Quantum corpuscular corrections to the Newtonian potential

Roberto Casadio
DIFA - University of Bologna
INFN-FLAG

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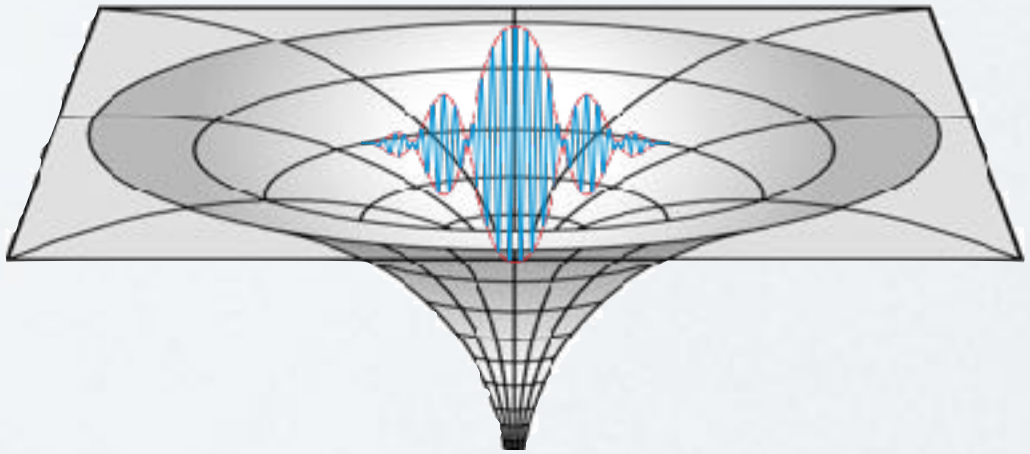


Plan of the talk

1. Motivation: understanding gravitational collapse and black hole formation in quantum theory

2. Results I: R.C., A. Giugno, A. Giusti, *Matter and gravitons in the gravitational collapse*, PLB 763 (2016)337 [arXiv: 1606.04744]

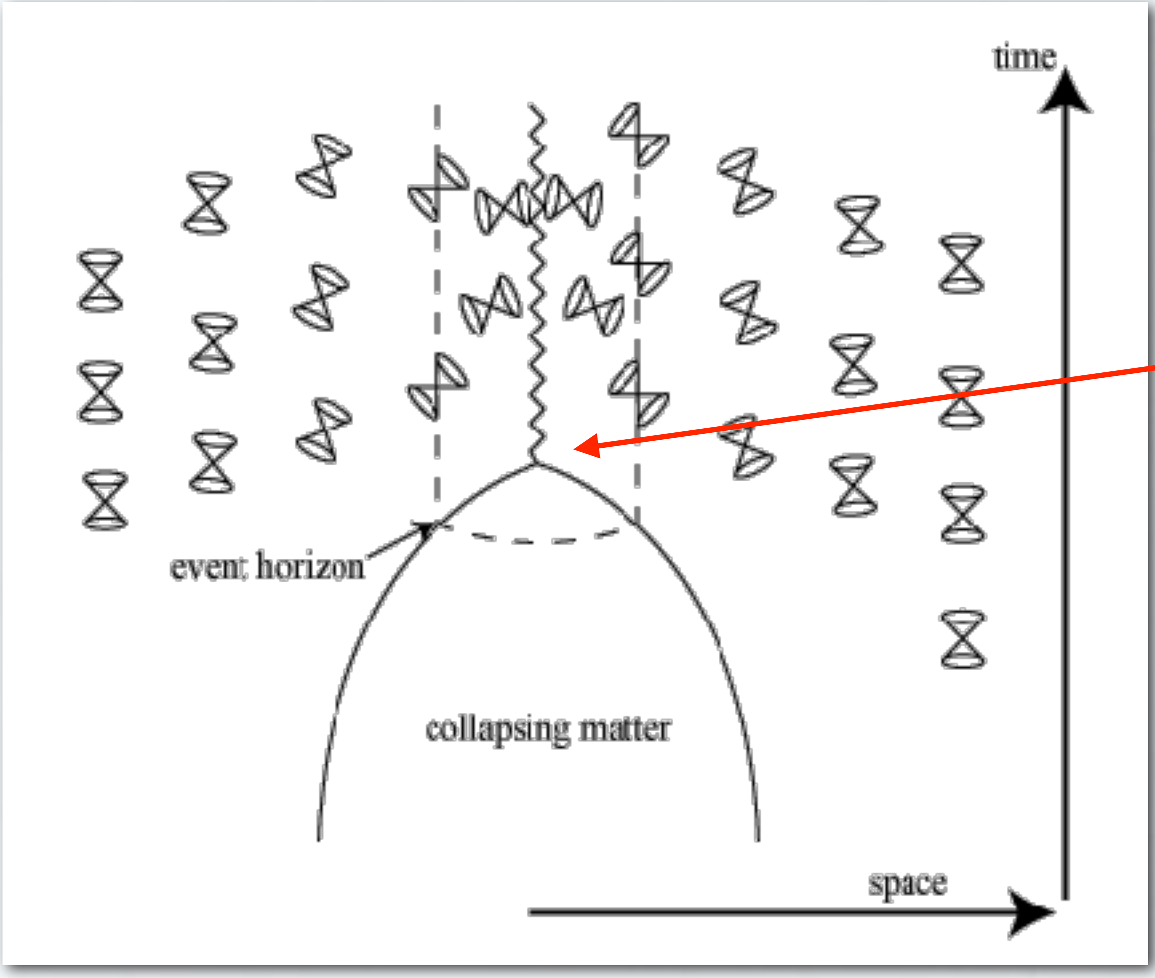
3. Results II: R.C., A. Giugno, A. Giusti, M. Lenzi, *Quantum corpuscular corrections to the Newtonian potential,* arXiv:1702.05918

4. Outlook

1) Gravitational collapse

Standard CLASSICAL picture = General Relativity
CLASSICAL matter and CLASSICAL* space-time

Oppenheimer-Snyder model



Central Singularity
(GR singularity theorems)

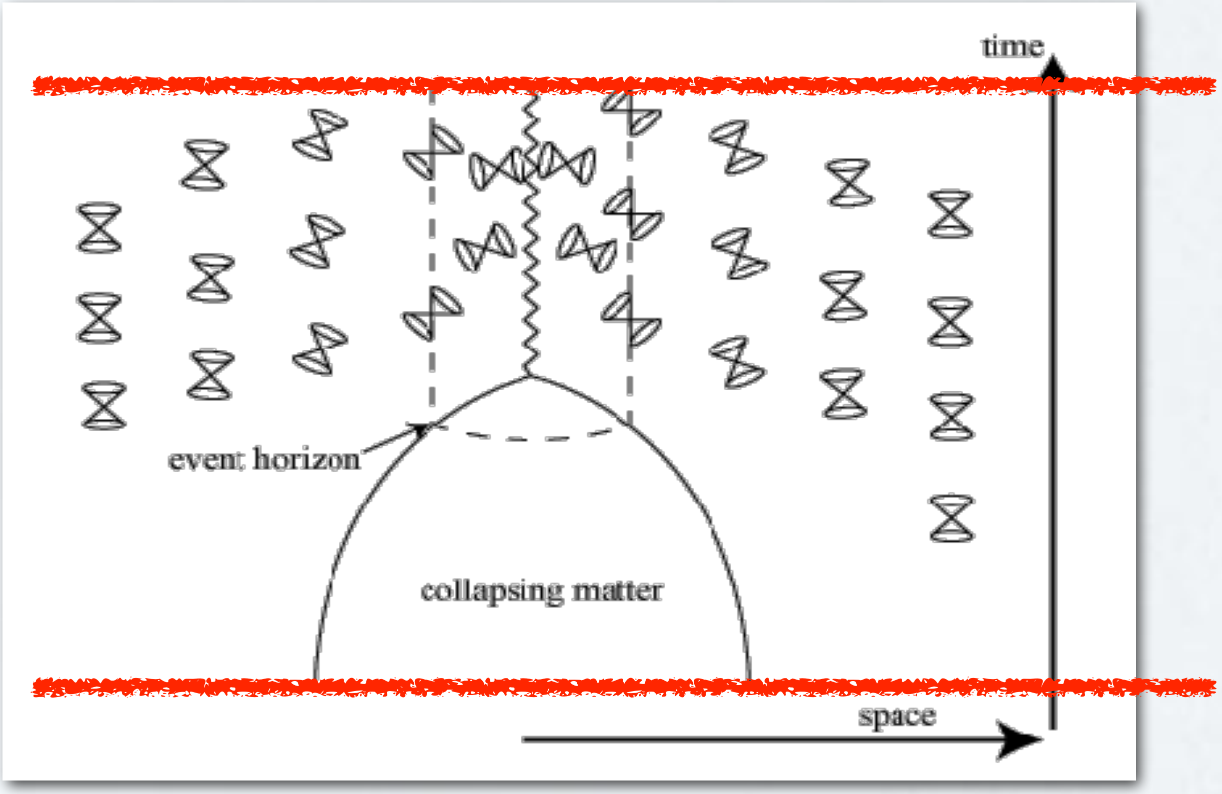
But matter is **QUANTUM**…!

* geometrical

1) Gravitational collapse

Standard SEMICLASSICAL picture = QFT on curved space-time

Background: CLASSICAL matter and CLASSICAL space-time
Foreground: QUANTUM particles



$$|0; t = +\infty\rangle$$

$$e^{-\frac{i}{\hbar} \int \hat{H}\, dt}$$

$$|0; t = -\infty\rangle$$

$$|0; t = +\infty\rangle = \sum \text{excitations} = \text{Hawking radiation}$$

Hawking radiation ~ physical paradoxes ~ perhaps wrong background
or Quantum Gravity…?

1) Gravitational collapse

Maybe we need **Quantum Gravity** … but what might it really be?

$$\sqrt{\hbar\, G_{\rm N}} = \ell_{\rm p} \simeq 10^{-35}\ {\rm m}$$

Numerology or…?

$$\sqrt{\hbar / G_{\rm N}} = M_{\rm p} \simeq 10^{19}\ {\rm GeV}$$

$$\frac{\hbar}{M_{\rm p}} \simeq G_{\rm N}\, M_{\rm p}$$

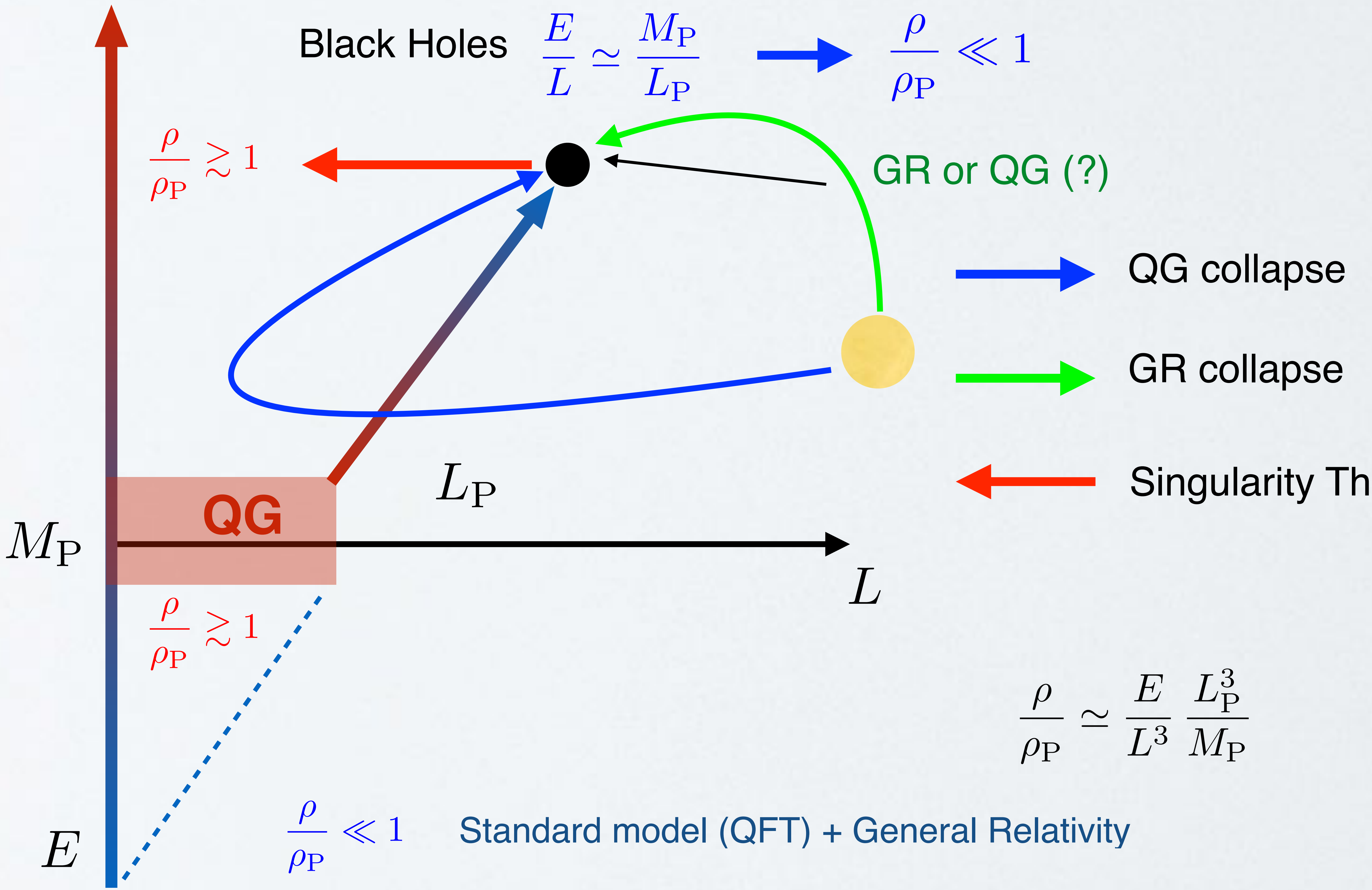
Q(FT) scale (Compton length)

(Strong) Gravity scale (Schwarzschild radius)

Quantum Gravity (Black Holes)

$$G_{\rm N} = \frac{\ell_{\rm p}}{m_{\rm p}}$$

1) Gravitational collapse



1) Gravitational collapse

Question: Quantum Gravity = Planck density or larger spatial scale?

1) Quantum Physics = Compton length or larger scale?

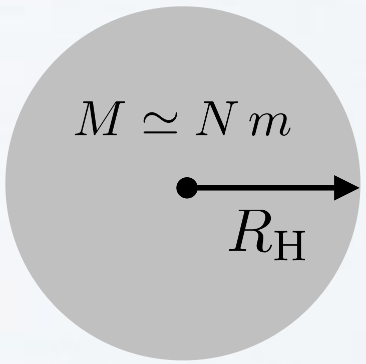
a) atoms and molecules
b) macroscopic condensates
c) …

4) Gravitation = gravitational radius or smaller scale?

a) asymptotic safety scenarios
b) non-commutative space-times
c) …

Poor man answer: Compare classical models with their quantum version

BH = self-sustained BEC of gravitons (effective mass $m = -\epsilon_G$)



$$U \simeq -m\, \frac{\ell_p}{m_p}\, \frac{M \sim N\, m}{r \sim \lambda_m \sim \frac{\ell_p\, m_p}{m}} \simeq -m\, N\, \left(\alpha \sim \frac{m^2}{m_p^2}\right)$$

$$E \simeq m + U \simeq 0$$ "marginally bound"

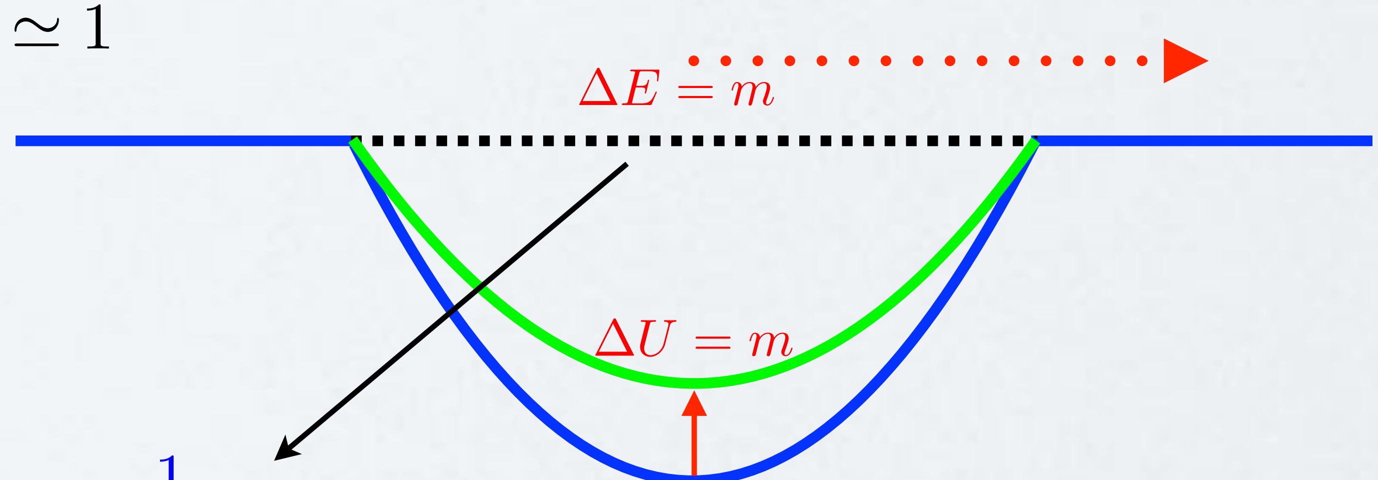
$$R_H \simeq \sqrt{N}\, \ell_p$$

$$m \simeq \frac{m_p}{\sqrt{N}}$$

N>>1

Hawking quantum

$$N\, \alpha \simeq 1$$



$$\Gamma \sim \frac{1}{N^2}\, N^2\, \frac{1}{\sqrt{N}\, \ell_p}$$

Hawking evaporation: $$\dot{M} \simeq m\, \frac{\dot{N}}{\sqrt{N}} \sim -m\, \frac{\Gamma}{\sqrt{N}} \sim -\frac{m_p^3/\ell_p}{M^2}$$

2) Baryons and Gravitons

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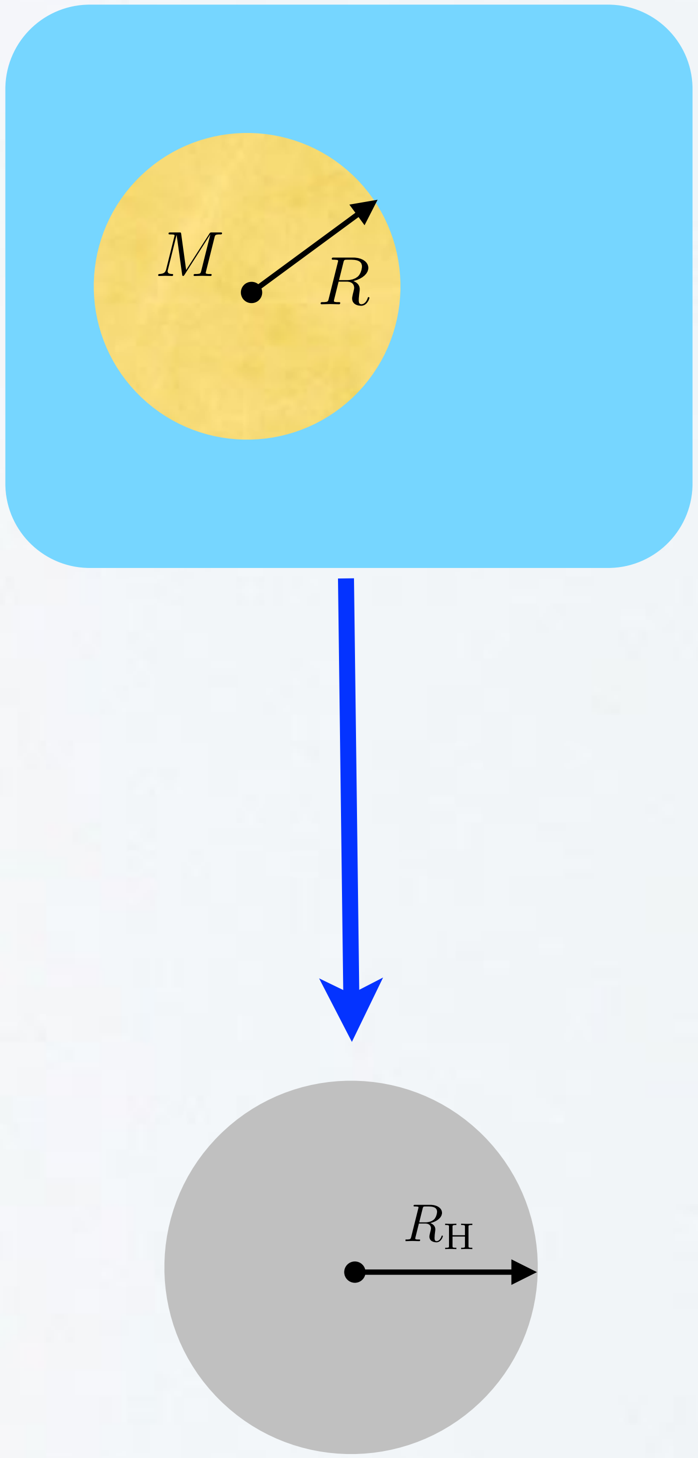
Star



Black Hole

"Energy balance": $H \equiv H_{\rm B} + H_{\rm G} = M$

$$H_{\rm B}(\infty) = \mu\, N_{\rm B} \simeq M$$

$$H = M + K_{\rm B}(R) + U_{\rm BG}(R) + U_{\rm BB}(R) + U_{\rm GG}(R)$$

kinetic ($K_{\rm B}$, $U_{\rm BG}$) — pressure ($U_{\rm BB}$, $U_{\rm GG}$)

$$U_{\rm BG}(R) \simeq N_{\rm B}\,\mu\,\phi_{\rm N}(R) \simeq -N_{\rm B}\,\mu\,\frac{\ell_{\rm p}\,M}{m_{\rm p}\,R} = -\frac{M^2\,\ell_{\rm p}}{m_{\rm p}\,R}$$

Newton

$$U_{\rm BG}(R) \simeq N_{\rm G}\,\epsilon_{\rm G}(R)$$

Coherent state

$$N_{\rm G} \simeq \frac{M^2}{m_{\rm p}^2} \sim \frac{R_{\rm H}^2}{\ell_{\rm p}^2}$$

$$\epsilon_{\rm G} \simeq -\frac{\ell_{\rm p}}{R}\,m_{\rm p}$$

$$U_{\rm GG}(R) \simeq N_{\rm G}\,\epsilon_{\rm G}(R)\,\phi_{\rm N}(R) \simeq N_{\rm G}\,\frac{M\,\ell_{\rm p}^2}{R^2}$$

← GR correction!

$$U_{\rm GG}(R_{\rm H}) \simeq -U_{\rm BG}(R_{\rm H}) \simeq M$$

→

$$R_{\rm H} \simeq \sqrt{N}\,\ell_{\rm p}$$

$$m \simeq \frac{m_{\rm p}}{\sqrt{N}}$$

"marginally bound"

$$K_{\rm B}(R_{\rm H}) + U_{\rm BB}(R_{\rm H}) \simeq 0$$

Einstein-Hilbert action:

$$S_{\rm EH} = \int d^4x\,\sqrt{-g}\left(\frac{m_{\rm p}}{16\,\pi\,\ell_{\rm p}}\,\mathcal{R} + \mathcal{L}_{\rm M}\right) \qquad \mathcal{L}_{\rm M} = \rho$$

1) Weak field: $g_{\mu\nu} = \eta_{\mu\nu} + \epsilon\,h_{\mu\nu}$

2) Static non-relativistic motion: $h_{\mu\nu} \simeq h_{00} = -2\,V$

3) De Donder gauge: $0 = 2\,\partial^\mu h_{\mu\nu} - \partial_\nu h \simeq \partial_t V$

4) Fierz-Pauli and some guessing …

Effective scalar action:

$$S[V] = 4\,\pi \int \epsilon\,dt \int_0^\infty r^2\,dr \left\{\frac{m_{\rm p}}{8\,\pi\,\ell_{\rm p}}\,V\,\triangle V - \rho\,V + \frac{\epsilon}{2}\left[\frac{m_{\rm p}}{4\,\pi\,\ell_{\rm p}}\,(V')^2 + V\,\rho\right]V\right\}$$

A) time measure → C) Post-Newtonian

B) Newtonian

C) Post-Newtonian

3) Post-Newtonian effective theory

ArXiv:1702.05918

A) time measure:

$$ds^2 = -\left(1 - \frac{2\,M}{\tilde{r}}\right) d\tilde{t}^2 + \left(1 - \frac{2\,M}{\tilde{r}}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2\, d\Omega^2$$

Radial geodesic fall:

$$\frac{d^2\tilde{r}}{d\tau^2} = -\frac{M}{\tilde{r}^2}$$

From area to distance:

$$dr = \frac{d\tilde{r}}{\sqrt{1 - \frac{2\,M}{\tilde{r}}}}$$

(… irrelevant)

i) From proper time to Schwarzschild time:

$$d\tau = \left(1 - \frac{2\,M}{\tilde{r}}\right) \frac{m}{E}\, d\tilde{t}$$

ii) From Schwarzschild time to static proper time:

$$dt = \left(1 - \frac{2\,M}{r}\right)^{1/2} d\tilde{t}$$

1) Weak field:

2) Static non-relativistic motion:

$$\frac{d^2 r}{d\tilde{t}^2} \simeq -\frac{d}{dr}\left(-\frac{M}{r} + \frac{2\,M^2}{r^2}\right)$$

Standard post-Newtonian from isotropic metric:

$$\frac{d^2 r}{dt^2} \simeq -\frac{d}{dr}\left(-\frac{M}{r} + \frac{M^2}{r^2}\right)$$

B) Newtonian potential:

$$H[V_{\rm N}] = -L[V_{\rm N}] = 4\,\pi \int_0^\infty r^2\, dr \left(-\frac{m_{\rm p}}{8\,\pi\,\ell_{\rm p}}\, V_{\rm N}\, \triangle V_{\rm N} + \rho\, V_{\rm N}\right)$$

Field equation: $\triangle V_{\rm N} = 4\,\pi\, \dfrac{\ell_{\rm p}}{m_{\rm p}}\, \rho$

Gravitational potential energy: $U_{\rm N}(r) = 2\,\pi \displaystyle\int_0^r \bar r^2\, d\bar r\, \rho(\bar r)\, V_{\rm N}(\bar r)$

Gravitational potential energy density :

$$J_V(r) = \frac{1}{4\,\pi\, r^2}\, \frac{d}{dr} U_{\rm N}(r) = -\frac{m_{\rm p}}{8\,\pi\,\ell_{\rm p}}\, \left[V_{\rm N}'(r)\right]^2$$

Gravitational potential self-energy: $U_{\rm GG} \sim \displaystyle\int r^2\, dr\, J_V\, V$

C) Post-Newtonian

$$L[V] = -4\,\pi \int_0^\infty r^2\,dr \left[\frac{m_{\rm p}}{8\,\pi\,\ell_{\rm p}}\,(1-4\,q_\Phi\,V)\,(V')^2 + q_{\rm B}\,V\,\rho\,(1-2\,q_\Phi\,V)\right]$$

Field equation:

$$(1-4\,q_\Phi\,V)\,\triangle V = 4\,\pi\,q_{\rm B}\,\frac{\ell_{\rm p}}{m_{\rm p}}\,\rho\,(1-4\,q_\Phi\,V) + 2\,q_\Phi\,(V')^2$$

$$V(r) = V_{(0)}(r) + q_\Phi\,V_{(1)}(r)$$

0 - order:

$$\triangle V_{(0)} = 4\,\pi\,q_{\rm B}\,\frac{\ell_{\rm p}}{m_{\rm p}}\,\rho \qquad q_{\rm B} = 1$$

1 - order:

$$\triangle V_{(1)} = 2\left(V'_{(0)}\right)^2$$

Potential energy:

$$U_{\rm BG} = 2\,\pi\,q_{\rm B}\int_0^\infty r^2\,dr\,\rho\left[V_{(0)} + q_\Phi\left(V_{(1)} - 4\,V_{(0)}^2\right)\right]$$

$$U_{\rm GG} = -3\,q_\Phi\,\frac{\ell_{\rm p}}{m_{\rm p}}\int_0^\infty r^2\,dr\,V_{(0)}\left(V'_{(0)}\right)^2$$

3) Post-Newtonian effective theory

ArXiv:1702.05918

Classical solutions:

0 - order: $\triangle V_{(0)} = 4\,\pi\, q_{\rm B}\, \frac{\ell_{\rm p}}{m_{\rm p}}\, \rho$ → ($\triangle j_0(k\,r) = -k^2\, j_0(k\,r)$) → $\tilde{V}_{(0)}(k) = -4\,\pi\, q_{\rm B}\, \frac{\ell_{\rm p}\,\tilde{\rho}(k)}{m_{\rm p}\,k^2}$

1 - order: $\triangle V_{(1)}(r) = 2\left[V'_{(0)}(r)\right]^2$

Ex1) Point-like source:

$$\rho = M_0\, \delta^{(3)}(\mathbf{x}) = \frac{M_0}{4\,\pi\, r^2}\,\delta(r)$$

(0 + 1) potential: $V \simeq -q_{\rm B}\,\frac{\ell_{\rm p}\, M}{m_{\rm p}\, r} + q_\Phi\, q_{\rm B}^2\, \frac{\ell_{\rm p}^2\, M^2}{m_{\rm p}^2\, r^2}$

$$M = M_0 + q_\Phi\, M_1$$

Arbitrary!

(0 + 1) energy: $U_{\rm BG} \simeq -q_{\rm B}^2\,\frac{\ell_{\rm p}\, M_0\, M}{2\, m_{\rm p}\, r_0} - q_{\rm B}^3\, q_\Phi\, \frac{3\,\ell_{\rm p}^2\, M^3}{2\, m_{\rm p}^2\, r_0^2}$

$\delta(r) \to \delta(r - r_0)$

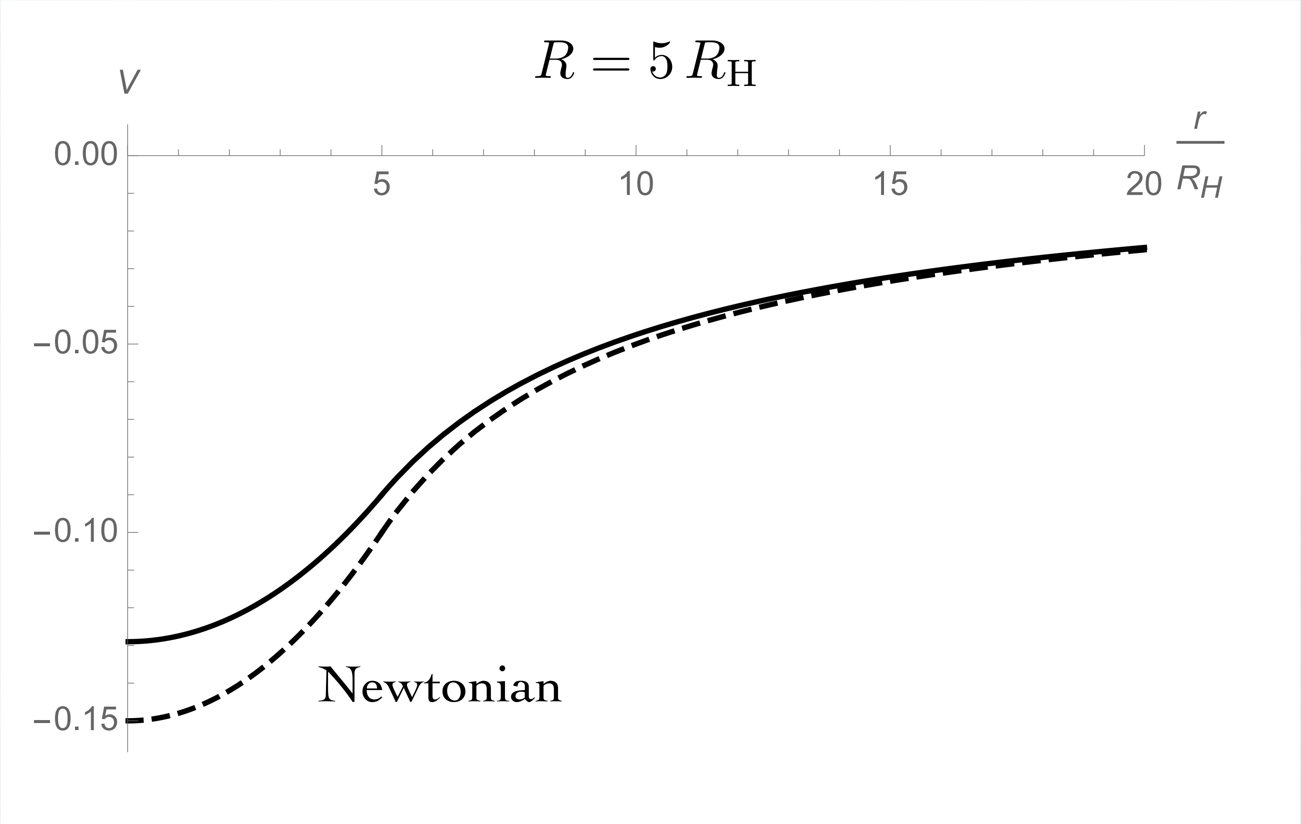
$$U_{\rm GG} \simeq q_{\rm B}^3\, q_\Phi\, \frac{3\,\ell_{\rm p}^2\, M^3}{2\, m_{\rm p}^2\, r_0^2}$$

Post-Newtonian too large!

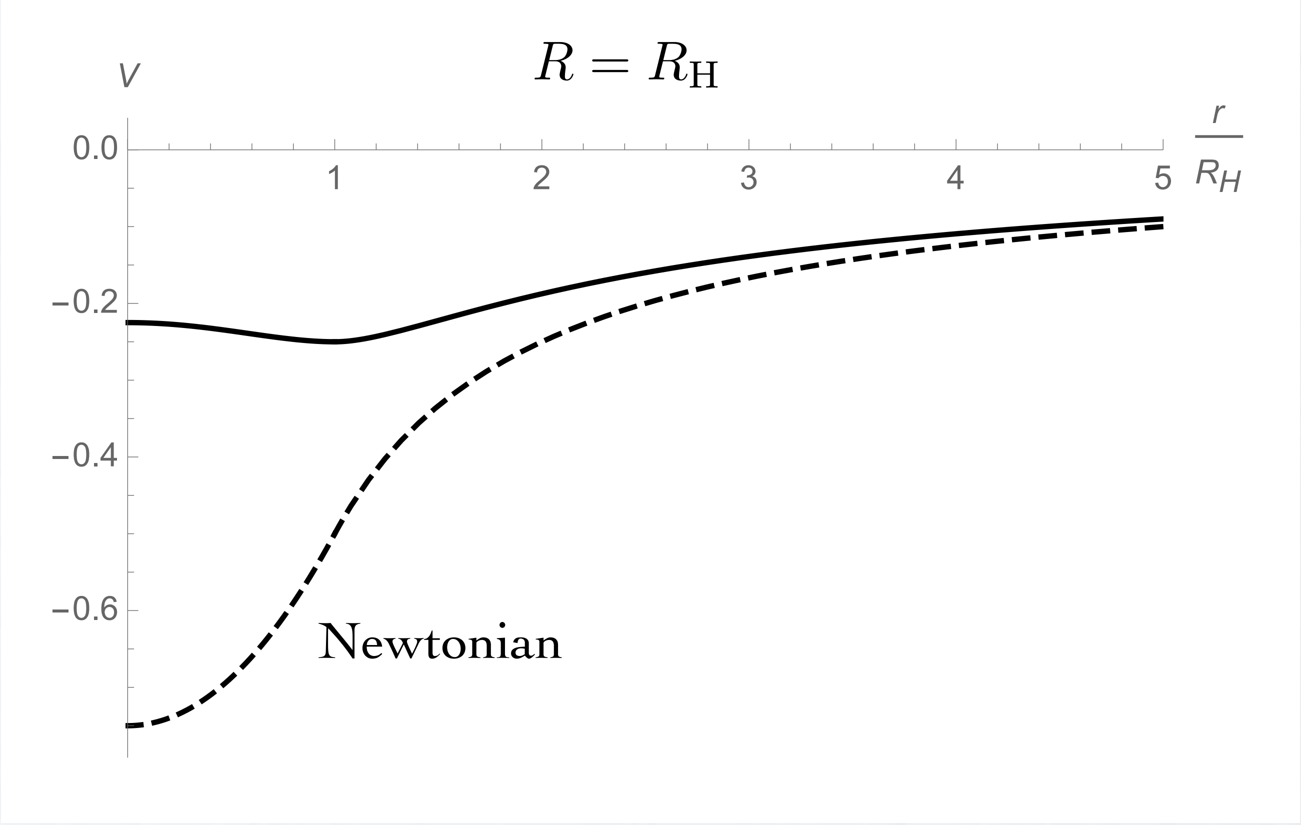
NO maximal packing for point source!

Ex2) Homogeneous star:

$$\rho(r) = \frac{3\,M_0}{4\,\pi\,R^3}\,\Theta(R-r)$$



(0 + 1) potential:



(0 + 1) energy:

$$U(R) \simeq -q_{\mathrm{B}}^2\,\frac{3\,\ell_{\mathrm{p}}\,M^2}{5\,m_{\mathrm{p}}\,R} + q_{\mathrm{B}}^3\,q_\Phi\,\frac{249\,\ell_{\mathrm{p}}^2\,M^3}{175\,m_{\mathrm{p}}^2\,R^2}$$

"marginally bound" for

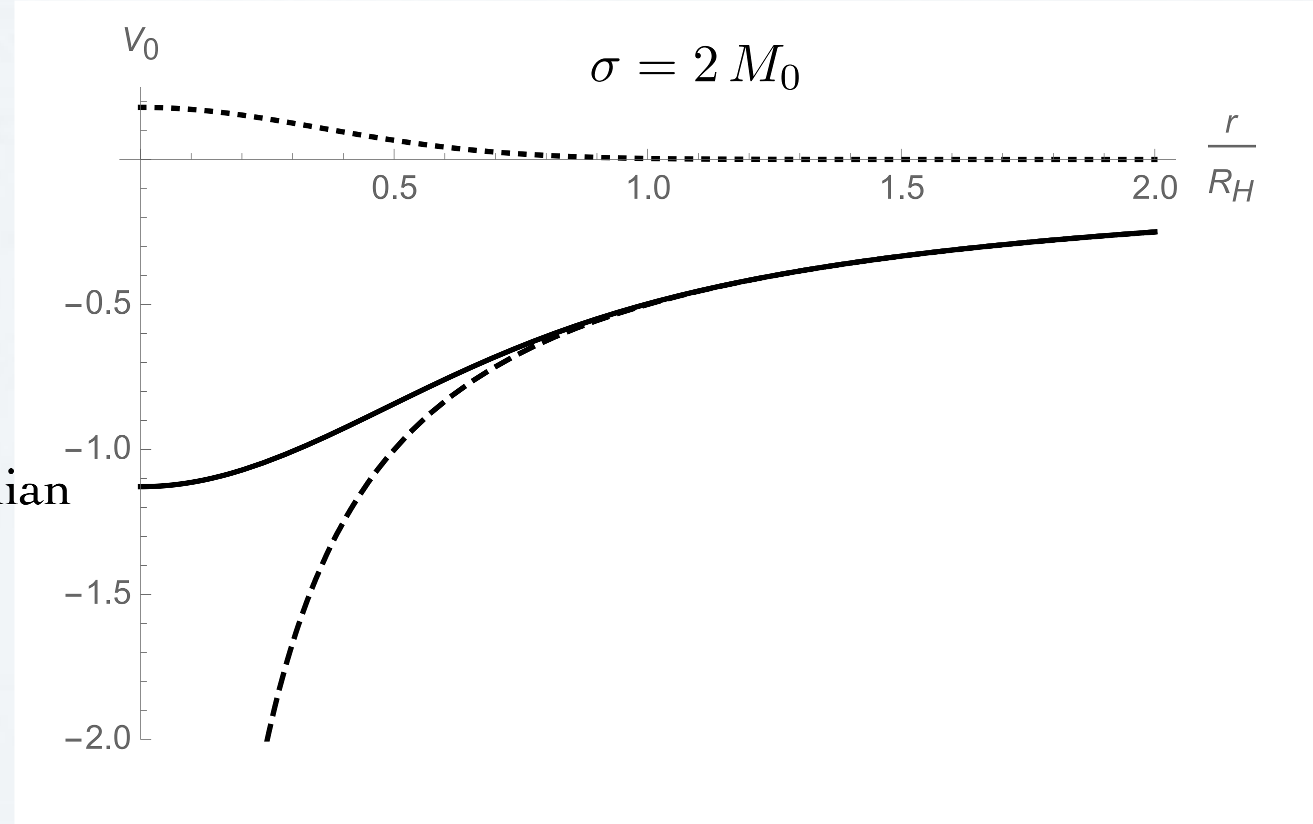
$$R \simeq 1.2\,R_{\mathrm{H}}$$

Ex3) Gaussian star:

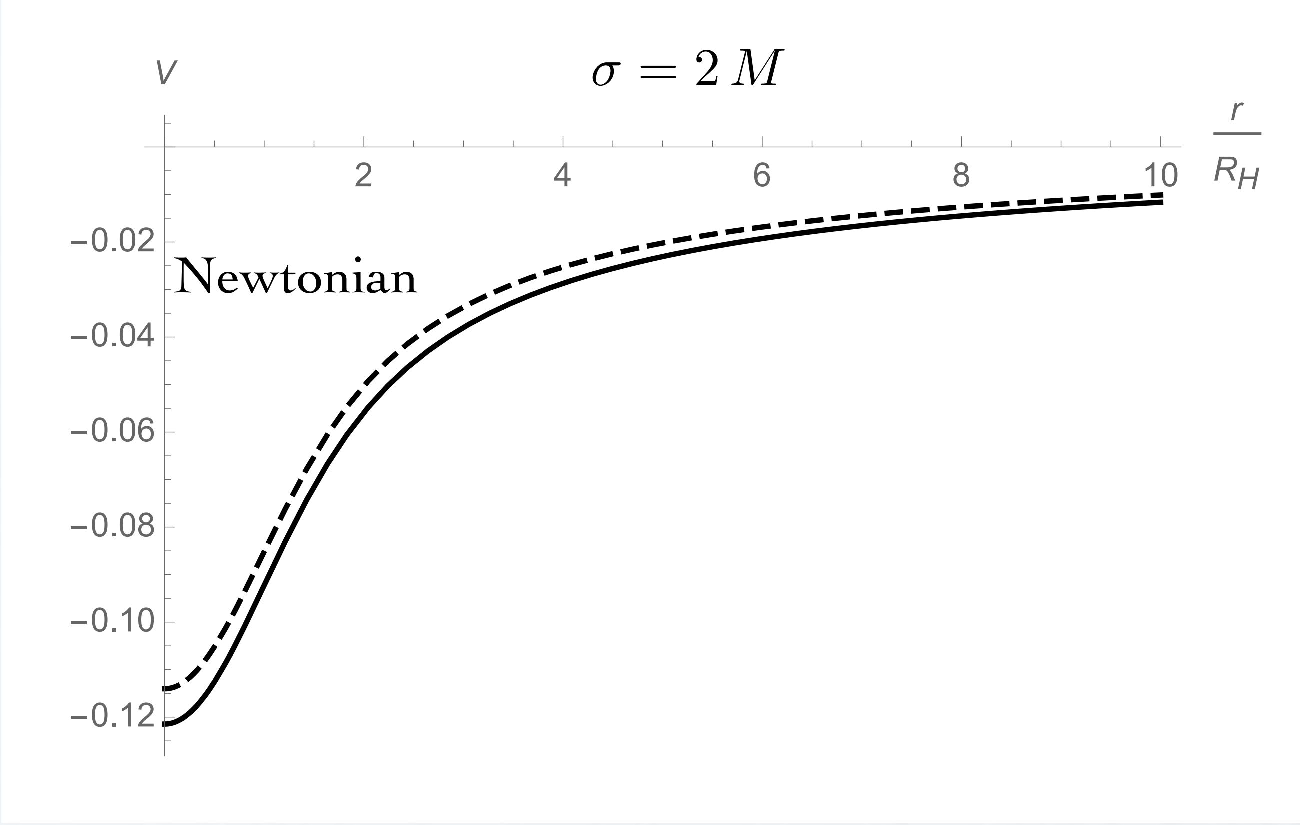
$$\rho(r) = \frac{M_0\, e^{-\frac{r^2}{\sigma^2}}}{\pi^{3/2}\,\sigma^3}$$

(0 + 1) potential:

Newtonian M_0



$M_0 \to M$



Quantum scalar field:

Field equation: $\Box\Phi = 0 \longrightarrow \hat{\Phi}(t,r) = \ell_{\rm p} \int_0^\infty \frac{k^2\, dk}{2\,\pi^2\,\sqrt{2\,k}}\, j_0(k\,r) \left(\hat{a}_k\, e^{i\,k\,t} + \hat{a}_k^\dagger\, e^{-i\,k\,t}\right)$

Commutators: $\left[\hat{a}_p, \hat{a}_k^\dagger\right] = \frac{2\,\pi^2}{k^2}\,\delta(p-k)$

Vacuum: $\hat{a}_k\,|0\rangle = 0$

Newtonian coherent state:

$$\hat{a}_k\,|g\rangle = e^{-i\,k\,t}\,g_k\,|g\rangle$$

$$g_k = \sqrt{\frac{k}{2}}\,\frac{\tilde{V}_c(k)}{m_{\rm p}} = -\frac{q_{\rm B}\,\tilde{\rho}(k)}{m_{\rm p}\,\sqrt{2\,k^3}}$$

Normalisation: $\langle g|g\rangle = 1$

$$\langle g|\,\hat{\Phi}(t,r)\,|g\rangle = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\, j_0(k\,r)\,\tilde{V}_c(k) = V_c(r)$$

3) Post-Newtonian effective theory

ArXiv:1702.05918

Normalisation ~ occupation number:

$$|g\rangle = e^{-\frac{N_G}{2}} \exp\left\{\int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\, g_k\, \hat{a}_k^\dagger\right\} |0\rangle \longrightarrow N_G = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\, g_k^2 = \langle g| \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\, \hat{a}_k^\dagger\, \hat{a}_k\, |g\rangle$$

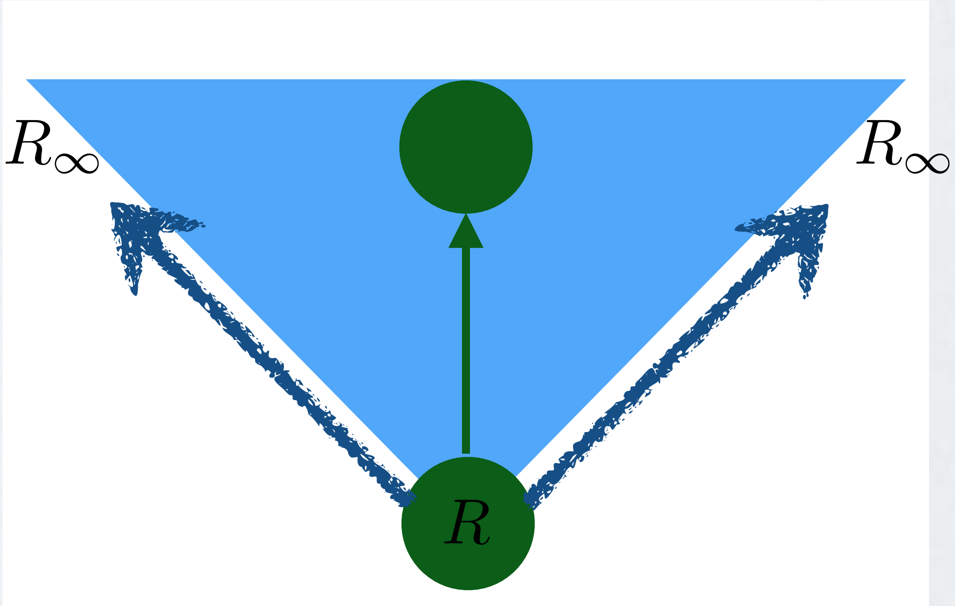
UV cut-off — Size of static field

Typically:
$$N_G \sim \frac{M^2}{m_p^2} \ln\left(\frac{\Lambda}{k_0}\right) \sim \frac{M^2}{m_p^2} \ln\left(\frac{R_\infty}{R}\right)$$

IR cut-off — Source size

$$\frac{dN_G}{N_G} \sim \frac{dM}{M} - \frac{1}{\ln(R_\infty/R)} \frac{dR}{R}$$

Dynamics…



3) Post-Newtonian effective theory

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Post-Newtonian corrections:

$$\triangle V_{(1)} = 2\,\frac{\ell_{\rm p}}{m_{\rm p}}\,\langle g|\left(\hat{\Phi}'\right)^2|g\rangle = 2\left(V'_{(0)}\right)^2 + \cancel{J_0}$$

Vacuum

New coherent state:

$$\hat{a}_k\,|g'\rangle \simeq g_k\,|g\rangle + q_\Phi\,\delta g_k\,|g\rangle$$

$$\sqrt{\frac{\ell_{\rm p}}{m_{\rm p}}}\,\langle g'|\,\hat{\Phi}\,|g'\rangle \simeq V_{(0)} + q_\Phi\,V_{(1)}$$

One-mode occupation: $\bar{k} \sim R^{-1}$ → $\delta g_{\bar{k}} \sim -\ell_{\rm p}\,\bar{k}^{5/2}\,g_{\bar{k}}^2$

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Typically:

$$\delta g_{\bar{k}} \sim \frac{R_{\rm H}}{R}\,g_{\bar{k}}$$

→ OK for large source

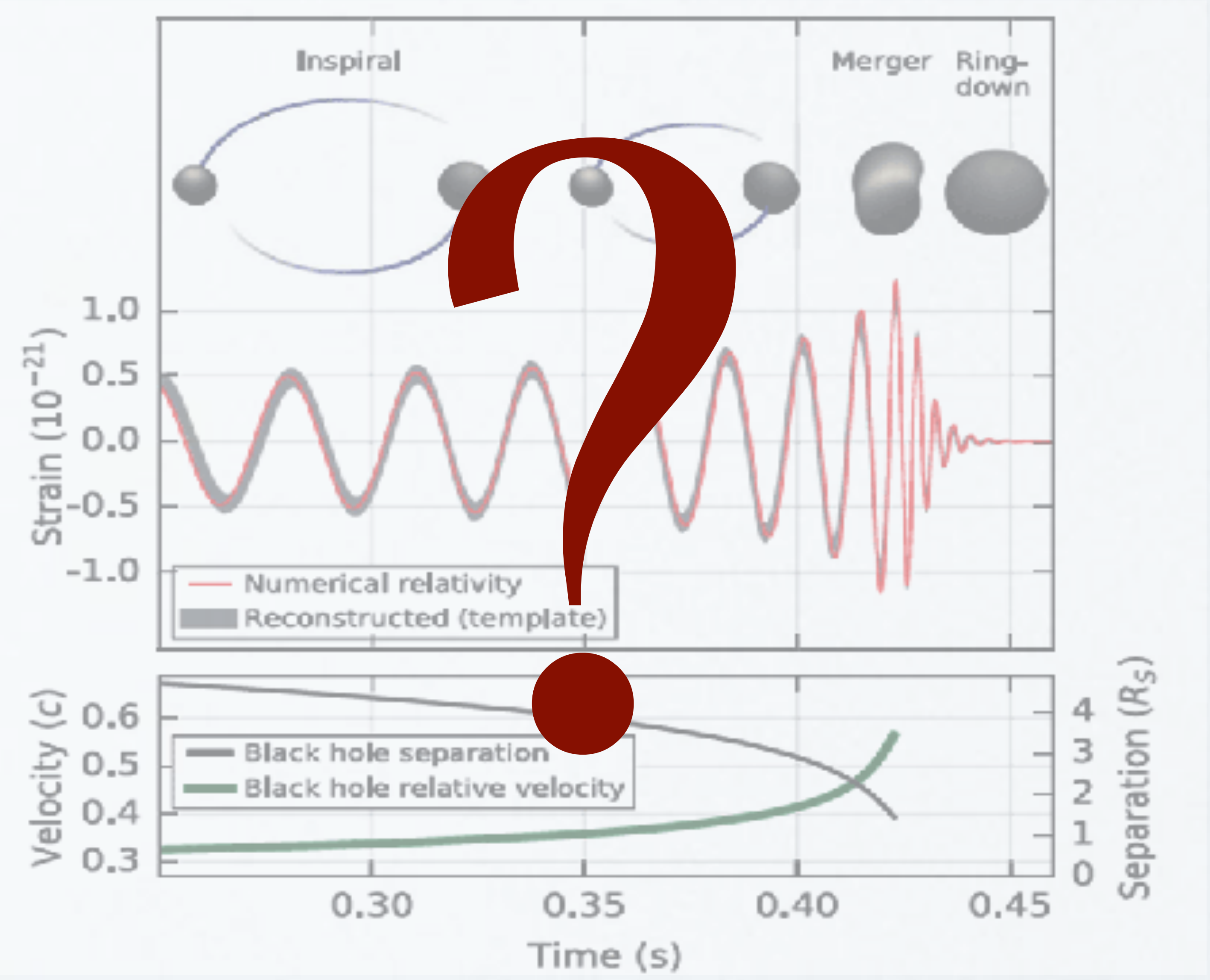
→ For black hole formation?

4) Outlook

1) Quantum post-Newtonian corrections:
    a. phenomenological consequences for (neutron) stars
    b. phenomenological consequences for black hole formation
2) Thermal fluctuations:
    a. quantum matter effects
    b. Hawking radiation
3) Inner quantum structure of black holes

Outlook

Black holes exist = ?extended quantum objects?
(new near horizon phenomenology?)



Thank you!