Statistical black hole entropy and AdS_2/CFT_1

Asymptotic symmetries and dual boundary theory

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17.5.2017

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Quantum gravity & BH thermodynamics

Looking for a quantum gravity theory

$D = 4$ metric degrees of freedom $=$ 10 components - 4 diffeos
-4 non-dynamical $=$ 2 d.o.f

- Gravity theory in $4D$ is **not a perturbative renormalizable theory** ($[G] = -2$ in mass units).
- We can interpret thoery of gravity as an *Effective field theory*:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \{-2\Lambda + \mathcal{R} + c_1 \mathcal{R}^2 + c_2 R^{\mu\nu} R_{\mu\nu} + c_3 R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + \dots\}$$

The theory needs an UV completion

- **No local observables** ;
- **The graviton is not composite (Weinberg-Witten theorem)**
- **Emergent spacetime** ...

BH Thermodynamics

- **Zero Law**: the surface gravity κ is costant over the horizon;
- **First law**:for any stationary black hole with mass M, angular momentum J and charge Q, it turns out to be
$$\delta M = \frac{\kappa}{8\pi G}\delta\mathcal{A} + \Omega\delta J + \phi\delta Q$$
where Ω is the angular velocity and ϕ is electrostatic potential.
- **Second law**: The Area $\mathcal{A}$ of the event horizon of a black hole never decreases
$$\delta\mathcal{A} \geq 0$$
- **Third law**:It is impossible to reduce, by any procedure, the surface gravity κ to zero in a finite number of steps.

The correspondence between thermodynamic and black hole mechanics is complete if we identify:

$$E \to M \qquad S \to \mathcal{A} \qquad T \to \kappa$$

- Moreover Bekenstein found:
$$S = \eta\frac{\mathcal{A}}{\hbar G}$$

Hawking radiation

$$T_{hawking} = \frac{\kappa}{2\pi} = \frac{\hbar c^3}{8\pi GM} \sim 6 \times 10^{-8} \frac{M}{M_\odot}$$

Can Hawking radiation be observed?

- For stellar mass black hole eight orders of magnitude smaller than cosmic microwave background;
- More important for primordial black holes;
- Analogue of Hawking radiation in condensed matter system.

The many derivations of Hawking radiation

- Canonical quantization in curved space time (Hawking, 1975);
- Path integral derivation (Hartle and Hawking, 1976);
- KMS condition (Bisognano and Wichmann,1976);
- Gravitational istantons(Gibbons and Hawking,1977);
- Tunneling trough the horizon (T.Damour and R.Ruffini,1976; M.K. Parikh and F.Wilczek, 2000);

Black hole entropy

$$S_{BH} = \frac{\mathcal{A}}{4\hbar G} \qquad S_{BH} \sim 10^{90} \left(\frac{M}{10^6 M_\odot} \right)^2$$

- *No hair theorem(s)*: Stationary, asymptotically, flat black hole solutions to general relativity coupled to electromagnetism that are nonsingular outside the event horizon are fully characterized by the parameters of mass, charge and spin.

$$S = -\sum_n p_n \ln p_n$$

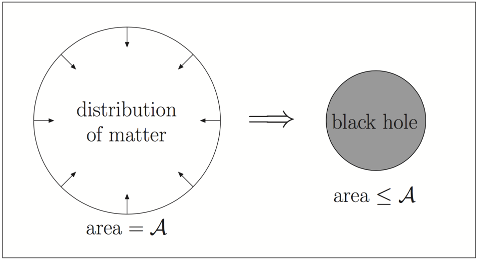
Why classical black holes have entropy?

- *Problem of universality*: A great many different models of black hole microphysics yeld the same thermodynamical proprieties;
- *Loss information paradox*: black holes evaporate, emitting Hawking radiation, which contains less information than the one that was originally in the spacetime, therefore information is lost.

AdS/CFT correspondence

Holographic principle

- **('t Hooft and Susskind)** A bulk theory with gravity describing a macroscopic region of space is equivalent to a boundary theory without gravity living on the boundary of that region;
- Susskind considered an approximately spherical distribution of matter that is not itself a black hole and that is contained in a closed surface of area $\mathcal{A}$



Let us suppose that the mass is induced to collapse to form a black hole, whose horizon area turns out to be smaller than $\mathcal{A}$. The black hole entropy is therefore smaller than $\frac{\mathcal{A}}{4}$ and the generalized second law implies the bound

$$S \leq \frac{\mathcal{A}}{4}$$

AdS/CFT correspondence

- The gauge/gravity correspondence (*duality*) is an exact relationship between any theory of quantum gravity in asymptotically AdS_{d+1} space (**the bulk**) and an ordinary CFT_d without gravity (**the boundary**) ;
- Each field ϕ propagating in a (d+1)-dimensional anti-de Sitter spacetime is related, through a one to one correspondence, to an operator O in a d-dimensional conformal field theory defined on the boundary of that space (**GKPW dictionary**).

$$Z_{grav}[\phi_0^i(x);\partial M] = \Big\langle \exp\left(-\sum_i \int d^d x \phi_0^i(x) O^i(x)\right) \Big\rangle_{CFT \text{ on } \partial M}$$

This is UV complete!!

- The mass of the bulk scalar is related to the scaling dimension of the CFT operator

$$m^2 = \Delta(d-\Delta), \quad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2 l^2}$$

- Thermal states in CFT are dual to black holes in quantum gravity

$$Z[\phi_0; M] = Z_{grav}[\phi_0, \text{boundary} = M]$$

Strong/Weak duality

Aside for certain examples, the corrispondence is well defined and useful only in certain limits. One realization which is understood in great details is:

$$\text{IIB strings on } AdS_5 \times S^5 \;=\; \text{Yang-Mills in 4d with } \mathcal{N} = 4 \,\text{supersymmetry}$$

The large symmetry group of 5d anti-de Sitter space matches precisely with the group of conformal symmetries of the N = 4 super Yang-Mills theory

- gravity side:

$$S_{IIB} \sim \int \sqrt{g}(\mathcal{R} + \mathcal{L}_{matter} + l_s^4 \mathcal{R}^4 + \dots)$$

  parameters: l_s, l_p, l_{AdS}

- *CFT side*: $SU(N)$ gauge fields + matter fields for supersymmetry.

  parameters: g_{YM}, N

$$\lambda = g_{YM}^2 N$$

- The mapping

$$\lambda \sim \frac{l_{AdS}^4}{l_s^4} \qquad \frac{l_{AdS}^{d-1}}{G_N} \sim N^2$$

Hamiltonian & symmetries

Asymptotic symmetries

GR is locally diff invariant, but it is not invariant under diff. that reach the boundary:

$$\int_M \delta_\xi(\sqrt{-g}\mathcal{L}) = \int_{\partial M} dA^\mu \xi_\mu \mathcal{L}$$

$$\text{Asymptotic Symmetry Group} = \frac{\text{symmetries}}{\text{trivial symmetries}}$$

where trivial symmetry is one whose associated vashining conserved charges.

Maxwell theory

$$S = -\frac{1}{4}\int d^4x (F_{\mu\nu}F^{\mu\nu} + A_\mu J^\mu{}_{matter})$$

The action is invariant under trasformations:

$$\delta A_\mu = \partial_\mu \Lambda(x), \qquad \delta\phi = i\Lambda(x)\phi$$

For $\Lambda = const.$

$$\frac{dQ}{dt} = 0 \qquad Q \sim \int_\Sigma d^3x\, J^0_{Matter} \sim \int_{\partial\Sigma} d^2x F_{tr} \quad ASG = U(1)_{global}$$

Hamiltonian in GR

In Hamiltonian formalism the global charges appears as canonical generators of the asymptotic symmetries of the theory. Let us use as canonical variable $h_{ij}(\vec{x},t)\pi_{ij}(\vec{x},t)$ and we parametrize the metric as:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dr)$$

Consider the action

$$I = \frac{1}{16\pi G}\int \sqrt{-g}(\mathcal{R} - 2\Lambda)d^4x + \frac{1}{8\pi G}\int \sqrt{\pm h}\,(\mathcal{K} - \mathcal{K}_0)d^3x,$$

$$I = \int_M d^4x \left[\pi^{ij}\dot{h}_{ij} - N\mathcal{H} - N^i\mathcal{H}_i\right] - \int_{\partial M} d^3x\sqrt{\sigma}u^\mu T_{\mu\nu}\xi^\mu$$

where T^{ij} is the **boundary (Brown-York) stress tensor** :

$$T^{ij} = \frac{1}{8\pi}(K^{ij} - h^{ij}\mathcal{K}) - (\text{background}) \qquad \delta I_{on-shell} = \frac{1}{2}\int_{\partial M}\sqrt{-h}T^{ij}\delta g_{ij}$$

wer can read off the Hamiltonian:

$$H[\xi] = \int_\Sigma d^3x\left[N\mathcal{H} + N^i\mathcal{H}_i\right] - \int_{\partial\Sigma} d^2x\sqrt{\sigma}u^\mu T_{\mu\nu}\xi^\mu$$

Asymptotic symmetries in GR

- The bulk term vanishes on-shell.
- The dynamics leaves ξ unspecified. This corresponds to a choice of time evolution:
$$\{H[\xi], X\} = \mathcal{L}_\xi X$$
- General spacetimes do not have isometries, so no local conserved quantities. Asymptotic symmetries allow to define global conserved charges.
- In General relativity *ASG* is generated by the conserved charges.
$$\{H[\xi], H[\eta]\} = H[[\xi, \eta]] + c(\xi, \eta)$$
- In Minkowski spacetime the ASG is Poincaré group.
- ASG leads to surprise. The isometry group of AdS_d is $SO(D-1, 2)$. A natural guess is that the asymptotic simmetry group is the same. This is not true for $D \leq 3$ (**Brown and Henneaux**)

2D Dilaton gravity

JT model

- In 2D dimensions, the curvature tensor has only one independent component, since all nonzero component can be obtained by simmetry

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\mathcal{R}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) \Longrightarrow G_{\mu\nu} = 0$$

- For formulating a theory endowed with a not trivial degree of freedom gravity theory coupled with a scalar:

$$S = \int d^2x\, \eta(\mathcal{R} - 2\lambda^2).$$

which admits BH solutions:

$$ds^2 = -(\lambda^2 x^2 - a^2)dt^2 + (\lambda^2 x^2 - a^2)^{-1}dx^2, \qquad \eta = \eta_0 \lambda x$$

- The action can be considered a dimensional reduction of an higher dimensional model:

$$ds^2_{(d+2)} = ds^2_{(2)} + \eta^{\frac{2}{d}}\, d\Omega^2(\kappa, d)$$

- Thermodynamics:

$$S = 2\pi\eta_0 a \quad M = \frac{a^2\eta_0\lambda}{2} \quad T = \frac{\lambda a}{2\pi}$$

Isometries

- AdS_2 ($a^2 = 0$) is a maximally symmetric space, it admits, therefore, three Killing vectors generating the $SO(1,2) \sim SL(2,R)$ group of isometries.

$$\chi^{(1)} = \frac{1}{\lambda}\frac{\partial}{\partial t}$$
$$\chi^{(2)} = t\frac{\partial}{\partial t} - x\frac{\partial}{\partial x},$$
$$\chi^{(3)} = \lambda(t^2 + \frac{1}{\lambda^4 x^2})\frac{\partial}{\partial t} - 2\lambda t x\frac{\partial}{\partial x}.$$

- For $a^2 \neq 0$ $SL(2,R)$ symmetry is realized in a different way

$$\delta\eta = \mathcal{L}_\chi \eta = \chi^\mu \partial_\mu \eta.$$

Symmetries of 2D spacetime are broken by the linear dilaton

$$\chi^\mu = F_0 \epsilon^{\mu\nu}\partial_\nu \eta, \quad SL(2,R) \to \mathcal{T}$$

- This symmetry breaking pattern will gives rise to a central charge!!

Asymptotic symmetries

- We define asymptotically AdS_2 if, for $x \to \infty$

$$g_{tt} = -\lambda^2 x^2 + \gamma_{tt} + o(x^{-2}) \quad g_{tx}(t) = \frac{\gamma_{tx}(t)}{\lambda^3 x^3} + o(x^{-5}) \quad g_{xx} = \frac{1}{\lambda^2 x^2} + \frac{\gamma_{xx}(t)}{\lambda^4 x^4} + o(x^{-6})$$

- This asymptotic form is preserved by:

$$\chi^t = \epsilon(t) + \frac{\ddot{\epsilon}(t)}{2\lambda^4 x^2} + \frac{\alpha^t(t)}{x^4} + o(x^{-5}),$$

$$\chi^x = -x\dot{\epsilon}(t) + \frac{\alpha^x(t)}{x} + o(x^{-2})$$

- The asymptotic behaviour of dilaton compatible with these trasformations is:

$$\eta = \eta_0 \left(\lambda\rho(t)x + \frac{\gamma_{\phi\phi}(t)}{2\lambda x} \right) + o(x^{-3})$$

- The boundary fields trasform as:

$$\begin{aligned}
\delta\gamma_{tt} &= \epsilon\dot{\gamma}_{tt} + 2\dot{\epsilon}\gamma_{tt} - \lambda^{-2}\dddot{\epsilon} - 2\lambda^2\alpha^x \\
\delta\gamma_{xx} &= \epsilon\dot{\gamma}_{xx} + 2\dot{\epsilon}\gamma_{xx} - 4\lambda^2\rho\alpha^x \\
\delta\gamma_{\phi\phi} &= \epsilon\dot{\gamma}_{\phi\phi} + \lambda^{-2}\ddot{\epsilon}\dot{\rho} + 2\lambda^2\rho\alpha^x \\
\delta\rho &= \epsilon\dot{\rho} + \dot{\epsilon}\rho.
\end{aligned}$$

- Using these results one finds:

$$J(\epsilon) = \frac{\eta_0}{\lambda}\epsilon\ddot{\bar{\rho}} + \epsilon M = \epsilon\Theta_{tt}$$

- Near the classical solution one finds:

$$\epsilon\delta_\omega\Theta_{tt} = \epsilon\left(\omega\dot{\Theta}_{tt} + 2\dot{\omega}\Theta_{tt}\right) - \frac{\eta_0}{\lambda}\epsilon\dddot{\omega}$$

- This equals the Dirac braket between the charges. Using Fourier series expansions for ϵ, Θ and ω one find:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{C}{12}m^3\delta_{m+n}$$

  with $C = 12\eta_0$ **(Virasoro algebra)**

Central charge and statistical entropy

- Since the physical states of quantum gravity on AdS_2 must form a representation of this algebra:

  quantum gravity on AdS_2 is a conformal field theory with central charge C

- The apparence of the central charge is related to a "soft" breaking of conformal symmetry by the introduction a macroscopic scale into the system. In other words it describes the way a specific system reacts to macroscopic length scales introduced, for instance, by boundary conditions.

$$< T_{cyl.}(w) >= -\frac{c\pi^2}{6L^2}$$

- Cardy formula

$$S = 2\pi\sqrt{\frac{cl_0}{6}}, \quad l_0 = \frac{M}{\lambda} \gg 1$$

  Using this formula one can easily reproduce the black hole entropy.

Boundary theory

Spontaneosly broken symmetry

- AdS_2 vacuum in (euclidean) Poincarré coordinates:

$$ds^2 = \frac{dt^2 + dz^2}{z^2}$$

we want to cutoff the space along a trajectory $(t(u), z(u))$:

$$g_{boundary} = \frac{1}{l^2} \qquad \frac{1}{l^2} = \frac{t'^2 + z'^2}{z^2} \to z = lt' + o(l^3)$$

(a) (b) (c)

Explicitly broken symmetry

- Introducing a dilaton:

$$\eta = \frac{\alpha + \gamma t + \delta(t^2 + z^2)}{z}$$

- The dilaton is diverging near the boundary:

$$\eta_b = \frac{\eta_r(u)}{l}$$

$$\frac{\alpha + \gamma t(u) + \delta(t(u)^2}{t'(u)} = \eta_r(u)$$

We can derive an effective action:

$$I = -\int du\, \eta_r(u) Sch(t, u) \qquad Sch(t, u) = -\frac{t''}{2t'^2} + (\frac{t''}{t'})^2$$

Pseudo Nambu Goldstone modes

- With this action we can reproduce the entropy:

$$\log Z = -I = 2\pi^2 DT \qquad D = \bar{\eta}_r$$

Thank you!!!