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Quantum Cosmology with Distorted Gravity

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Plan of the Talk

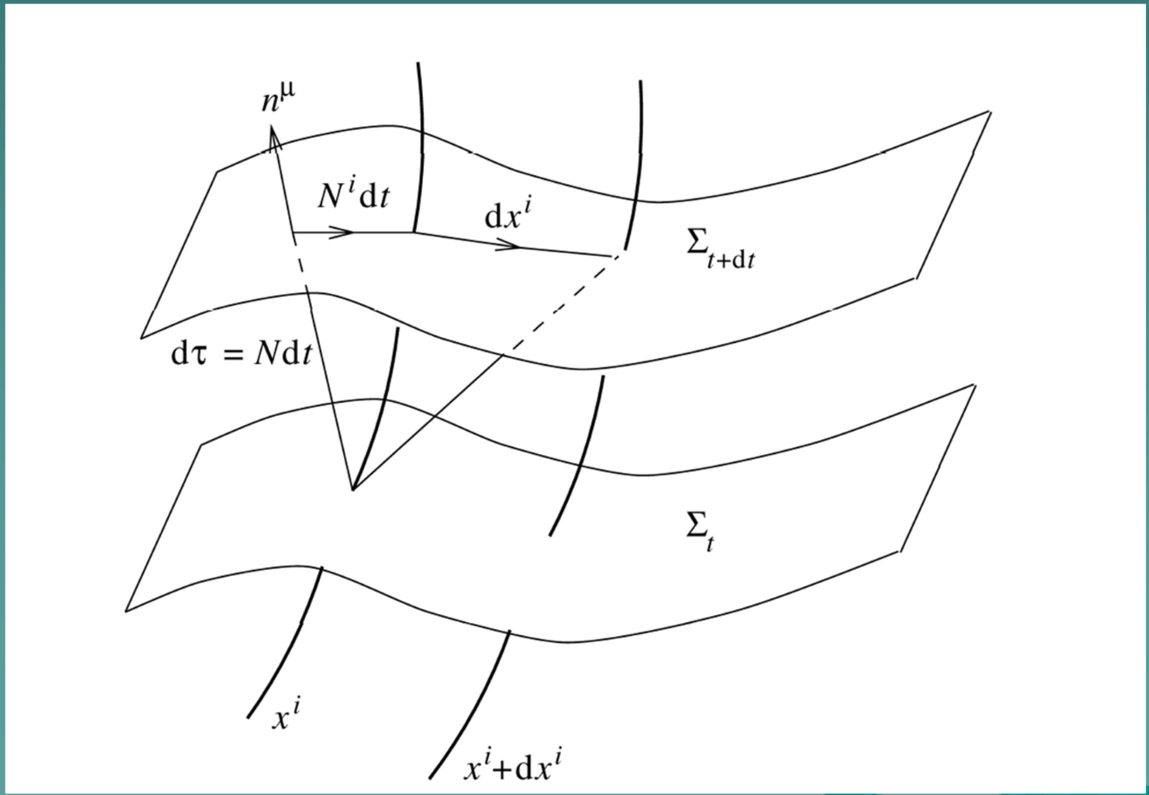
- Building the Wheeler-DeWitt Equation
- The Wheeler-DeWitt Equation as a Sturm-Liouville problem
- Relaxing the Lorentz Symmetry in a MSS approach for a FLRW model.
- The Cosmological Constant as a Zero Point Energy Computation in the Gravity's Rainbow context
- Conclusions and Outlooks

Relevant Action for Quantum Cosmology

$$S = \frac{1}{2\kappa}\int_{\mathfrak{M}} d^4x\sqrt{-g}\left({}^4R - 2\Lambda\right) + 2\int_{\partial\mathfrak{M}} d^3x\sqrt{g^{(3)}}K + S_{matter} \qquad \kappa = 8\pi G$$

$G \rightarrow$ Newton's Constant

$\Lambda \rightarrow$ Cosmological Constant



Relevant Action for Quantum Cosmology

$$S = \frac{1}{2\kappa}\int_{\mathfrak{M}} d^4x\sqrt{-g}\left({}^4R - 2\Lambda\right) + \frac{1}{\kappa}\int_{\partial\mathfrak{M}} d^3x\sqrt{g^{(3)}}K + S_{matter}$$

ADM Decomposition

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^k N_k & N_j \\ N_i & g_{ij}^{(3)} \end{pmatrix} \qquad g^{\mu\nu} = \begin{pmatrix} -\frac{1}{N^2} & \frac{N^j}{N^2} \\ \frac{N^i}{N^2} & g^{ij(3)} - \frac{N^i N^j}{N^2} \end{pmatrix}$$

$$\mathrm{ds}^2 = g_{\mu\nu}dx^\mu dx^\nu = -N^2 dt^2 + g_{ij}\left(N^i dt + dx^i\right)\left(N^j dt + dx^j\right)$$

N is the lapse function N_i is the shift function

$$K_{ij} = -\frac{1}{2N}\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i \qquad K = K^{ij}g_{ij}$$

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^k N_k & N_j \\ N_i & g_{ij}^{(3)} \end{pmatrix} \qquad g^{\mu\nu} = \begin{pmatrix} -\dfrac{1}{N^2} & \dfrac{N^j}{N^2} \\ \dfrac{N^i}{N^2} & g^{ij(3)} - \dfrac{N^i N^j}{N^2} \end{pmatrix} \qquad \mathrm{ds}^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$S = \frac{1}{2\kappa} \int_{\Sigma\times I} dt d^3x N \sqrt{g^{(3)}} \left(K^{ij} K_{ij} - K^2 + {}^3R - 2\Lambda \right) + S_{\partial(\Sigma\times I)} + S_{matter}$$

Legendre Transformation $\rightarrow$ $H = \int_\Sigma d^3x \left(N_i \mathcal{H}^i + N \mathcal{H} \right) + H_{\partial\Sigma}$

$\mathcal{H} = (2\kappa) G_{ijkl} \pi^{ij} \pi^{kl} - \dfrac{\sqrt{g}}{2\kappa} \left({}^3R - 2\Lambda \right) = 0$ Classical Constraint $\rightarrow$ Invariance by time reparametrization

$\mathcal{H}^i = 2\pi^{ij}{}_{|j} = 0$ Classical Constraint $\rightarrow$ Gauss Law

Wheeler-De Witt Equation

B. S. DeWitt, Phys. Rev.**160**, 1113 (1967).

$$\left[(2\kappa)G_{ijkl}\pi^{ij}\pi^{kl}-\frac{\sqrt{g}}{2\kappa}(R-2\Lambda)\right]\Psi\left[g_{ij}\right]=0$$

- G_{ijkl} is the super-metric,
- R is the scalar curvature in 3-dim.

Example:WDW for Tunneling

$$ds^2=-N^2dt^2+a^2(t)\,d\Omega_3^2$$

$$H\Psi[a]=\left[-\frac{\partial^2}{\partial a^2}-\frac{q}{a}\frac{\partial}{\partial a}+\frac{9\pi^2}{4G^2}\left(a^2-\frac{\Lambda}{3}a^4\right)\right]\Psi[a]=0$$

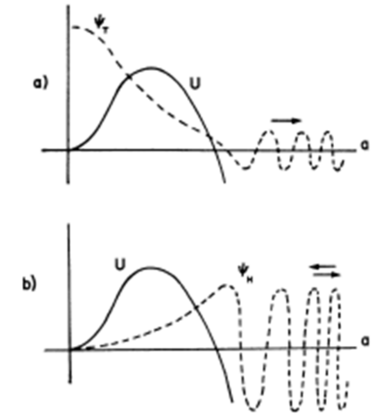


FIG. 2. (a) Tunneling and (b) Hartle-Hawking wave functions for the one-dimensional minisuperspace model describing a de Sitter space. The "potential" $U(a)$ is shown by a solid line and the wave functions by dashed lines.

Formal Schrödinger Equation with zero eigenvalue whose solution is a linear combination of Airy's functions (q=-1 Vilenkin Phys. Rev. D 37, 888 (1988).) containing expanding solutions

Wheeler-De Witt Equation

B. S. DeWitt, Phys. Rev.**160**, 1113 (1967).

$$H\Psi[a]=\left[-\frac{1}{a^q}\frac{\partial}{\partial a}\left(a^q\frac{\partial}{\partial a}\right)+\frac{9\pi^2}{4G^2}\left(a^2-\frac{\Lambda}{3}a^4\right)\right]\Psi[a]=E\Psi[a]\Leftrightarrow E=0$$

E=0 is highly degenerate

Sturm-Liouville Eigenvalue Problem

$$\left[\frac{d}{dx}\left(p(x)\frac{d}{dx}\right)+q(x)+\lambda w(x)\right]y(x)=0$$

$$\int_a^b w(x)\,y^*(x)\,y(x)\,dx\leftrightarrow \text{Normalization with weight } w(x)\ \rightarrow\int_0^\infty a^{q+4}\Psi^*(a)\Psi(a)\,da$$

$$p(x)\rightarrow a^q(t)\quad q(x)\rightarrow-\left(\frac{3\pi}{2G}\right)^2 a^{q+2}(t)\quad w(x)\rightarrow a^{q+4}(t)\quad y(x)\rightarrow\Psi[a]\quad \lambda\rightarrow\left(\frac{3\pi}{2G}\right)^2\left(\frac{\Lambda}{3}\right)$$

Wheeler-De Witt Equation

B. S. DeWitt, Phys. Rev.**160**, 1113 (1967).

$$H\Psi[a]=\left[-\frac{1}{a^q}\frac{\partial}{\partial a}\left(a^q\frac{\partial}{\partial a}\right)+\frac{9\pi^2}{4G^2}\left(a^2-\frac{\Lambda}{3}a^4\right)\right]\Psi[a]=0$$

Sturm-Liouville Eigenvalue Problem → Variational procedure

$$\lambda=\min_{y(x)}\frac{-\int_a^b y^*(x)\left[\frac{d}{dx}\left(p(x)\frac{d}{dx}\right)+q(x)\right]y(x)\,dx}{\int_a^b w(x)\,y^*(x)\,y(x)\,dx}\rightarrow$$

Rayleigh-Ritz
Variational Procedure

$$y(a)=y(b)=0$$

Wheeler-De Witt Equation

B. S. DeWitt, Phys. Rev.**160**, 1113 (1967).

$$H\Psi[a]=\left[-\frac{1}{a^q}\frac{\partial}{\partial a}\left(a^q\frac{\partial}{\partial a}\right)+\frac{9\pi^2}{4G^2}\left(a^2-\frac{\Lambda}{3}a^4\right)\right]\Psi[a]=0$$

Sturm-Liouville Eigenvalue Problem → *Variational procedure*

$$\frac{\Lambda}{3}\left(\frac{3\pi}{2G}\right)^2=\min_{\Psi(a)}\frac{\int_0^\infty\Psi^*(a)\left[-\frac{d}{da}\left(a^q\frac{d}{da}\right)+\left(\frac{3\pi}{2G}\right)^2a^{q+2}\right]\Psi(a)\,da}{\int_0^\infty a^{q+4}\Psi^*(a)\Psi(a)\,da}\rightarrow$$

Rayleigh-Ritz Variational Procedure

$\Psi(\infty)=0$ $\Psi(0)=0\leftarrow$ De Witt Condition

$\Psi(0)\neq 0$ for example for q=0

$\Psi(a)=\exp\left(-\beta a^2\right)\rightarrow$ No Solution

Relaxing Lorentz symmetry

Hořava-Lifshitz theory → UV Completion, problems with scalar graviton in IR

Varying Speed of Light Cosmology→ Solve problems in the Inflationary phase (horizon,flatness, particle production)

Gravity's Rainbow→ Like VSL. Moreover it allows finite calculation to one loop. The set of the Rainbow's functions is too large. A selection procedure is necessary

At low energy all these models describe GR

Gravity’s Rainbow

Doubly Special Relativity

G. Amelino-Camelia, Int.J.Mod.Phys. D 11, 35 (2002); gr-qc/001205.
G. Amelino-Camelia, Phys.Lett. B 510, 255 (2001); hep-th/0012238.

$$E^2 g_1^2\left(E/E_P\right) - p^2 g_2^2\left(E/E_P\right) = m^2$$

$$\lim_{E/E_P \to 0} g_1\left(E/E_P\right) = \lim_{E/E_P \to 0} g_2\left(E/E_P\right) = 1$$

Curved Space Proposal → *Gravity’s Rainbow*
[J. Magueijo and L. Smolin, Class. Quant. Grav. 21, 1725 (2004) arXiv:gr-qc/0305055].

$$ds^2 = -\frac{N(r)\,dt^2}{g_1^2\left(E/E_P\right)} + \frac{dr^2}{\left(1-\frac{b(r)}{r}\right) g_2^2\left(E/E_P\right)} + \frac{r^2}{g_2^2\left(E/E_P\right)} d\theta^2 + \frac{r^2}{g_2^2\left(E/E_P\right)} \sin^2\theta\, d\varphi^2$$

$N(r) = \exp\left(-2\Phi(r)\right)$ $\quad$ $\Phi(r)$ is the redshift function

$b(r)$ is the shape function $\quad$ Condition $\to b(r_0) = r_0 \quad r \in [r_0, +\infty)$

Gravity's Rainbow ⟶ Application to Inflation

[R. Garattini and M. Sakellariadou, Phys. Rev. D 90 (2014) 4, 043521; arXiv:1212.4987 [gr-qc]]

$$ds^2 = -\frac{N^2(t)}{g_1^2(E/Ep)}dt^2 + \frac{a^2(t)}{g_2^2(E/Ep)}d\Omega_3^2 \quad \Leftrightarrow \text{ Distorted } \textit{FLRW metric}$$

$$\left[16\pi G\frac{g_1^2(E/E_{\rm Pl})}{g_2^3(E/E_{\rm Pl})}\tilde{G}_{ijkl}\tilde{\pi}^{ij}\tilde{\pi}^{kl} - \frac{\sqrt{\tilde{g}}}{16\pi G g_2(E/E_{\rm Pl})}\left(\tilde{R} - \frac{2\Lambda}{g_2^2(E/E_{\rm Pl})}\right)\right]\Psi(a) = 0\,.$$

$$\left[-\frac{\partial^2}{\partial a^2} - \frac{q}{a}\frac{\partial^2}{\partial a} + \left(\frac{3\pi g_2(E/E_P)}{2Gg_1(E/E_P)}\right)^2 a^2\left(1 - \frac{\Lambda a^2}{3g_2^2(E/E_P)}\right)\right]\Psi(a) = 0$$

$$\left[-\frac{\partial^2}{\partial a^2} - \frac{q}{a}\frac{\partial^2}{\partial a} + \left(\frac{3\pi}{2G}\right)^2 a^2\left(1 - \frac{\Lambda_{eff}a^2}{3}\right)\right]\Psi(a) = 0 \qquad \Lambda_{eff} = \Lambda\left(1 + \frac{4G}{\Lambda\pi}V(\phi)\right)$$

Gravity's Rainbow ⟶ Application to Hořava-Lifshitz theory

[R. Garattini and E.N.Saridakis, Eur.Phys.J. C75 (2015) 7, 343; arXiv:1411.7257 [gr-qc]]

$$\left[16\pi G\frac{g_1^2\left(E/E_{\rm Pl}\right)}{g_2^3\left(E/E_{\rm Pl}\right)}\tilde{G}_{ijkl}\tilde{\pi}^{ij}\tilde{\pi}^{kl}-\frac{\sqrt{\tilde{g}}}{16\pi G g_2\left(E/E_{\rm Pl}\right)}\left(\tilde{R}-\frac{2\Lambda}{g_2^2\left(E/E_{\rm Pl}\right)}\right)\right]\Psi(a)=0\,.$$

$$\left[-\frac{\partial^2}{\partial a^2}-\frac{q}{a}\frac{\partial^2}{\partial a}+\left(\frac{3\pi g_2\left(E/E_P\right)}{2Gg_1\left(E/E_P\right)}\right)^2 a^2\left(1-\frac{\Lambda a^2}{3g_2^2\left(E/E_P\right)}\right)\right]\Psi(a)=0$$

But we can go beyond this…indeed if $E\equiv E\left(a(t)\right)$ then

$$K^{ij}K_{ij}-\lambda K^2=3g_1^2\left(E/E_P\right)\frac{1-3\lambda}{N^2(t)}\left(\frac{\dot{a}}{a}\right)^2 f\left(a(t),a\right)\quad\text{where}\quad f\left(a(t),a\right)=1-2a(t)A(t)+A^2(t)a^2(t)$$

$$A(t)=\frac{1}{g_2\left(E\left(a(t)\right)/E_P\right)E_P}\frac{dg_2\left(E\left(a(t)\right)/E_P\right)}{dE}\frac{dE}{da}$$

Gravity's Rainbow ⟹ Application to Hořava-Lifshitz theory

[R. Garattini and E.N.Saridakis, Eur.Phys.J. C75 (2015) 7, 343; arXiv:1411.7257 [gr-qc]]

If we fix $g_1^2\left(E/E_P\right)f\left(a(t),a\right)=1$

$$g_2^2\left(E/E_P\right)=1-c_1\frac{E^2\left(a(t)\right)}{E_P^2}-c_2\frac{E^4\left(a(t)\right)}{E_P^4}$$

then using the "normal" dispersion relation $E^2=\dfrac{k^2}{a^2(t)}$ and $E_P^2=\dfrac{k^2}{a_P^2}=\dfrac{k^2}{l_P^2}=\dfrac{k^2}{G}$

$$g_2^2\left(E/E_P\right)=1-\frac{16b\pi G}{a^2(t)}-\frac{256\pi^2G^2}{a^4(t)}=1-\frac{16b\pi R}{R_0}-\frac{256\pi^2R^2}{R_0^2}\leftarrow$$

Potential part of the Projectable Hořava-Lifshitz theory without detailed balanced Condition z=3

$$E_P^2=G^{-1},\qquad c_1=16b\pi\qquad\text{and}\qquad c_2=256c\pi^2.$$

It is possible to build a map
Also for SSM

Gravity's Rainbow ⟹ Application to Hořava-Lifshitz theory

[R. Garattini and E.N.Saridakis, Eur.Phys.J. C75 (2015) 7, 343; arXiv:1411.7257 [gr-qc]]

$$\mathcal{L}_{Pp} = N\sqrt{g}\Big\{g_0\kappa^{-1} + g_1 R + \kappa\left(g_2 R^2 + g_3 R^{ij}R_{ij}\right) + \kappa^2\left(g_4 R^3 + g_5 R R^{ij}R_{ij} + g_6 R^i_j R^j_k R^k_i + g_7 R\nabla^2 R + g_8 \nabla_i R_{jk}\nabla^i R^{jk}\right)\Big\},$$

$$\mathcal{L}_P = N\sqrt{g}\left[g_0\kappa^{-1} + g_1\frac{6}{a^2(t)} + \frac{12\kappa}{a^4(t)}\left(3g_2 + g_3\right) + \frac{24\kappa^2}{a^6(t)}\left(9g_4 + 3g_5 + g_6\right)\right].$$

$g_0\kappa^{-1} = 2\Lambda \quad g_1 = -1$

$$\begin{cases} 3g_2 + g_3 = b \\ 9g_4 + 3g_5 + g_6 = c \end{cases}$$

$b = c = 0 \rightarrow GR$

Applying the Rayleigh-Ritz procedure we can find candidate eigenvalues depending on the combination of the coupling constants

[R. G., P.R.D 86 123507 (2012) 7, 343; arXiv:0912.0136 [gr-qc]]

(a) g_0 is large and determined by the set of coupling constants (g_2, g_3). For example, it could be fine-tuned to Planck era values and therefore to the order of 10^{120}. This implies that $b = 6g_2 + g_3 \ll 1$. This can be achieved if $g_2, g_3 \ll 1$ or $6g_2 \simeq -g_3$. When $g_2, g_3 \ll 1$ we fall into a perturbative regime, but when $6g_2 \simeq -g_3$ this could not be the case. In this respect, this version of HL theory and the version with the detailed balanced condition behave in the same way except (eventually) for the smallness of the coupling constants g_2 and g_3.

(b) g_0 is of the order of unity or less and determined by the set of coupling constants (g_2, g_3). It can be fine-tuned to the values obtained from observation. This implies that $b = 6g_2 + g_3 \simeq 1$, which means that the set (g_2, g_3) can be in the perturbative region. For example, it is sufficient to take the couple $(g_2 = 1/12, \quad g_3 = 1/2)$. Indeed, since in case (a) we started with $g_0 \simeq 10^{120}$, we do not need to obtain $g_0 \simeq 10^{-120}$ as a final result.

(c) g_0 is large and determined by the set of coupling constants (g_4, g_5, g_6) in Eq. (62). It can be fine-tuned to be of the order of 10^{120}. This can be realized when the combination $c = 9g_4 + 3g_5 + g_6 < 0$, and even if $1 + 16\tilde{c}c \neq 0$ nothing prevents us from considering the case in which $1 + 16\tilde{c}c$ is small. Since we are dealing with three coupling constants, it is not trivial to have a discussion similar to the cases in (a) and (b), as we have more combinations. However, when one of the constants vanishes one can repeat the same analysis as case (a), and one discovers three other sub-subcases

MSS in a VSL Cosmology

R.G. and M.De Laurentis, arXiv:1503.03677

$$ds^2 = -N^2(t)\, c^2(t)\, dt^2 + a^2(t)\, d\Omega_3^2,$$

$$c(t) = c_0 \left(\frac{a(t)}{a_0}\right)^{\alpha}$$

Albrecht, Barrow, Harko, Maguejio, Moffat..

The WDW equation becomes

$$\left(-\frac{\partial^2}{\partial a^2} - \frac{q}{a}\frac{\partial}{\partial a} + U_c(a)\right)\Psi(a) = 0,$$

$$U_c(a) = \left(\frac{3\pi}{2G\hbar}\right)^2 a^2 c^6(t)\left(1 - \frac{\Lambda}{3}a^2\right) = \left(\frac{3\pi c_0^3}{2G\hbar a_0^{3\alpha}}\right)^2 a^{2+6\alpha}\left(1 - \frac{\Lambda}{3}a^2\right).$$

MSS in a VSL Cosmology

R.G. and M.De Laurentis, arXiv:1503.03677

$$\frac{\int \mathcal{D}a a^q \Psi^*(a)\left[-\frac{\partial^2}{\partial a^2}-\frac{q}{a}\frac{\partial}{\partial a}+\left(\frac{3\pi}{2l_P^2 a_0^{3\alpha}}\right)^2 a^{2+6\alpha}\right]\Psi(a)}{\int \mathcal{D}a a^q \Psi^*(a)\left[a^{4+6\alpha}\right]\Psi(a)} = 3\Lambda\left(\frac{\pi}{2l_P^2 a_0^{3\alpha}}\right)^2,$$

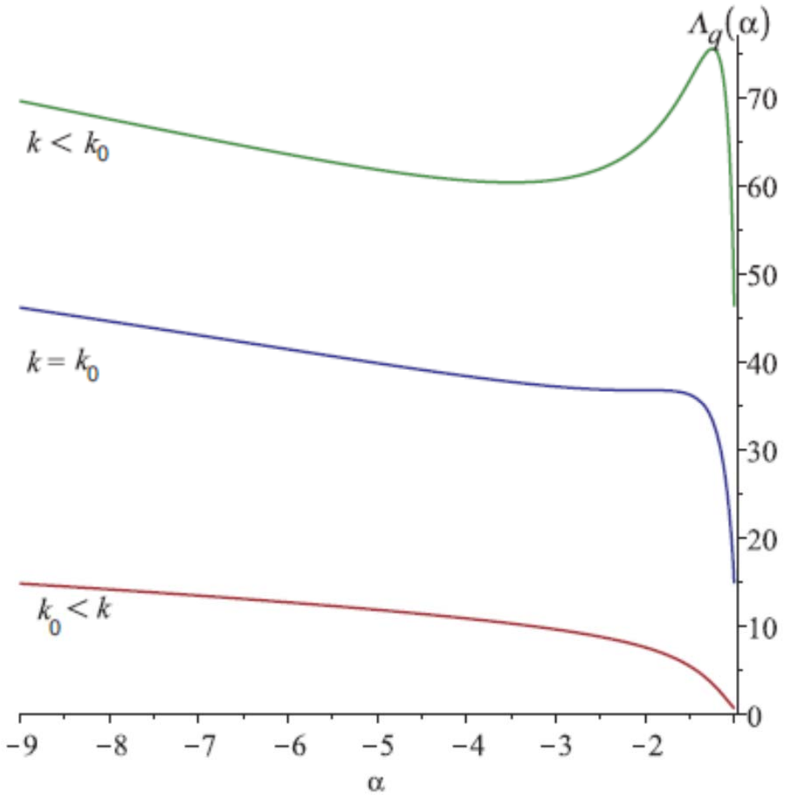
Setting $a_0 = kl_P$

$$c(t) = c_0\left(\frac{a(t)}{a_0}\right)^\alpha$$

$$\Psi(a) = a^{-\frac{q+1}{2}}(\beta a)^{-3\alpha}\exp\left(-\frac{\beta a^4}{2}\right)$$

$q=1$	$k_0 = 0.5779378002$	$\bar{\alpha} = -2.007150679$
$q=0$	$k_0 = 0.5843673484$	$\bar{\alpha} = -1.988596177$
$q=-1$	$k_0 = 0.6030705325$	$\bar{\alpha} = -1.940190188$

$$c(E/E_{Pl}) = \frac{dE}{dp} = c_0\frac{g_2(E/E_{Pl})}{g_1(E/E_{Pl})},$$



A Brief Mention to GUP

[R. Garattini and Mir Faizal; **N.P. B 905 (2016) 313** arXiv:1510.04423 [gr-qc]]

Deformed Momentum

$$\pi_a = \tilde{\pi}_a(1 - \alpha||\tilde{\pi}_a|| + 2\alpha^2||\tilde{\pi}_a||^2)$$

Deformed U.P.

$$\Delta a \Delta \pi_a = 1 - 2\alpha < \pi_a > + 4\alpha^2 < \pi_a^2 >$$

Trial Wave Function

$$\Psi(x) = x^\beta \exp\left(-\frac{\beta x^4}{2}\right)$$

$$\frac{\int_0^{+\infty} dx x^{\beta/2} \exp\left(-\frac{\beta x^4}{2}\right)\left[-\frac{d^2}{dx^2} + 5\alpha_0^2 \frac{d^4}{dx^4}\right] x^{\beta/2} \exp\left(-\frac{\beta x^4}{2}\right)}{\int_0^{+\infty} dx x^\beta \exp(-\beta x^4)} = \tilde{\Lambda}\frac{3\pi^2}{4}$$

Flat space

α_0	β_m	$\tilde{\Lambda}_{\alpha_0}(\beta_m)$
1	1.053	29.24
10	1.017	246.29
20	1.012	481.46

Higher Order Derivative

Generalization

From Mini-SuperSpace
to
Field Theory in 3+1
Dimensions

The Cosmological Constant
as a Zero Point Energy
Calculation

Wheeler-De Witt Equation

B. S. DeWitt, Phys. Rev.**160**, 1113 (1967).

$$\frac{1}{V}\frac{\int D\mu[h]\Psi^*[h]\int_\Sigma d^3x\widehat{\Lambda}_\Sigma\Psi[h]}{\int D\mu[h]\Psi^*[h]\Psi[h]}=-\frac{\Lambda}{\kappa}$$

Induced Cosmological "Constant"

$$D\mu[h]=D\left[h_{ij}^{\perp}\right]D\left[\xi_j^T\right]D[h]J$$

Solve this infinite dimensional PDE with a Variational Approach without matter fields contribution

Ψ is a trial wave functional of the gaussian type

Schrödinger Picture

Spectrum of Λ depending on the metric

Energy (Density) Levels

Eliminating Divergences using Gravity's Rainbow

[R.G. and G.Mandanici, Phys. Rev. D 83, 084021 (2011), arXiv:1102.3803 [gr-qc]]

One loop Graviton Contribution

$$\Delta_2 = \underbrace{\left(\Delta h\right)_{ij} - 4R_{ia}h_j^a + Rh_{ij}}_{\text{Modified Lichnerowicz operator}}$$

$$\left(\Delta h\right)_{ij} = \underbrace{\Delta h_{ij} - 2R_{ijkl}h^{jl} + R_{ia}h_j^a + R_{ja}h_i^a}_{\text{Standard Lichnerowicz operator}}$$

$$\left(\Delta_2 \tilde{h}^{\perp}\right)_{ij} = \frac{\mathrm{E}^2}{g_2^2\left(E\right)}\tilde{h}_{ij}^{\perp}$$

$$\hat{\Lambda}_{\Sigma}^{\perp} = \frac{g_2^3\left(E\right)}{4V}\int_{\Sigma} d^3x\sqrt{\tilde{\tilde{g}}}\,\tilde{G}^{ijkl}\left[\left(2\kappa\right)\frac{g_1^2\left(E\right)}{g_2^3\left(E\right)}\tilde{K}^{-1,\perp}\left(x,x\right)_{ijkl} + \frac{1}{2\kappa g_2\left(E\right)}\left(\tilde{\Delta}_2\tilde{K}^{\perp}\left(x,x\right)\right)_{ijkl}\right]$$

$$\tilde{K}\left(\vec{x},\vec{y}\right)_{ijkl} := \sum_{\tau}\frac{\tilde{h}\left(\vec{x}\right)_{ij}^{(\tau)\perp}\,\tilde{h}\left(\vec{y}\right)_{kl}^{(\tau)\perp}}{2\lambda\left(\tau\right)g_2^4\left(E\right)} \quad \text{(Propagator)}$$

Eliminating Divergences using Gravity's Rainbow

[R.G. and G.Mandanici, Phys. Rev. D 83, 084021 (2011), arXiv:1102.3803 [gr-qc]]

$$\begin{cases} m_1^2(r) = \dfrac{6}{r^2}\left(1-\dfrac{b(r)}{r}\right)+\dfrac{3b'(r)}{2r^2}-\dfrac{3b(r)}{2r^3} \\ m_2^2(r) = \dfrac{6}{r^2}\left(1-\dfrac{b(r)}{r}\right)+\dfrac{b'(r)}{2r^2}+\dfrac{3b(r)}{2r^3} \end{cases}$$

We can define an r-dependent radial wave number

$$k^2(r,l,E_{nl}) = \frac{E_{nl}^2}{g_2^2(E/E_P)} - \frac{l(l+1)}{r^2} - m_i^2(r) \qquad r \equiv r(x)$$

$$\frac{\Lambda}{8\pi G} = -\frac{1}{3\pi^2}\sum_{i=1}^{2}\int_{E^*}^{+\infty} E_i g_1(E/E_P) g_2(E/E_P) \frac{d}{dE_i}\sqrt{\left(\frac{E_i^2}{g_2^2(E/E_P)} - m_i^2(r)\right)^3}\, dE_i$$

Standard Regularization

$$\frac{\Lambda}{8\pi G} = -\frac{1}{16\pi^2}\int_{\sqrt{m_i^2(r)}}^{+\infty} \frac{\omega_i^2}{\left(\omega_i^2 - m_i^2(r)\right)^{\varepsilon-\frac{1}{2}}}\, d\omega_i$$

Gravity's Rainbow and the Cosmological Constant

R.G. and G.Mandanici, Phys. Rev. D 83, 084021 (2011), arXiv:1102.3803 [gr-qc]

Popular Choice...... $\rightarrow$ Not Promising

$$g_1\left(E/E_P\right)=1-\eta\left(\frac{E}{E_P}\right)^n$$

$$g_2\left(E/E_P\right)=1$$

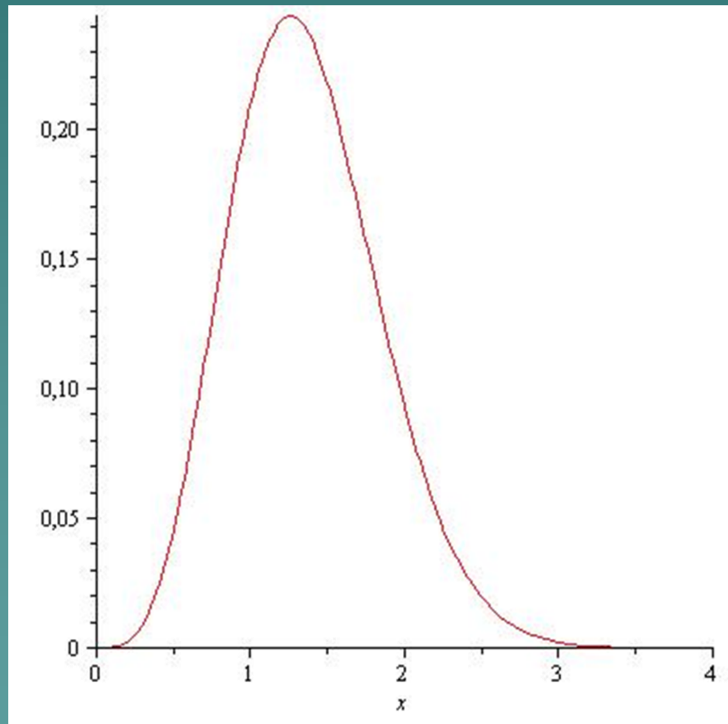
Failure of Convergence

$$g_1\left(E/E_P\right)=\exp\left(-\alpha\frac{E^2}{E_P^2}\right)\left(1+\beta\frac{E}{E_P}\right)$$

$$g_2\left(E/E_P\right)=1$$

Minkowski - de Sitter - Anti-de Sitter

$$m_1^2\left(r\right)=m_2^2\left(r\right)=m_0^2\left(r\right)\rightarrow x=\sqrt{m_0^2\left(r\right)/E_P^2}$$



Conclusions and Outlooks

- The Wheeler De Witt equation can be considered as a Sturm-Liouville Problem→ Rayleigh-Ritz Variational procedure.
- In ordinary GR, we need a cut-off or a regularization/renormalization scheme.
- Application of Gravity's Rainbow can be considered to compute divergent quantum observables.
- Neither Standard Regularization nor Renormalization are required. This also happens in NonCommutative geometries. A tool for ZPE Computation
- A connection between Horava-Lifshits theory without detailed balanced condition and with projectability and Gravity's Rainbow seems possible, at least in a FLRW metric. This is expected also for a VSL
- Repeating the above procedure for a SSM
- Technical Problems with Kerr and other complicated metrics. Comparison with Observation.