



H I G H
E N E R G Y
P H Y S I C S
C O L L O Q U I A



Quasi-real photon exchange in transverse single-spin asymmetries for $\ell p^\uparrow \rightarrow h X$

Carlo Flore (UniCa & INFN - CA)

Outline

- Introduction:

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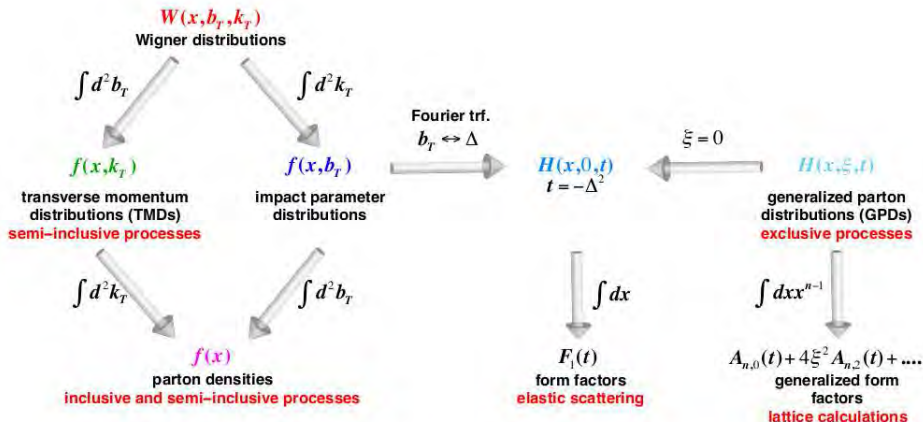
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- Conclusions

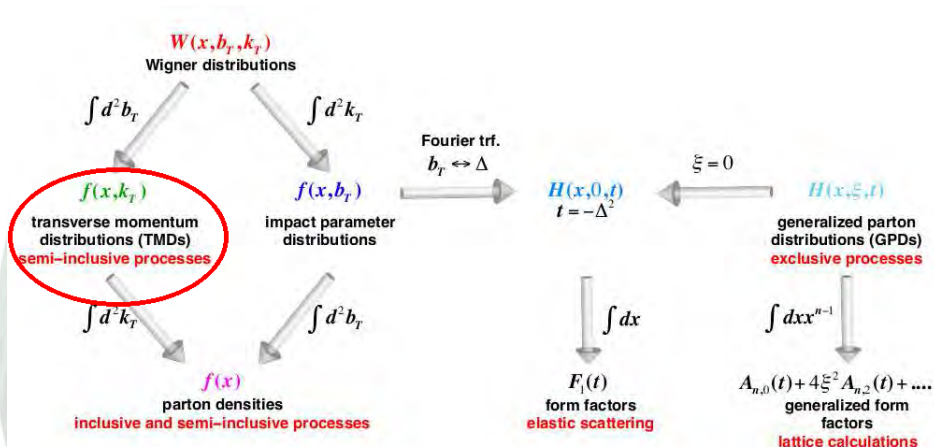
Introduction: TMDs and SSAs definitions

From Umberto's HEP Colloquium..



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- inclusion of \mathbf{k}_\perp effects \Rightarrow from 3 to 8 independent TMD-PDF:

parton pol.

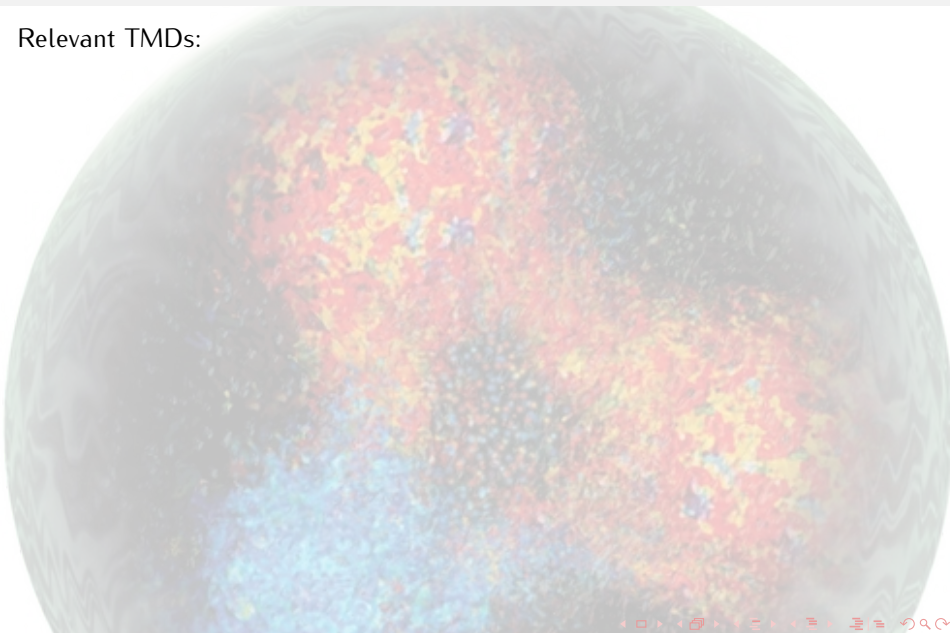
	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	$h_1 \quad h_{1T}^\perp$

nucleon pol.

collinear T-odd

Introduction: TMDs and SSAs definitions

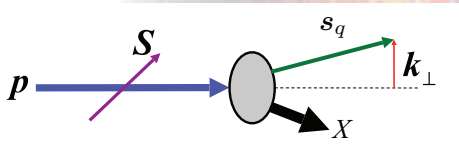
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Sivers function



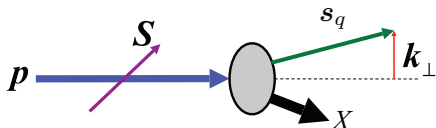
$$f_{q/p\uparrow}(x, k_{\perp}) - f_{q/p\uparrow}(x, -k_{\perp}) \\ = \Delta^N f_{q/p\uparrow}(x, k_{\perp}) \frac{(\hat{\mathbf{P}} \times \mathbf{k}_{\perp}) \cdot \mathbf{S}}{|\mathbf{k}_{\perp}|}$$

$$\Delta^N f_{q/p\uparrow}(x, k_{\perp}) = -\frac{2|\mathbf{k}_{\perp}|}{M} f_{1T}^{\perp q}(x, k_{\perp})$$

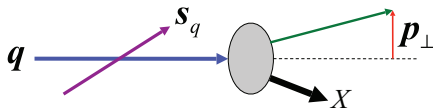
Introduction: TMDs and SSAs definitions

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Collins function



$$f_{q/p^\uparrow}(x, k_\perp) - f_{q/p^\uparrow}(x, -k_\perp) = \Delta^N f_{q/p^\uparrow}(x, k_\perp) \frac{(\hat{\mathbf{p}} \times \mathbf{k}_\perp) \cdot \mathbf{S}}{|\mathbf{k}_\perp|}$$

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$$D_{h/q^\uparrow}(z, p_\perp) - D_{h/q^\uparrow}(z, -p_\perp) = \Delta^N D_{h/q^\uparrow}(z, p_\perp) \frac{(\hat{\mathbf{k}} \times \mathbf{p}_\perp) \cdot \mathbf{S}_q}{|\mathbf{p}_\perp|}$$

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = \frac{2|\mathbf{p}_\perp|}{zM_h} H_1^{\perp q}(z, p_\perp)$$

Introduction: TMDs and SSAs definitions

Sivers & Collins effects play role in the Single Spin Asymmetry (SSA):

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\Delta\sigma}{2d\sigma^{\text{unp}}} \quad (1)$$

where $d\sigma^{\uparrow(\downarrow)}$ is the polarized (differential) cross section:

$$d\sigma^{(A,S_A)+(B,S_B)\rightarrow C+X} = \sum_{a,b,c,d,\{\lambda\}} \rho_{\lambda_a,\lambda'_a}^{a/A,S_A} \hat{f}_{a/A,S_A}(x_a, \mathbf{k}_{\perp a}) \otimes \rho_{\lambda_b,\lambda'_b}^{b/B,S_B} \hat{f}_{b/B,S_B}(x_b, \mathbf{k}_{\perp b}) \otimes \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \hat{M}_{\lambda'_c,\lambda'_d;\lambda'_a,\lambda'_b}^* \otimes \hat{D}_{\lambda_c,\lambda'_c}^{\lambda_c,\lambda_c}(z, \mathbf{k}_{\perp c}); \quad (2)$$

with:

- (i) $\rho_{\lambda_a,\lambda'_a}^{a/A,S_A}$ ($\rho_{\lambda_b,\lambda'_b}^{b/B,S_B}$): helicity density matrix for parton a (b);
- (ii) $\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$: helicity amplitude $ab \rightarrow cd$;
- (iii)

$$\frac{d\hat{\sigma}^{ab\rightarrow cd}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2} \frac{1}{4} \sum_{\lambda_a,\lambda_b,\lambda_c,\lambda_d} |\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}|^2; \quad (3)$$

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Main motivations:

- a test for TMD factorization for inclusive processes;

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 - Anselmino, Boglione, D'Alesio, Melis, Murgia & Prokudin: proposed as a test for TMD factorization (PRD, 2010); predictions for SSA @ LO and comparison w/ HERMES data (PRD, 2014);

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(HERMES Collaboration)

[26–28]. The present data sample is dominated by the kinematic regime $Q^2 \approx 0 \text{ GeV}^2$ of quasireal photoproduction where the cross section is largest and where the hadronic component of the photon plays an important role. Generally speaking, in this kinematic

the hadrons. After correction for trigger efficiency, about 98% of all hadrons belong to the 'anti-tagged' category. The fraction of these

WW approximation or why physics is often like a pork



Physics is like sex: sure, it may give
some practical results, but that's not
why we do it.

— *Richard P. Feynman* —

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Physics is like pork:
you don't throw away anything!

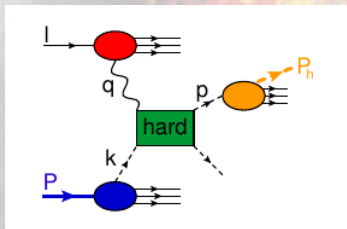
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Weizsäcker-Williams or equivalent photon approximation

Original idea: Weizsäcker & Williams (WW) in the 1930s;

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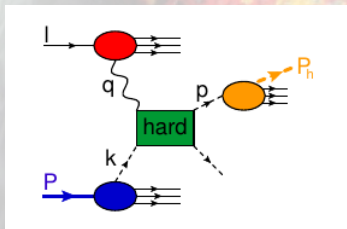
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Lepton as a source of real photons

By evaluating the fundamental vertex $\ell \rightarrow \ell' \gamma$, with ℓ and ℓ' almost collinear ($Q^2 \sim 0$), one can define:

$$f_{\gamma/\ell}(y) = \frac{\alpha}{2\pi} \left(\frac{1 + (1-y)^2}{y} \right) \ln \frac{s}{m_\ell^2}.$$

New partonic channels:

- (i) $q\gamma \rightarrow qg$;
- (ii) $g\gamma \rightarrow q\bar{q}$.

WW contribution to SSA for $\ell p^\uparrow \rightarrow hX$

We assume the following factorization formula:

$$\sigma^{\text{WW}}(\ell p \rightarrow hX) = \int dy f_{\gamma|\ell}(y) \sigma(\gamma p \rightarrow hX)$$

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using the following WW distribution (Vogelsang et.al./2015):

$$f_{\gamma|e}(y, \mu) = \frac{\alpha}{2\pi} \frac{1 + (1-y)^2}{y} \left[\ln \left(\frac{\mu^2}{y^2 m_\ell^2} \right) - 1 \right] + \mathcal{O}(\alpha^2),$$

μ : factorization scale.

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We write:

$$A_N = \frac{d\Delta\sigma^{\text{LO}} + d\Delta\sigma^{\text{WW}}}{2[d\sigma^{\text{LO}} + d\sigma^{\text{WW}}]} \quad (4)$$

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Master equation:

$$\begin{aligned} E_h \frac{d\sigma_{\text{WW}}^{(p,S)\ell \rightarrow hX}}{d^3P_h} &= \sum_{a,c,d,\{\lambda\}} \int \frac{dx dy dz}{16\pi^2 xyz^2 s} d^2k_\perp d^3p_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}_c) J(p_\perp) \\ &\times \delta(\hat{s} + \hat{t} + \hat{u}) \rho_{\lambda_a, \lambda'_a}^{a/p,S} \hat{f}_{a/p,S}(x, \mathbf{k}_\perp) \rho_{\lambda_\gamma, \lambda'_\gamma}^{y/\ell} f_{y/\ell}(y) \\ &\times \hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_\gamma} \hat{M}_{\lambda'_c, \lambda'_d; \lambda'_a, \lambda'_\gamma}^* D_{\lambda_c, \lambda'_c}^{\lambda_h, \lambda_h}(z, \mathbf{p}_\perp), \end{aligned} \quad (5)$$

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- Note: $f_{y/\ell}(y)$ is consistently adopted as a function of only y and not also of \mathbf{k}_\perp

WW contribution to SSA for $\ell p^\uparrow \rightarrow hX$

Schematically:

$$d\sigma^{\text{WW}}(S) = \sum_{a,c,d} \int \frac{dx dy dz}{16\pi^2 xyz^2 s} d^2k_\perp d^3p_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}_c) J(p_\perp) \delta(s + \hat{t} + \hat{u}) \Sigma(S)^{ay \rightarrow cd} \quad (6)$$

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Numerator

$$d\Delta\sigma^{\text{WW}} = \sum_{a,c,d} \int \frac{dx dy dz}{16\pi^2 xyz^2 s} d^2\mathbf{k}_\perp d^3\mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}_c) J(p_\perp) \times \delta(s + \hat{t} + \hat{u}) [\Sigma(\uparrow) - \Sigma(\downarrow)]^{a\gamma \rightarrow cd} \quad (7)$$

Denominator

$$2d\sigma^{\text{WW}} = \sum_{a,c,d} \int \frac{dx dy dz}{16\pi^2 xyz^2 s} d^2\mathbf{k}_\perp d^3\mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}_c) J(p_\perp) \times \delta(s + \hat{t} + \hat{u}) [\Sigma(\uparrow) + \Sigma(\downarrow)]^{a\gamma \rightarrow cd} \quad (8)$$

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with

$$\begin{aligned} \sum_{a,c,d} [\Sigma(\uparrow) \pm \Sigma(\downarrow)]^{a\gamma \rightarrow cd} &= [\Sigma(\uparrow) \pm \Sigma(\downarrow)]^{q\gamma \rightarrow qg} + [\Sigma(\uparrow) \pm \Sigma(\downarrow)]^{q\gamma \rightarrow gq} \\ &+ [\Sigma(\uparrow) \pm \Sigma(\downarrow)]^{\bar{q}\gamma \rightarrow \bar{q}g} + [\Sigma(\uparrow) \pm \Sigma(\downarrow)]^{\bar{q}\gamma \rightarrow g\bar{q}} \\ &+ [\Sigma(\uparrow) \pm \Sigma(\downarrow)]^{g\gamma \rightarrow q\bar{q}} + [\Sigma(\uparrow) \pm \Sigma(\downarrow)]^{g\gamma \rightarrow \bar{q}q} \end{aligned} \quad (9)$$

WW contribution to SSA for $\ell p^\uparrow \rightarrow h X$

$$\begin{aligned}
 [\Sigma(\uparrow) - \Sigma(\downarrow)]^{q\gamma \rightarrow qg} &= f_{\gamma/\ell}(y) \left\{ \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \cos \phi \left[|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 \right]^{q\gamma \rightarrow qg} D_{h/q}(z, p_\perp) \right. \\
 &+ h_{1q}(x, k_\perp) \left[\hat{M}_1^0 \hat{M}_2^0 \right]^{q\gamma \rightarrow qg} \Delta^N D_{h/q^\uparrow}(z, p_\perp) \cos(\phi' + \phi_q^h) \\
 &\left. - \frac{k_\perp^2}{2M^2} h_{1T}^{\perp q}(x, k_\perp) \left[\hat{M}_1^0 \hat{M}_2^0 \right]^{q\gamma \rightarrow qg} \Delta^N D_{h/q^\uparrow}(z, p_\perp) \cos(2\phi - \phi' - \phi_q^h) \right\}
 \end{aligned}$$

$$\begin{aligned}
 [\Sigma(\uparrow) + \Sigma(\downarrow)]^{q\gamma \rightarrow qg} &= f_{\gamma/\ell}(y) f_{q/p}(x, k_\perp) \left[|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 \right]^{q\gamma \rightarrow qg} D_{h/q}(z, p_\perp) \\
 &- \frac{k_\perp}{M} h_1^{\perp q}(x, k_\perp) \left[\hat{M}_1^0 \hat{M}_2^0 \right]^{q\gamma \rightarrow qg} \Delta^N D_{h/q^\uparrow}(z, p_\perp) \cos(\phi - \phi' - \phi_q^h)
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WW contribution to SSA for $\ell p^\uparrow \rightarrow hX$

For jet production:

$$E_j \frac{d\sigma_{WW}^{(p,S)\ell \rightarrow \text{jet} X}}{d^3\mathbf{P}_j} = \sum_{a,c,d,\{\lambda\}} \int \frac{dx dy}{16\pi^2 xys} d^2\mathbf{k}_\perp \delta(\hat{s} + \hat{t} + 0) \quad (10)$$
$$\times \rho_{\lambda_a, \lambda'_a}^{a/p,S} \hat{f}_{a/p,S}(x, \mathbf{k}_\perp) \rho_{\lambda_\gamma, \lambda'_\gamma}^{\gamma/\ell} f_{\gamma/\ell}(y) \hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_\gamma} \hat{M}_{\lambda'_c, \lambda'_d; \lambda'_a, \lambda'_\gamma}^*$$

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and

$$d\Delta\sigma_{\text{jet}}^{\text{WW}} = \sum_{a,c,d} \int \frac{dx dy}{16\pi^2 xys} d^2\mathbf{k}_\perp \delta(\hat{s} + \hat{t} + 0) [\Sigma(\uparrow) - \Sigma(\downarrow)]_{\text{jet}}^{a\gamma \rightarrow cd} \quad (11)$$

$$2d\sigma^{\text{WW}} = \sum_{a,c,d} \int \frac{dx dy}{16\pi^2 xys} d^2\mathbf{k}_\perp \delta(\hat{s} + \hat{t} + 0) [\Sigma(\uparrow) + \Sigma(\downarrow)]_{\text{jet}}^{a\gamma \rightarrow cd} \quad (12)$$

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- Note: replace $D_{h/q,g}(z, p_\perp) \rightarrow 1$ and $\Delta^N D_{h/q^\uparrow}(z, p_\perp) \rightarrow 0$ (no fragmentation process! - pure Sivvers effect)

Phenomenological results

We computed predictions for



EIC

unpolarized cross sections + SSAs

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Comparison w/ HERMES data:

$$d\sigma = d\sigma_{UU} [1 + S_T A_{UT}^{\sin\psi} \sin\psi],$$

with

$$\sin\psi = \mathbf{S}_T \cdot (\hat{\mathbf{P}}_T \times \hat{\mathbf{k}}),$$

so

$$A_{UT}^{\sin\psi}(x_F, P_T) = A_N^{p^\uparrow \ell \rightarrow hX}(-x_F, P_T) \quad (13)$$

Phenomenological results

- Parametrizations for unpolarized TMDs:

$$f_{a/p}(x, k_{\perp}) = f_{a/p}(x) \frac{1}{\pi \langle k_{\perp}^2 \rangle} \exp \left\{ -\frac{k_{\perp}^2}{\langle k_{\perp}^2 \rangle} \right\},$$

$$D_{h/c}(z, p_{\perp}) = D_{h/c}(z) \frac{1}{\pi \langle p_{\perp}^2 \rangle} \exp \left\{ -\frac{p_{\perp}^2}{\langle p_{\perp}^2 \rangle} \right\},$$

with $\langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2$ and $\langle p_{\perp}^2 \rangle = 0.2 \text{ GeV}^2$;

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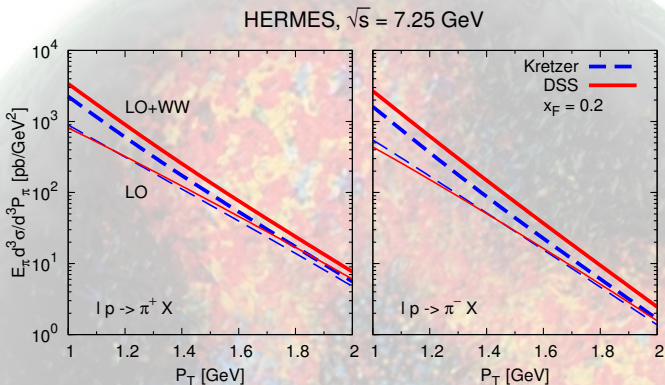
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- Collinear PDFs set: GRV98;
- Collinear FFs sets: Kretzer, De Florian+Sassot+Stratmann (DSS) (different role of gluon FF);
- Tested new extraction of Sivers function: $\langle k_{\perp}^2 \rangle = 0.57 \text{ GeV}^2$ and $\langle p_{\perp}^2 \rangle = 0.12 \text{ GeV}^2$ (CTEQ6L PDFs were used).

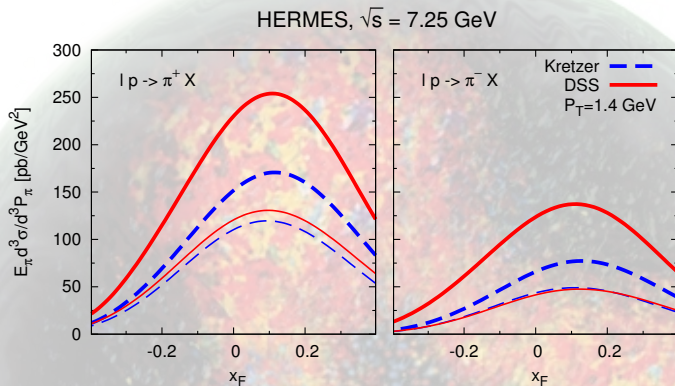
Phenomenological results - unpolarized cross sections



Note:

- $x_F = P_L / P_L^{\max} \equiv 2P_L / \sqrt{s}$;
- WW contribution more important at small P_T .

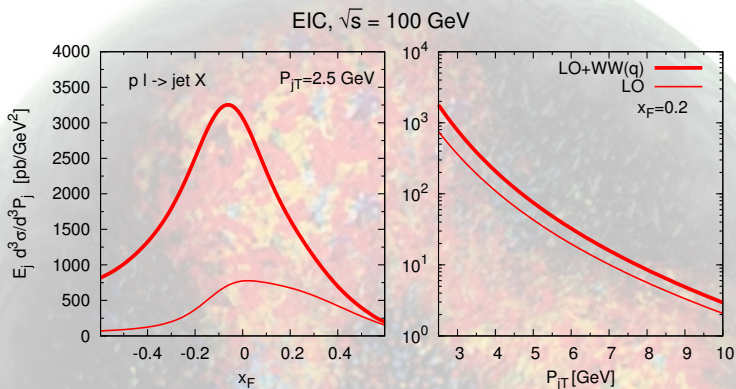
Phenomenological results - unpolarized cross sections



Note:

- for large positive x_F (final hadron produced in backward proton hemisphere) $|\hat{u}| \ll |\hat{t}|$;
- favoured region for WW contribution! LO: $1/Q^2 \equiv 1/\hat{t}^2$, WW: $1/Q^2 \equiv 1/\hat{s}\hat{u}$.

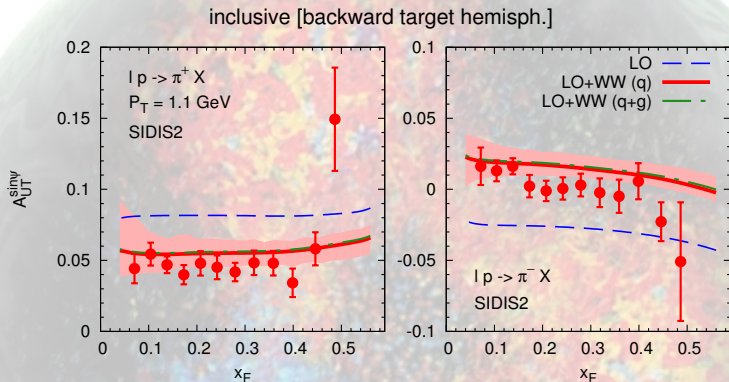
Phenomenological results - unpolarized cross sections



Note:

- here, $x_F > 0$ corresponds to the forward proton hemisphere;
- pure Sivers effect - no fragmentation process!

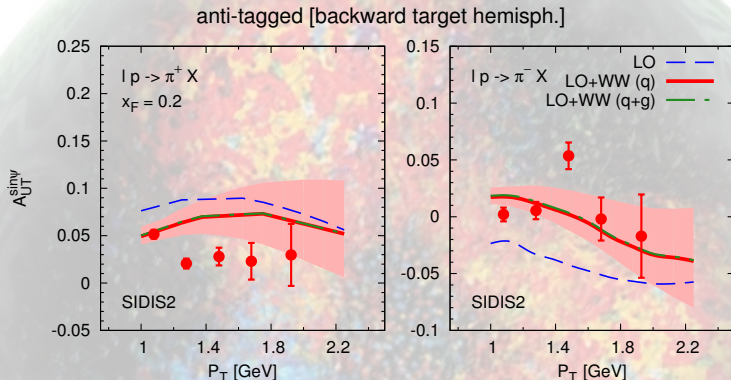
Phenomenological results - comparison with HERMES data



Note:

- improved description of data;
- gluon Sivvers function contribution practically negligible.

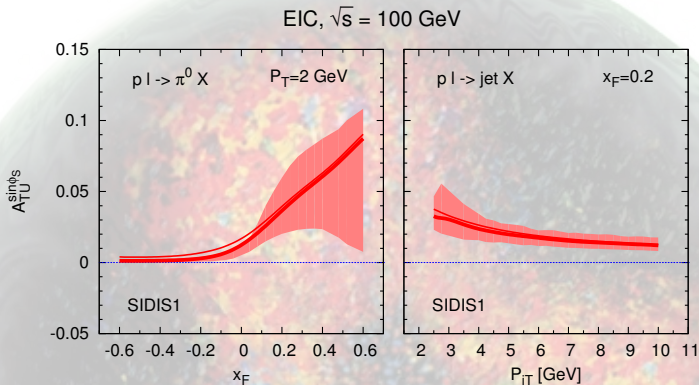
Phenomenological results - comparison with HERMES data



Note:

- same considerations as inclusive data;
- some discrepancies for π^+ .

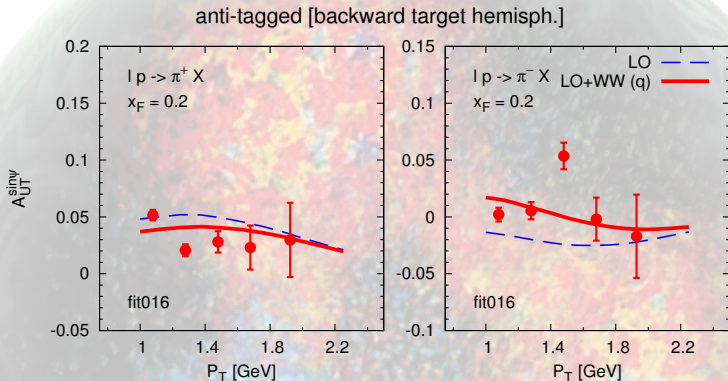
Phenomenological results - predictions for EIC



Note:

- gluon Sivers effect completely negligible;
- WW contribution does not change LO behaviour (both enters with the same structure in the SSA);
- interesting flat P_T behaviour - also measurable!

Phenomenological results - comparison with HERMES data (2)



Note:

- only Siverts effect is showed;
- different widths: $\langle k_{\perp}^2 \rangle = 0.57 \text{ GeV}^2$ and $\langle p_{\perp}^2 \rangle = 0.12 \text{ GeV}^2$;
- CTEQ6L PDFs were used for the new fit of Siverts function

Conclusions

- computed, in the spirit of a TMD approach, WW contribution;
- found new theoretical expression within helicity formalism;
- predictions for unpolarized cross sections and SSAs for ongoing and future experiments were given;
- found that WW contribution plays a huge role in $\ell p^\uparrow \rightarrow hX$;
- very good description of HERMES data, both for inclusive and untagged data (new extraction of Sivers enhanced the agreement with the data);
- future measurements, especially at EIC, would test TMD approach;
- a further step towards a deeper understanding of the origin of SSAs for inclusive processes and towards a unified TMD picture.



BACKUP

WW contribution - Helicity amplitudes

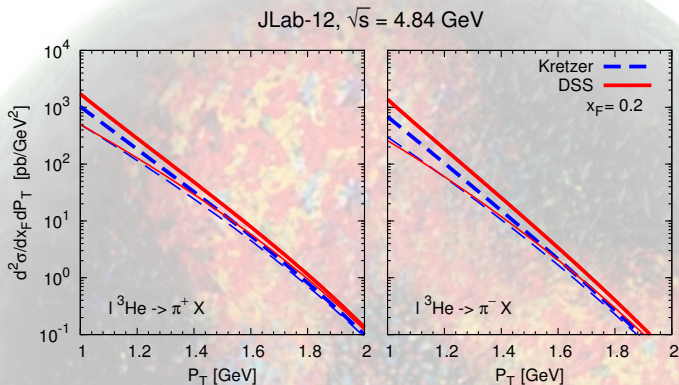
Besides an overall factor $16\pi^2\alpha\alpha_s$:

$$\left[|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2\right]^{q\gamma \rightarrow qg} = \frac{16}{3}e_q^2 \frac{\hat{s}^2 + \hat{u}^2}{-\hat{s}\hat{u}}, \quad \left[\hat{M}_1^0 \hat{M}_2^0\right]^{q\gamma \rightarrow qg} = \frac{16}{3}e_q^2,$$

$$\left[|\hat{M}_1^0|^2 + |\hat{M}_3^0|^2\right]^{q\gamma \rightarrow gq} = \frac{16}{3}e_q^2 \frac{\hat{s}^2 + \hat{t}^2}{-\hat{s}\hat{t}},$$

$$\left[|\hat{M}_2^0|^2 + |\hat{M}_3^0|^2\right]^{g\gamma \rightarrow q\bar{q}} = 2e_q^2 \frac{\hat{u}^2 + \hat{t}^2}{-\hat{u}\hat{t}}.$$

Phenomenological results - unpolarized cross sections

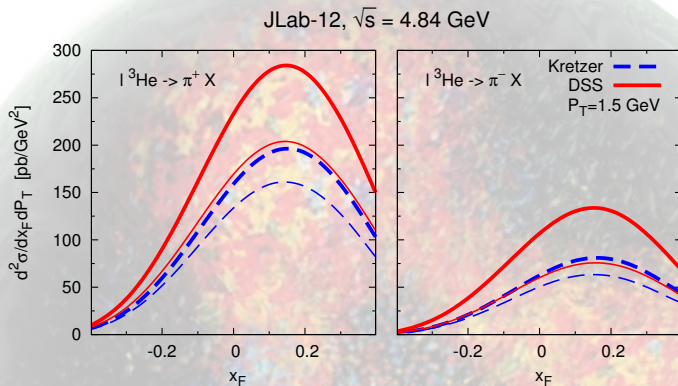


Note:

$$\frac{d^2\sigma}{dx_F dP_T} = \frac{2\pi P_T}{\sqrt{x_F^2 + x_T^2}} E_\pi \frac{d^3\sigma}{d^3\mathbf{P}_\pi},$$

where $x_T = 2P_T/\sqrt{s}$.

Phenomenological results - unpolarized cross sections

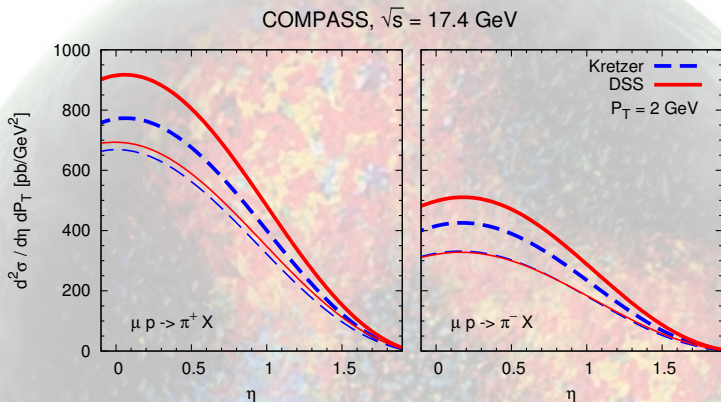


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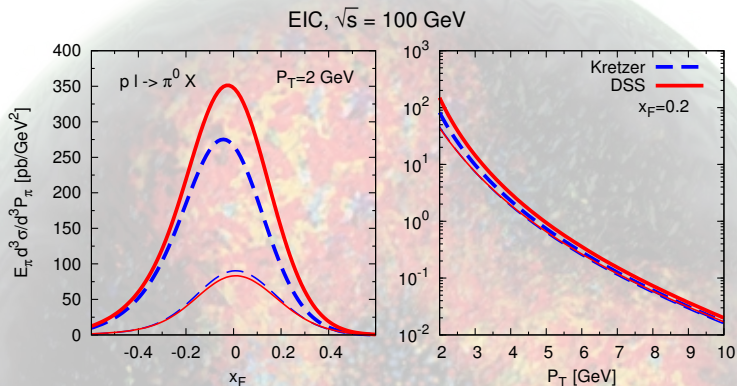
Phenomenological results - unpolarized cross sections



Note:

$$\frac{d^2\sigma}{d\eta dP_T} = 2\pi P_T E_\pi \frac{d^3\sigma}{d^3\mathbf{P}_\pi}$$

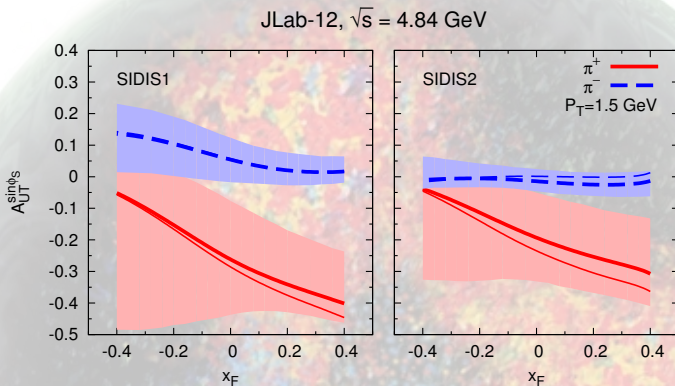
Phenomenological results - unpolarized cross sections



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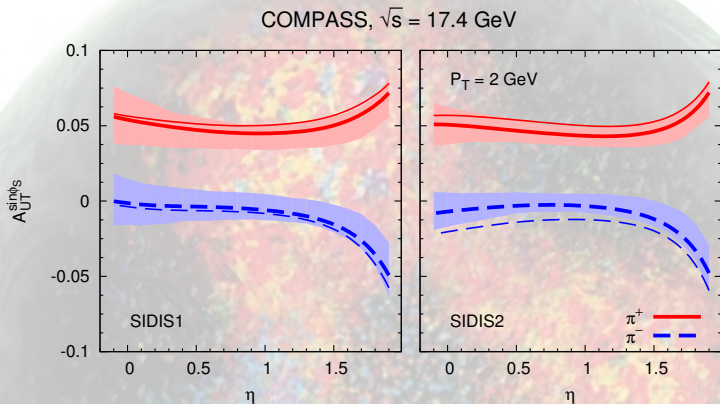
Phenomenological results - predictions for JLAB



Note:

- wider uncertainty bands due to large- x region probed at moderate energies;
- extractions of Sivers function are unconstrained at large x .

Phenomenological results - predictions for COMPASS



Note:

- sizeable SSA for π^+ ;
- data would be useful as a clear test of TMD approach.

References

- Article: arxiv:1701.01148v1 [hep-ph];
- QCD Evolution 2016 proceedings: PoS(QCDEV2016)002.