The Sound and the Noise of Black Holes

a High Energy Physics Colloquium

Edgardo Franzin

Cagliari, 4th July 2016

Outline

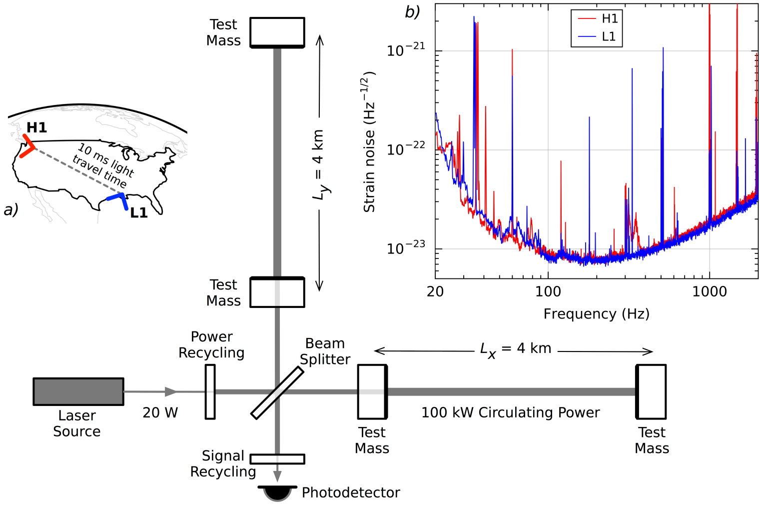
- The experimental setup and the discovery
- Event horizons and light rings in Schwarzschild
- Black hole perturbations
  or How to produce gravitational waves
- Are we listening to black holes?

A bit of History

1916	Einstein predicts gravitational waves
1936	Einstein contradicts himself: gravitational waves don't exist
1950s–60s	Theoretical arguments about existence of gravitational waves
1960	Weber starts building detectors
1969	Weber announces first detection of gravitational waves Physics Focus 16 (2005) 19
1970s	Consensus that gravitational waves exist and doubts on Weber's findings
1974–79	Hulse-Taylor binary pulsar (Nobel prize in 1993)
1990	Construction of the Laser Interferometer Gravitational-wave Observatory begins
2002–11	Joint observations of LIGO, TAMA300 (Japan), GEO600 (Germany) and Virgo (Italy)
2015	Advanced LIGO starts
2016	LIGO detects gravitational waves caused by the collision of black holes

LIGO and GW150904

aLIGO consists of two giant laser interferometers, one in Hanford, WA, and one in Livingston, LA.

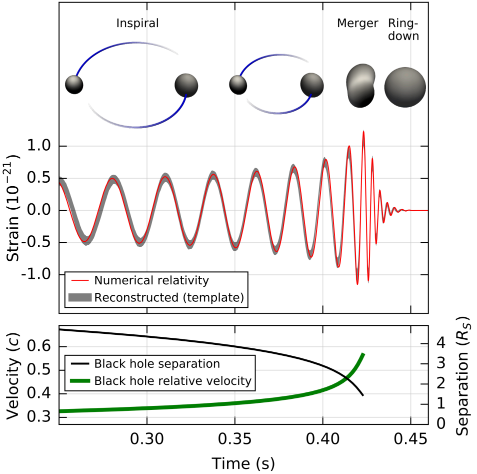


Credits: The LIGO Collaboration

(Advanced) Laser Interferometer Gravitational-wave Observatory

- How does it work?
  When a gravitational wave passes by, the arms of the interferometer alternately lengthen and shrink and the laser beams take a different time to travel through the arms.
- How sensitive?
  The difference between the two arm lengths is proportional to the gravitational-wave strain. For what we can detect, we expect the strain to be about 1/10 000th the width of a proton!
- Why two detectors?
  To successfully detect a gravitational wave, LIGO needs to combine astounding sensitivity with an ability to isolate real signals from environmental effects and instrument noise. Only a real gravitational wave signal would appear in both detectors.
  With two or more detectors we can also triangulate the direction on the sky from which a gravitational wave arrives.

Each signal is analysed very promptly, looking for evidence of a gravitational-wavelike pattern but without modeling the precise details of the waveform. The gravitational-wave strain data are then compared with an extensive bank of theoretically predicted waveforms. GW150914 [LIGO/Virgo collab. (2016)] has been produced by the merger of two black holes with masses of about $36M_\odot$ and $29M_\odot$ into a spinning black hole with mass of about $62M_\odot$. The coalescence converted about $3M_\odot$ into gravitational-wave energy, emitted in a fraction of a second.



Credits: The LIGO Collaboration

All the observations are consistent with the predictions of General Relativity.

The Schwarzschild Spacetime

The Schwarzschild Spacetime

The gravitational field outside a spherical mass is Schwarzschild (1916)

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right).$$

The surface $r = 2M$ is an event horizon — the defining property of black holes.

Most of the experimental tests of GR are based on the Schwarzschild geometry for $r > 2M$: the precession of planetary orbits, the bending of light, accretion discs around black holes, …

Trajectories of Photons and Light Ring

The geodesics in the Schwarzschild geometry, (equatorial plane)

$$\left(1-\frac{2M}{r}\right)\dot{t}=E, \quad \left(1-\frac{2M}{r}\right)\dot{t}^2-\left(1-\frac{2M}{r}\right)^{-1}\dot{r}^2-r^2\dot{\phi}^2=0, \quad r^2\dot{\phi}=L,$$

can be combined to give the energy and the shape equations,

$$\dot{r}^2+\frac{L^2}{r^2}\left(1-\frac{2M}{r}\right)=E^2, \quad \frac{\mathrm{d}^2u}{\mathrm{d}\phi^2}+u=3Mu^2 \ \ (u\equiv 1/r).$$

The only possible radius for a circular photon orbit is $r=3M$, the light ring — a boundary within which photons can be trapped in circular orbits.

Black Hole Perturbations

or How to produce gravitational waves

Linearized Einstein Equations

The Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi T_{\mu\nu},$$

can be solved exactly for a very few cases.

Wave-like solutions to the Einstein equations can be found in a spacetime with very modest curvature,

$$g_{\mu\nu} = \mathring{g}_{\mu\nu} + h_{\mu\nu}, \quad \text{with } |h_{\mu\nu}| \ll 1.$$

The linearized Einstein field equations (in an appropriate gauge) are,

$$\Box h_{\mu\nu} = -16\pi T_{\mu\nu}.$$

In vacuum $\Box h_{\mu\nu} = 0$, *i.e.* the metric perturbations propagate as waves distorting the background spacetime.

Metric perturbations were first consired to analyse the stability of black holes, and then to study the gravitational field of particles falling into black holes.

The Schwarzschild solution is the only spherically symmetric, asymptotically flat solution of Einstein equations in vacuum (Birkhoff's theorem). As a result, perturbations must have complete angular dependence.

The metric perturbations can be expanded in terms of tensor spherical harmonics that behave differently under parity transformation, (in the Regge-Wheeler gauge)

$$h_{\mu\nu}^{\text{odd}} = \begin{pmatrix} 0 & 0 & -h_0(t,r)\csc\theta\frac{\partial}{\partial\varphi} & h_0(t,r)\sin\theta\frac{\partial}{\partial\theta} \\ 0 & 0 & -h_1(t,r)\csc\theta\frac{\partial}{\partial\varphi} & h_1(t,r)\sin\theta\frac{\partial}{\partial\theta} \\ \star & \star & 0 & 0 \\ \star & \star & 0 & 0 \end{pmatrix} Y_{\ell m}(\theta,\varphi),$$

$$h_{\mu\nu}^{\text{even}} = \begin{pmatrix} e^{\nu(r)}H_0(t,r) & H_1(t,r) & 0 & 0 \\ \star & e^{-\nu(r)}H_2(t,r) & 0 & 0 \\ 0 & 0 & r^2K(t,r) & 0 \\ 0 & 0 & 0 & r^2K(t,r)\sin^2\theta \end{pmatrix} Y_{\ell m}(\theta,\varphi).$$

The Regge-Wheeler and the Zerilli Equations

Let us assume harmonic dependance and consider odd metric perturbations. Regge & Wheeler (1957)

Einstein equations do not depend on m and they can be recasted in a Schrödinger-like wave equation,

$$\frac{d^2\psi}{dr_*^2} - \left[\omega^2 - \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right)\right]\psi = 0,$$

where the "tortoise" coordinate $r_* \equiv r + 2M \log\left(\frac{r}{2M} - 1\right)$, is particularly suited to study the propagation of perturbations near the black hole horizon since it is placed at $-\infty$.

The Regge-Wheeler potential has a maximum just outside the event horizon, at $r \approx 3.3M$.

Let us now consider even metric perturbations. In this case we obtain the Zerilli equation, Zerilli (1970)

$$\frac{d^2\psi}{dr_*^2} - \left[\omega^2 - \left(1 - \frac{2M}{r}\right)\frac{18M^3 + 18M^2 r\Lambda + 6Mr^2\Lambda^2 + 2r^3\Lambda^2(1+\Lambda)}{r^3(3M + r\Lambda)^2}\right]\psi = 0,$$

where $\Lambda = (\ell - 1)(\ell + 2)/2$.

The Regge-Wheeler and Zerilli equations describe the response of the black hole to external perturbations, *i.e.* they tell us about the vibrational modes of the spacetime.

The quasi-normal modes (QNM) are complex values of ω for which we have solutions of the equations such that we have pure outgoing-waves at infinity, and pure ingoing-waves at the event horizon. Kokkotas & Schmidt (1999); Berti, Cardoso & Starinets (2009)

Some properties of the Schwarzschild QNMs: Chandrasekhar & Detweiler (1975)

- All the QNMs are damped modes, *i.e.* Schwarzschild is linearly stable against perturbations.
- The damping time depends on the mass, *i.e.* $\omega_n \sim 1/M$ and is shorter for higher modes.
- The QNMs in black holes are isospectral, *i.e.* even and odd perturbations have the same complex eigenfrequencies.

For a Kerr black hole, the calculations are way too involved, but the entire QNM spectrum is characterized only by the black hole mass and angular momentum.

Gravitational Field of a Particle Falling into a Black Hole

For a test particle of mass $\mu_p \ll M$ and energy E falling into a black hole, there is also a "source term". Zerilli (1970) In the Regge-Wheeler gauge,

$$S_\ell = \frac{4\mu_p\sqrt{\ell+\frac{1}{2}}\left(1-\frac{2M}{r}\right)\left(2\mathrm{i}E\Lambda\sqrt{E^2-1+\frac{2M}{r}}-\omega\left(\Lambda r+3M\right)\right)\mathrm{e}^{\mathrm{i}\omega T(r)}}{\omega\sqrt{E^2-1+\frac{2M}{r}}\left(\Lambda r+3M\right)^2},$$

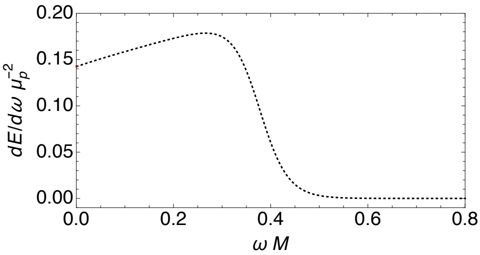
where $T(r)$ is the coordinate time, $\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{E}{\left(1-\frac{2M}{r}\right)\sqrt{E^2-1+\frac{2M}{r}}}$.

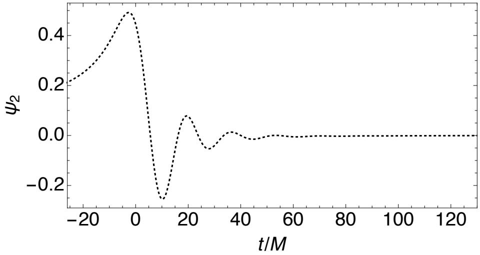
The energy flux emitted in gravitational waves is

$$\frac{\mathrm{d}E}{\mathrm{d}\omega} = \frac{1}{32\pi} \sum_{\ell \geqslant 2} \frac{(\ell+2)!}{(\ell-2)!} \omega^2 \left| \psi_\ell(\omega, r \to \infty) \right|^2 .$$

The time-domain wavefunction can be recovered via

$$\Psi_\ell(t, r) = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}\omega \, \mathrm{e}^{-\mathrm{i}\omega t} \psi_\ell(\omega, r).$$





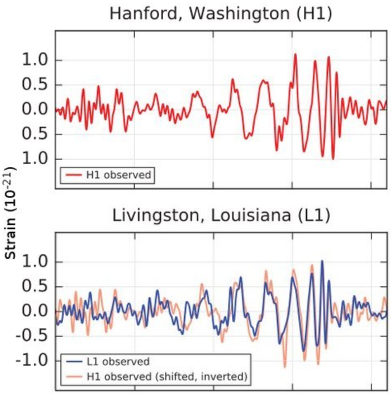
Are We Listening to Black Holes?

The detection of GW150914 and GW151226 enormously strengthen the evidence for stellar-mass black holes, whose existence is already supported by various indirect observations in the electromagnetic band.

But, is there any evidence for horizons?

Electromagnetic observations cannot probe the existence of event horizons, they may probe only the existence of light rings. Abramowicz, Kluźniak & Lasota (2002)

The gravitational wave ringdown signal might arguably provide the only conclusive proof of the existence of an event horizon in dark, compact objects.



Credits: The LIGO Collaboration

Black hole mimickers include:

- Boson stars
- Wormholes
- Gravastars

These objects and black holes are dark and then indistinguishable to electromagnetic instruments.

Is there any evidence for *horizonless* compact objects? Do we have solid physical motivations?

Ultra-compact objects are generically unstable. [Cardoso et al. (2014)]

It was commonly believed that the ringdown is dominated by the QNMs of the final object.

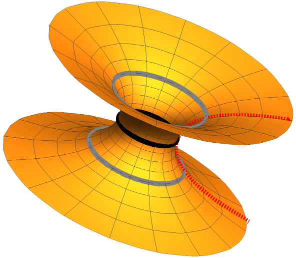
For black holes, the QNMs are deeply related to the boundary conditions required at the event horizon, but for a horizonless object, the boundary conditions change completely, and so the QNM structure. So, is the ringdown waveform dominated by the quasinormal modes?

In principle, the ringdown phase should not depend on the presence of a horizon as long as the final object has a light ring. Cardoso, EF & Pani (2016)

For a black hole, the ingoing condition at the horizon simply takes the ringdown waves and 'carries' them inside the black hole. In this case, the black hole QNMs *incidentally* describe also the ringdown phase.

For another horizonless object, its relaxation should consist on the usual light ring ringdown modes (which are no longer QNMs), followed by the proper modes of vibration of the object itself.

We study the gravitational radiation emitted by a point particle in radial motion towards a traversable wormhole. Morris, Thorne & Yurtsever (1988); Visser (1996)



Two copies of the Schwarzschild spacetime with the same mass M,

$$ds^2 = -F dt^2 + F^{-1} dr^2 + r^2 d\Omega^2, \quad \text{where } F = 1 - 2M/r,$$

glued together at the throat $r = r_0 > 2M$.

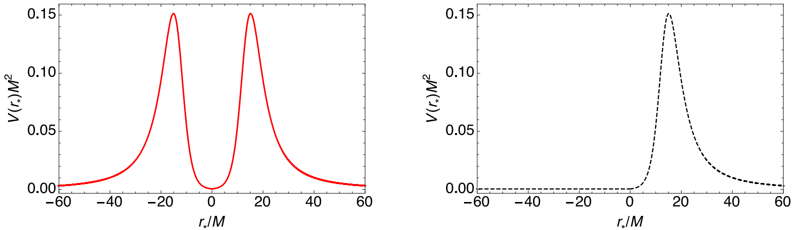
Consider an infalling particle with mass $\mu_p \ll M$ and energy E.

In the point-particle limit, the Einstein equations reduce to a pair of Zerilli equations, Zerilli (1970)

$$\frac{\mathrm{d}^2\psi_\ell(\omega, r)}{\mathrm{d}r_*^2} + \left(\omega^2 - V_\ell(r)\right)\psi_\ell(\omega, r) = S_\ell \, .$$

The junction conditions for ψ_ℓ at the throat depend on the properties of the matter confined in the thin shell. Pani et al. (2009) Here we assume that the microscopic properties of the shell are such that ψ_ℓ and $\mathrm{d}\psi_\ell/\mathrm{d}r_*$ are continuous at the throat.

Effective $\ell = 2$ potential for a static traversable wormhole with $r_0 = 2.001M$ and for a black hole



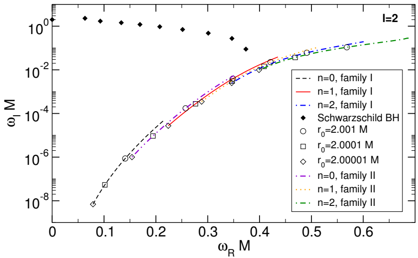
The effective potential is Z_2 symmetric and develops another barrier at $r_* < 0$.

For any $r_0 \lesssim 3M$, wormholes can support long-lived modes trapped between the two potential wells near the light rings.

The QNMs of the wormhole are defined by the eigenvalue problem associated with the Zerilli equation with $S_\ell = 0$ supplemented by regularity boundary conditions.

- At the asymptotic boundaries of both universes we require $\psi_\ell \sim \mathrm{e}^{\pm i\omega r_*}$.
- At the throat we impose continuity of $\mathrm{d}\psi_\ell/\mathrm{d}r_*$, i.e., either $\mathrm{d}\psi_\ell(0)/\mathrm{d}r_* = 0$ or $\psi_\ell(0) = 0$.

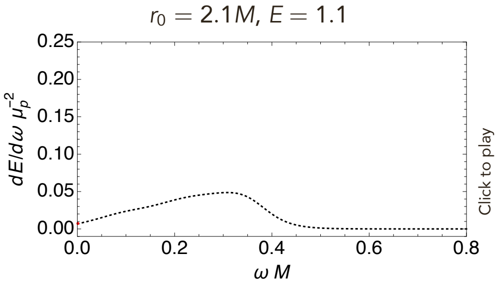
We find two families of QNMs, $\omega = \omega_R + \mathrm{i}\,\omega_I$, for different values of the throat location r_0.
In the black hole limit $r_0 \to 2M$, the spectrum is dramatically different from that of Schwarzschild. The QNMs of the wormhole approach the real axis and become long lived.

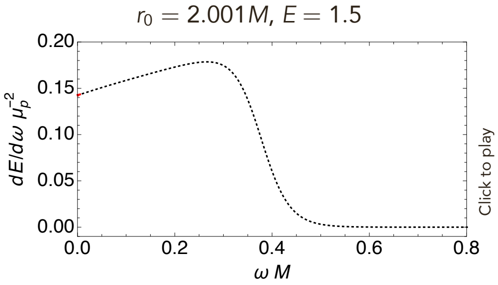


Quadrupolar Gravitational Waves Energy Spectra

The energy flux emitted in gravitational waves is

$$\frac{\mathrm{d}E}{\mathrm{d}\omega} = \frac{1}{32\pi} \sum_{\ell \geqslant 2} \frac{(\ell+2)!}{(\ell-2)!} \omega^2 \left|\psi_\ell(\omega, r \to \infty)\right|^2 .$$

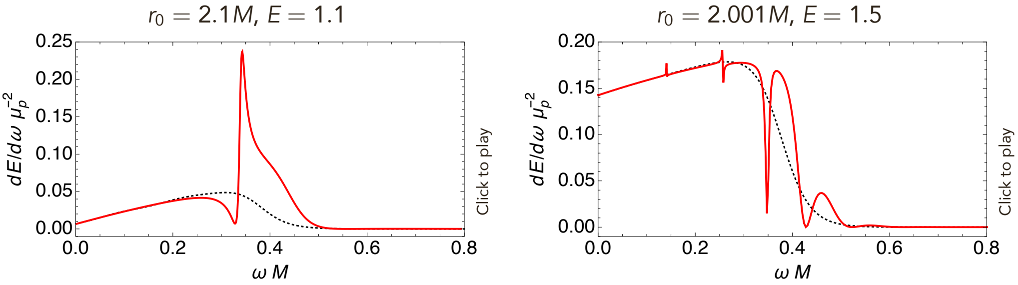




Quadrupolar Gravitational Waves Energy Spectra

The energy flux emitted in gravitational waves is

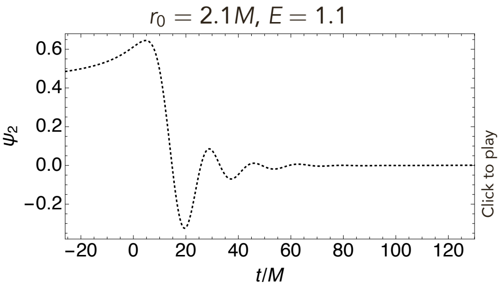
$$\frac{dE}{d\omega} = \frac{1}{32\pi} \sum_{\ell \geqslant 2} \frac{(\ell+2)!}{(\ell-2)!} \omega^2 \left|\psi_\ell(\omega, r \to \infty)\right|^2 .$$

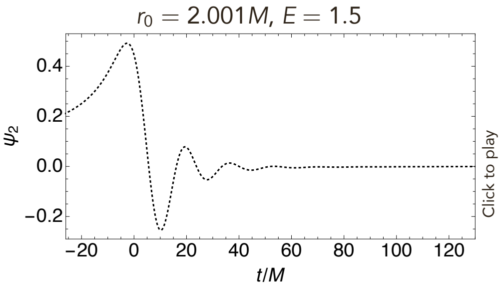


The spectra coincide only at low frequencies, but are generically very different.

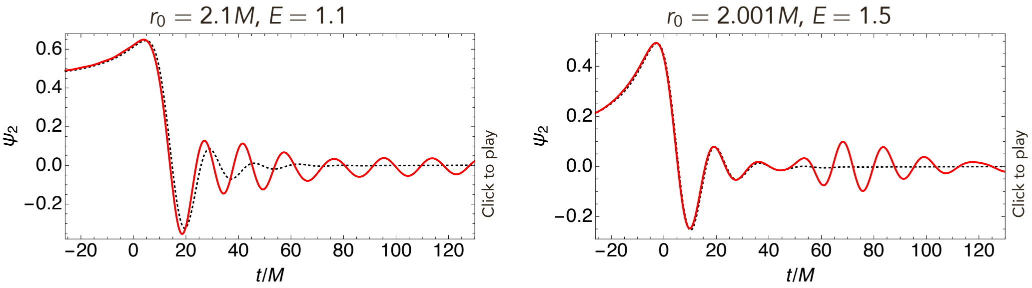
In the black hole limit, the long-lived QNMs of the wormhole can be excited and correspond to narrow, Breit-Wigner resonances in the spectrum. Pons et al. (2002); Berti, Cardoso & Pani (2009)

Given the drastically differences between the QNM and energy spectra of a wormhole and those of a black hole, one might be tempted to expect a completely different ringdown signal.





Given the drastically differences between the QNM and energy spectra of a wormhole and those of a black hole, one might be tempted to expect a completely different ringdown signal.



The differences in the energy spectra do not leave any trace in the initial ringdown waveform.

The QNMs of the wormhole contain low energy and get excited only at late times.

Summary

Gravitational wave astronomy has just begun.

A highly counterintuitive phenomenon: the initial ringdown signal chiefly depends on the properties of the light ring of the final object.

The actual QNMs of the object are excited only at late times and typically do not contain a significant amount of energy.

If future gravitational waves detections by aLIGO, aVIRGO and KAGRA will be able to extract the late-time ringdown signal, we could rule out exotic alternatives to black holes and test quantum effects at the horizon scale. Giddings (2016)

As it stands, the single event GW150914 does not provide the final evidence for horizons, leaves room for alternative theories of gravity and ECOs, Chirenti & Rezzolla (2016); Yunes, Yagi & Pretorius (2016) but strongly supports the existence of light rings.

Grazie!

See you in fall for the next cycle of Colloquia