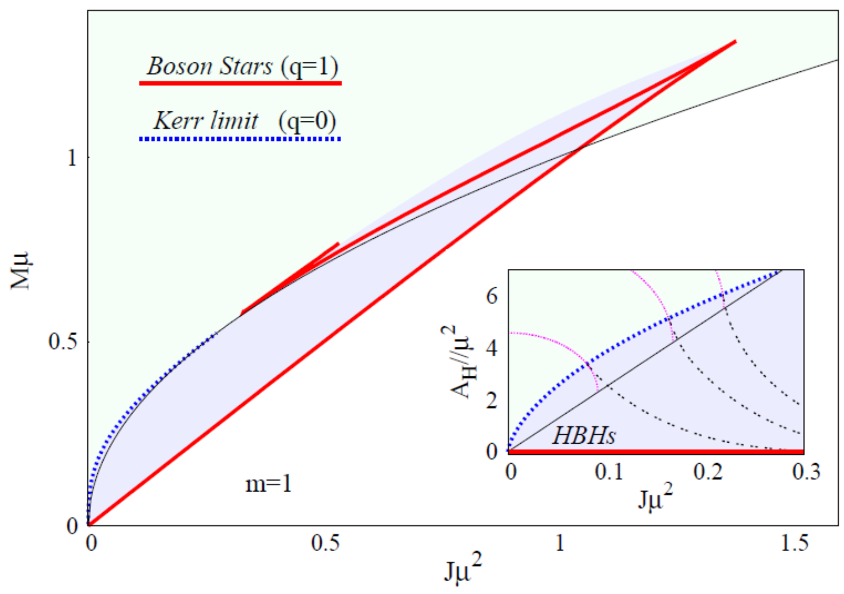
Final state I: almost-hairy BHs

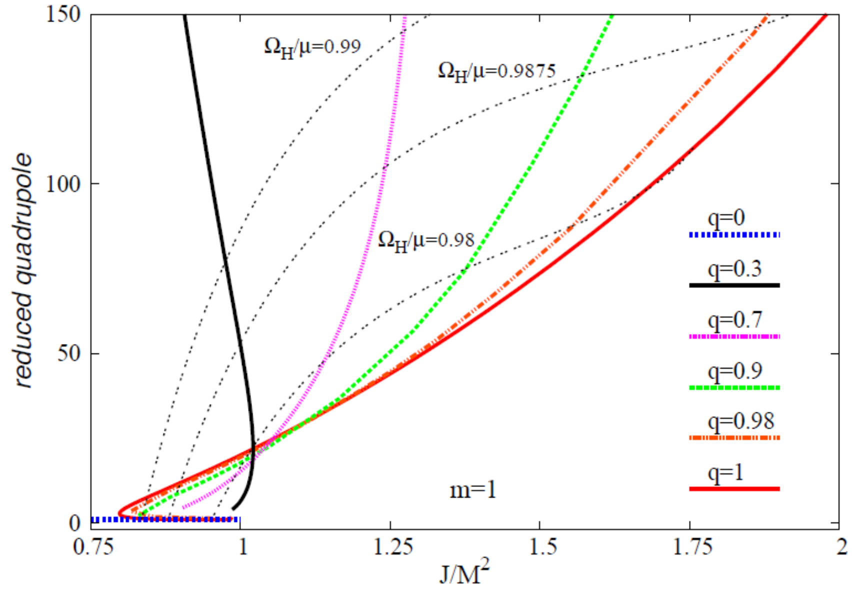


$$\Psi = e^{-i\omega t} Y_{lm}(\theta,\, \phi)\psi(r)$$
$$T^{\mu\nu} = -g^{\mu\nu}\left(\Psi^{*}_{,\alpha}\Psi^{,\alpha} + \mu^2\Psi^{*}\Psi\right) + \Psi^{*,\mu}\Psi^{,\nu} + \Psi^{,\mu}\Psi^{*,\nu}$$

Okawa et al PRD89, 104032 (2014)

Final state II: hairy black holes?





Herdeiro, Radu, *PRL112: 221101 (2014)*

Are themselves unstable in parts of the parameter space

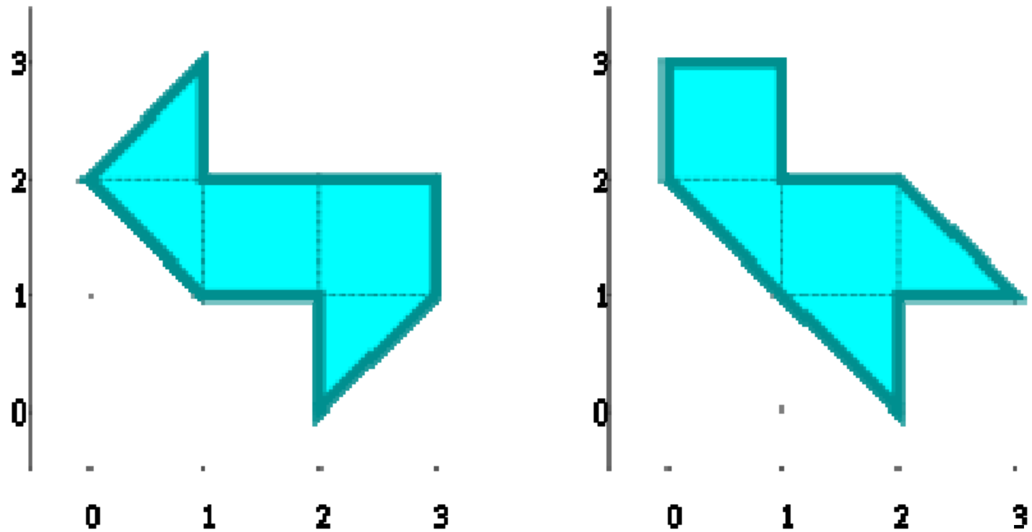
Brito, Cardoso, Pani, arXiv:1501.06570

“Can one hear the shape of a drum?”

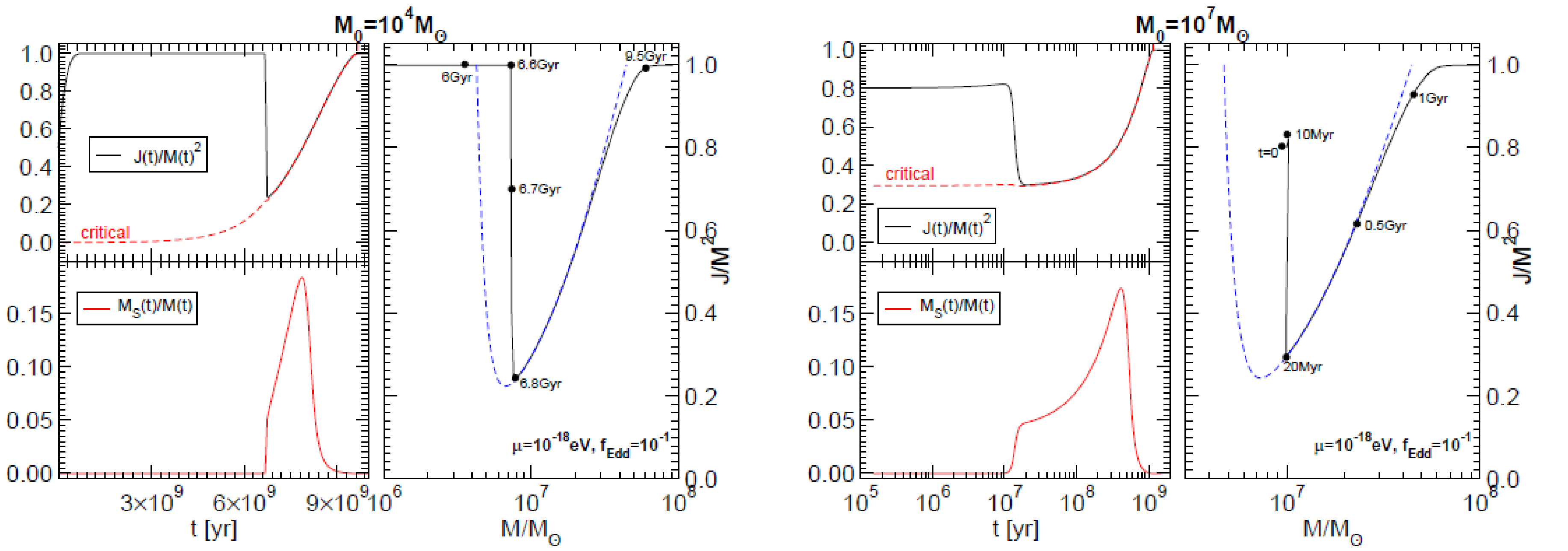
Mark Kac, American Mathematical Monthly, *1966*

$$A = (2\pi)^d \lim_{R\to\infty} \frac{N(R)}{R^{d/2}}$$

H. Weyl 1911



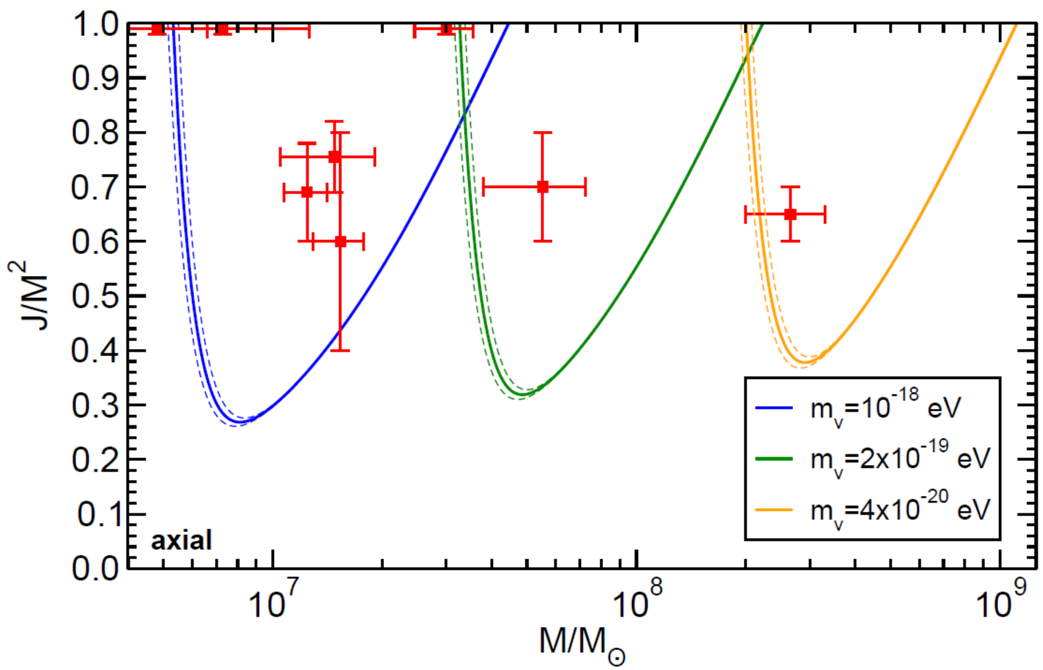
Gordon, Webb & Wolpert, Inventiones Mathematicae 1992



Brito, Cardoso, Pani arXiv:1411.0686

Bounding the boson mass

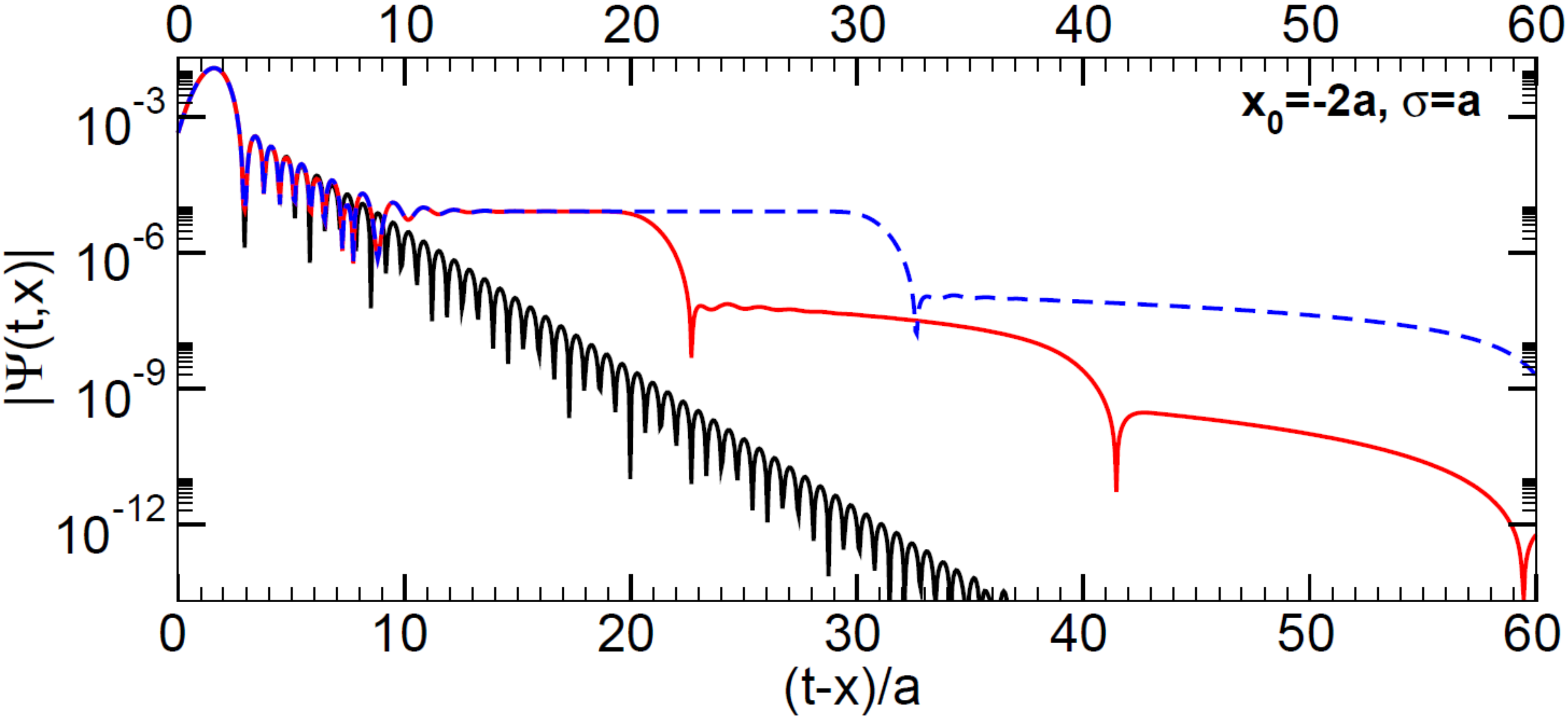
Pani et al PRL109, 131102 (2012)



Bound on photon mass is model-dependent: details of accretion disks or intergalactic matter are important...but gravitons interact very weakly!

$$m_g < 5 \times 10^{-23}\,\mathrm{eV}$$

Brito et al PRD88:023514 (2013); Review of Particle Physics 2014


0
10
20
30
40
50
60
10^{-3}
10^{-6}
10^{-9}
10^{-12}
$|\Psi(t,x)|$
x_0=-2a, σ=a
(t-x)/a

Accretion:

$$\dot{M}_{\rm ACC} \equiv f_{\rm Edd}\dot{M}_{\rm Edd} \sim 0.02 f_{\rm Edd}\frac{M(t)}{10^6 M_\odot} M_\odot {\rm yr}^{-1}$$

$$\dot{J}_{\rm ACC} \equiv \frac{L(M,J)}{E(M,J)}\dot{M}_{\rm ACC}$$

Gravitational-wave emission:

$$\dot{E}_{\rm GW} = \frac{484+9\pi^2}{23040}\left(\frac{M_S^2}{M^2}\right)(M\mu)^{14}$$

$$\dot{J}_{\rm GW} = \frac{1}{\omega_R}\dot{E}_{\rm GW}$$

Yoshino, Kodama PTEP 2014 (043E02); Brito et al CQG32 (2015) 13, 134001

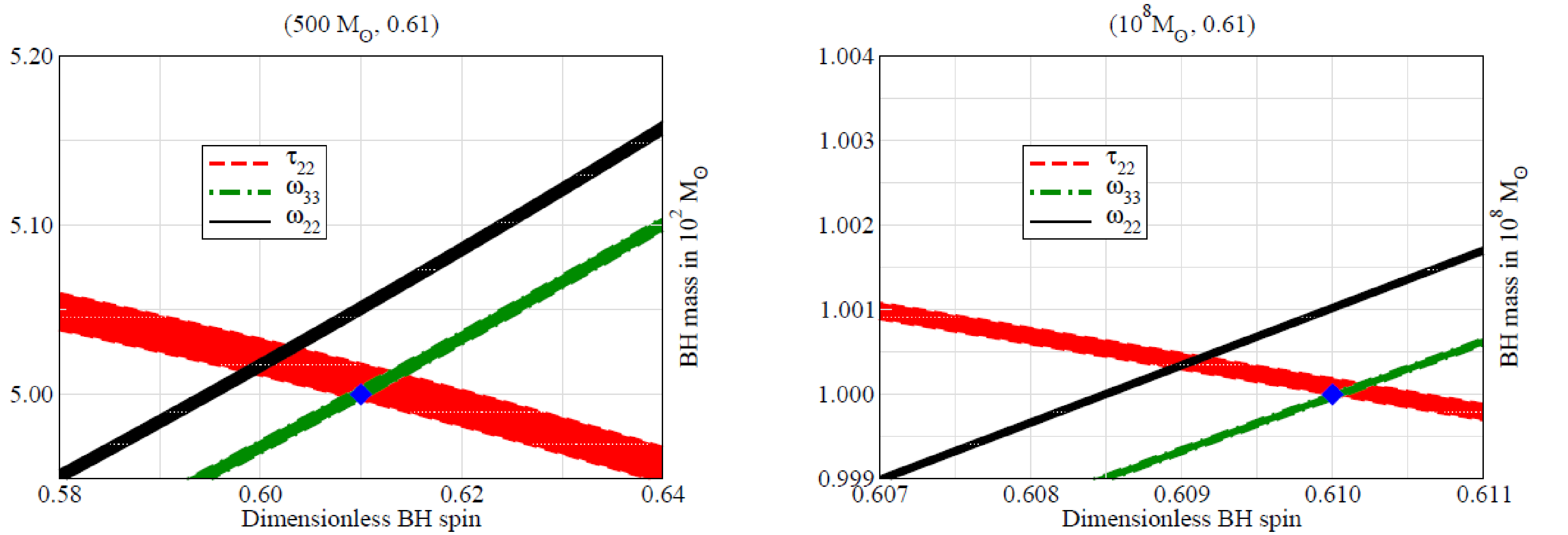


Figure 10. Projections in the (M, j)-plane of the 90% confidence limits on ω_{22}, τ_{22} and ω_{33} (blue, blue dotted and red lines, respectively) for non-GR injections of $M = 500\,M_\odot$ (left at 125 Mpc; with $\Delta\hat{\omega}_{22} = -0.01$, SNR $= 2\,867$) and and $M = 10^8\,M_\odot$ (right at 1 Gpc; with $\Delta\hat{\omega}_{22} = -0.001$, SNR $= 115\,130$). The injected value is denoted in each case by a diamond. ET and NGO (Gossan S, Veitch J and Sathyaprakash 2012)