Resummation in PDF fits

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LHC, New Physics, and the pursuit of Precision

LHC as a **discovery machine**

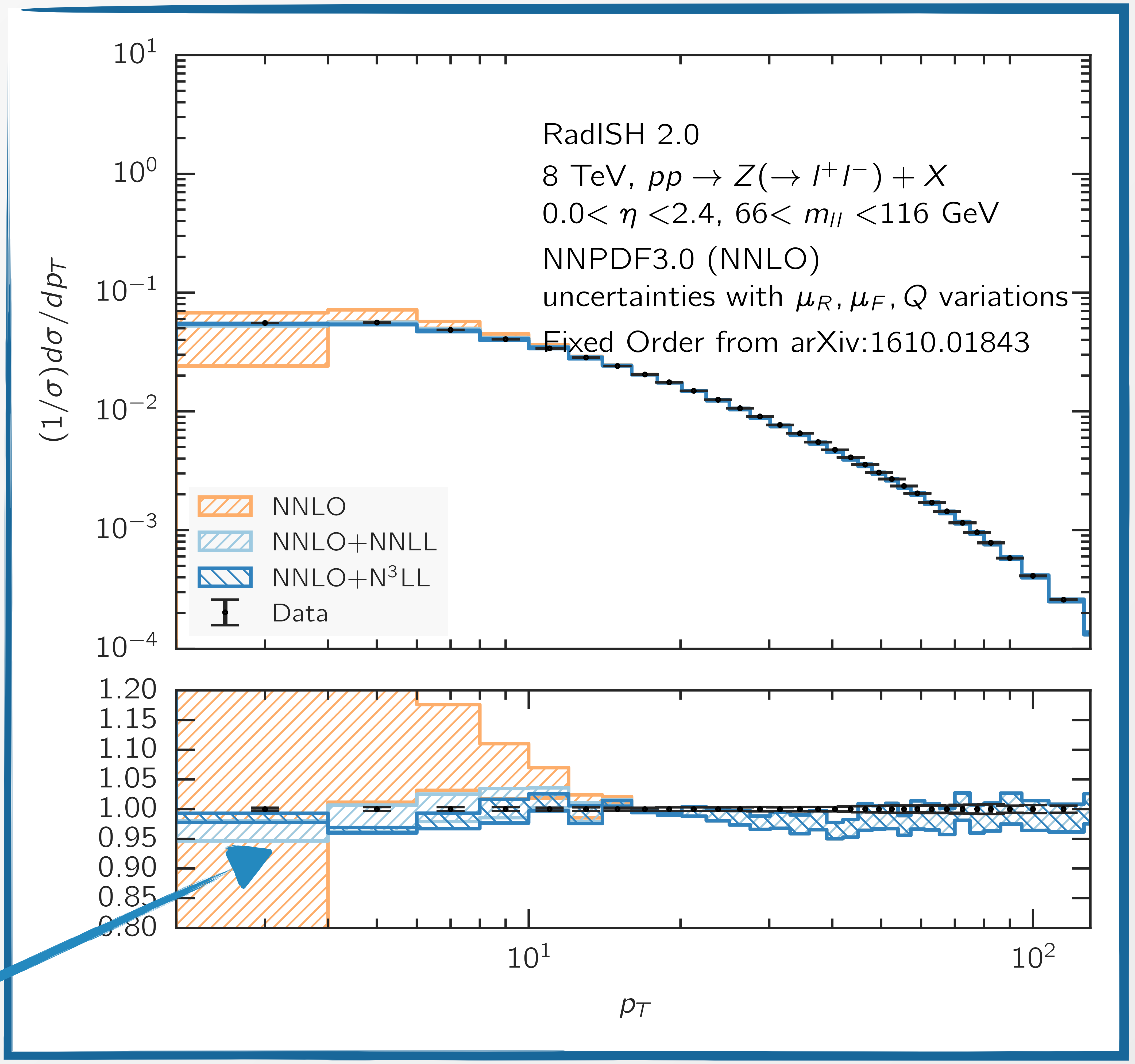
- Higgs Boson ✓
- BSM particles ✗ (~~never~~ as of today)

Focus in LHC run II

- Measurement of the Standard Model parameters with **very high precision**
- Signals of New Physics **beyond the Standard Model**

A theorist's Quest:

- New BSM scenarios to be tested
- New techniques to enhance signal/background ratio and isolate tiny deviations from SM predictions
- Development of **accurate** and **precise** theoretical predictions



Goal: 1% accuracy in theoretical predictions

[Bizon,Monni,Re,LR,Torielli et al, in preparation]

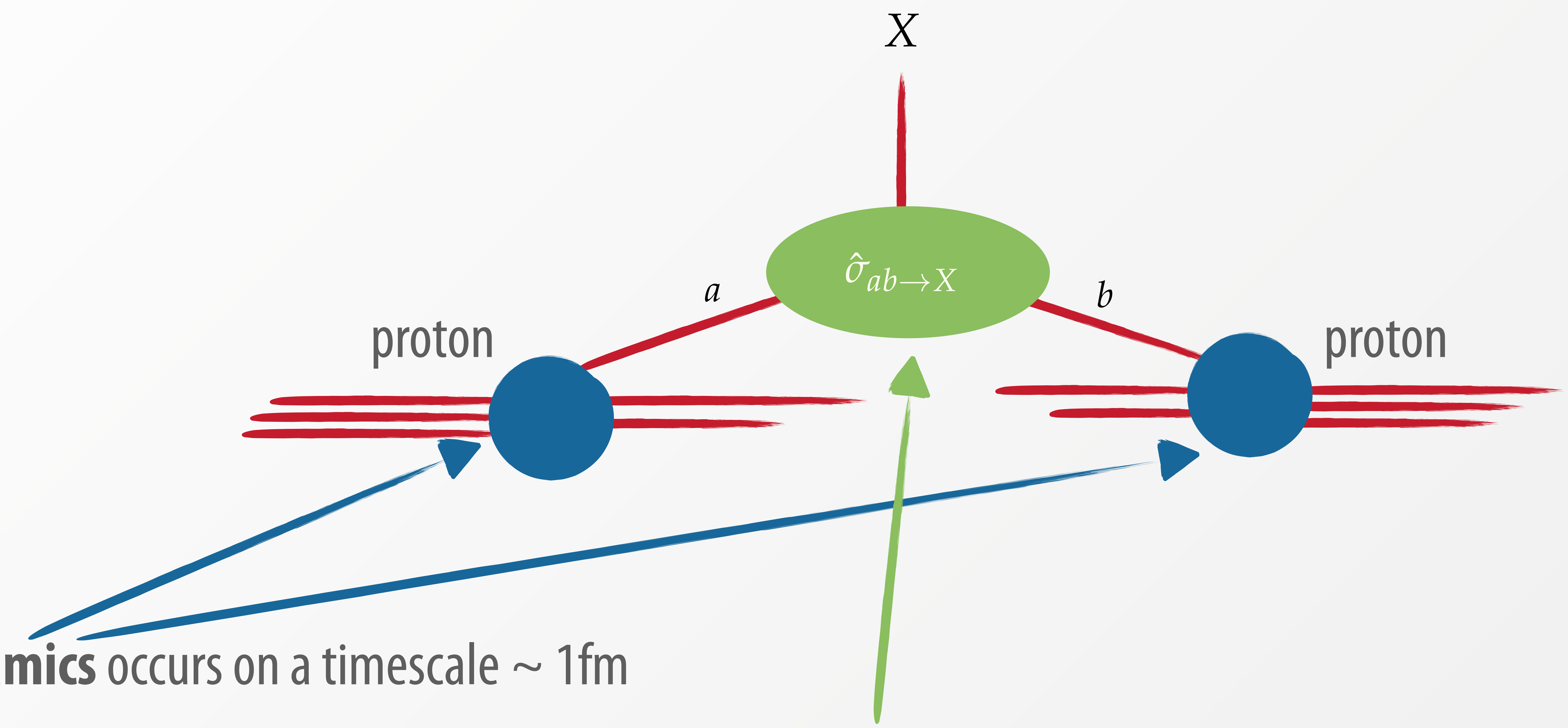
LHC, New Physics, and the pursuit of Precision

Large **HADRON** Collider

↓

A crucial ingredient in the physics precision programme at the LHC is the **accurate understanding** of the **internal structure of the initial state hadrons**

Factorization

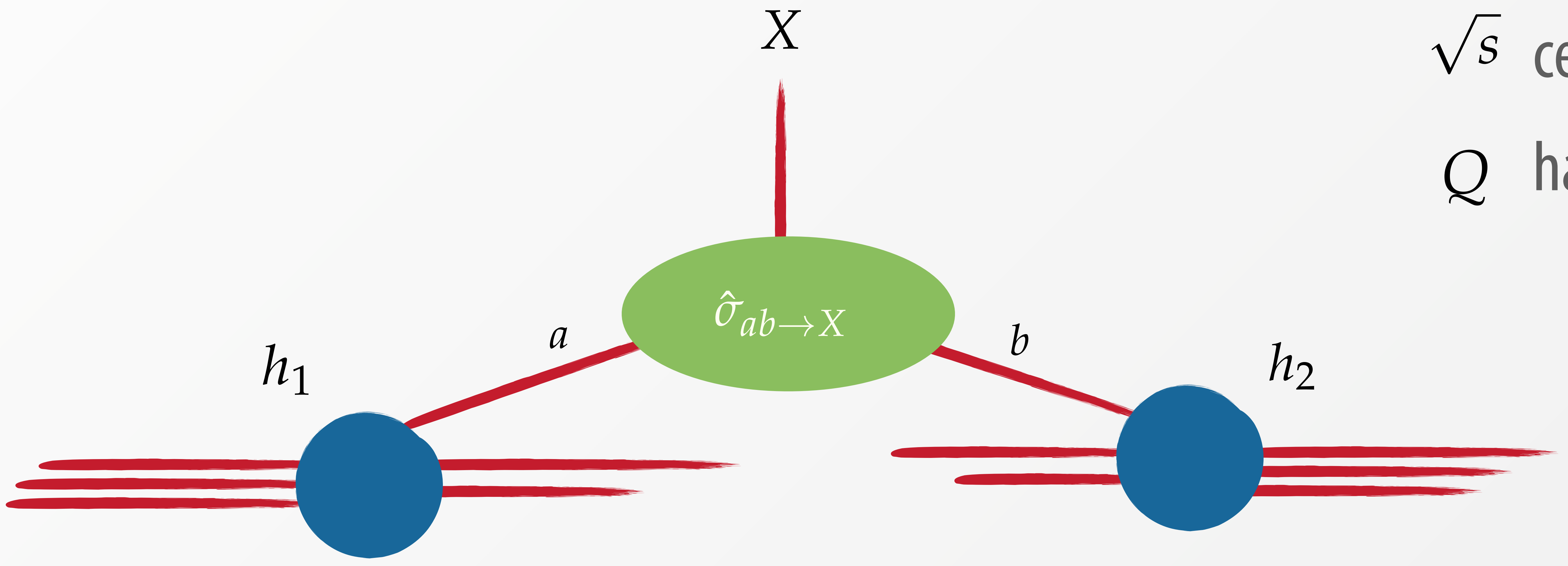


Proton's dynamics occurs on a timescale ~ 1fm

Production of a heavy particle e.g. Higgs Production (**hard process**) occurs on timescale $1/M_X \sim 1/100$ GeV ~ 0.002 fm

Large separation between scales allows to separate the hard process and treat it **independently** from the hadronic dynamics: **collinear factorization**

Factorization

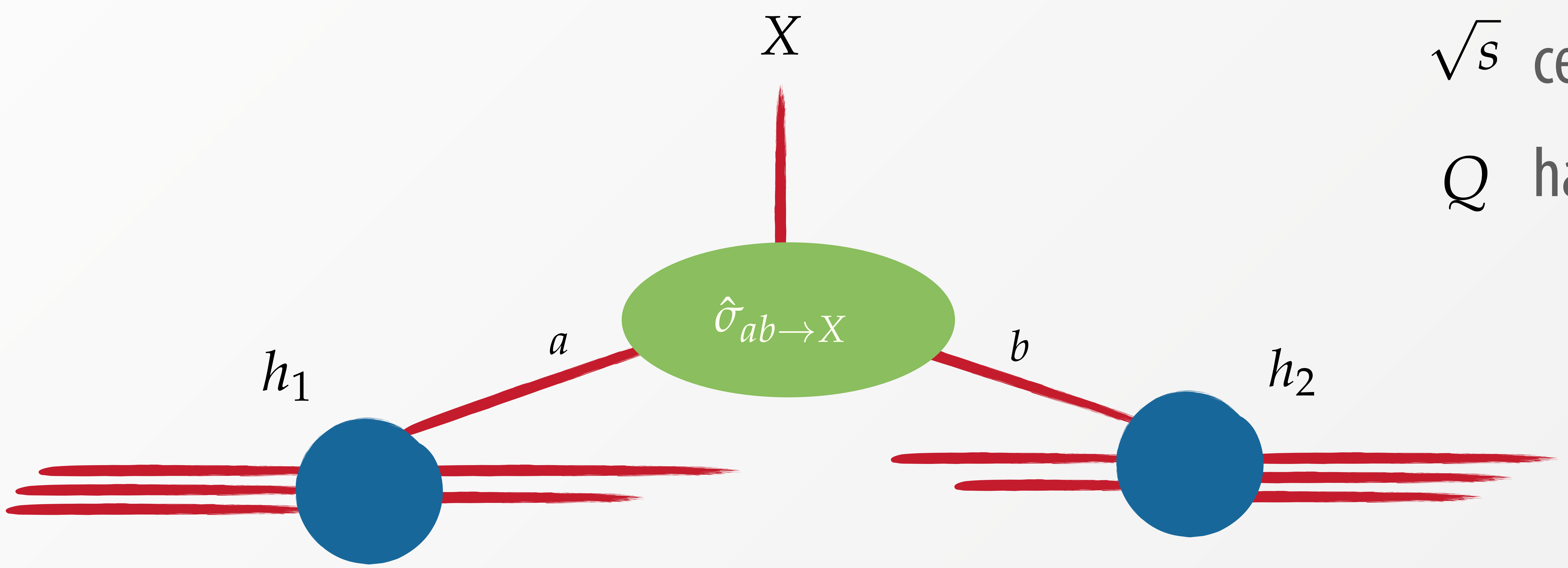


$\sqrt{s}$ centre-of-mass energy

Q hard scale of the process

$$\sigma_X(s,Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1,Q^2) f_{b/h_2}(x_2,Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s)$$

Factorization



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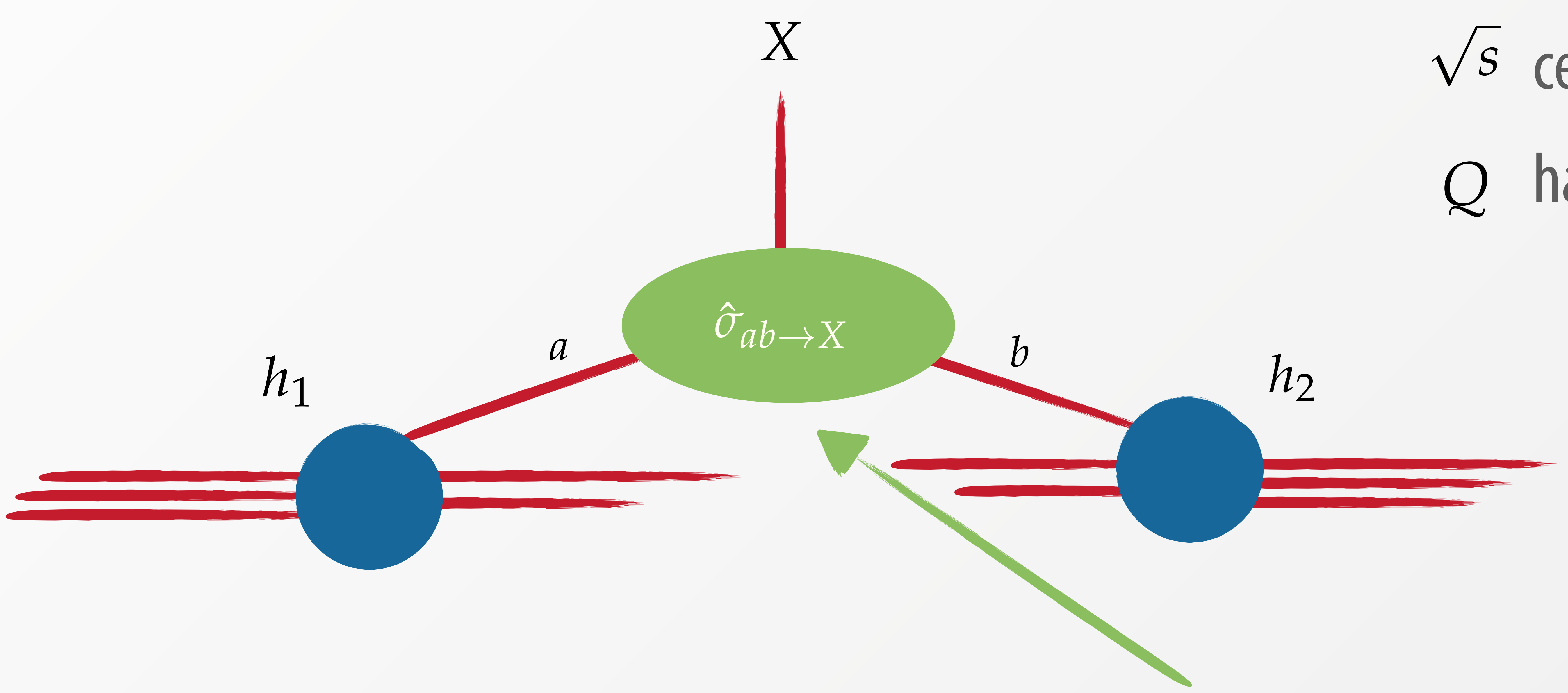
Factorization



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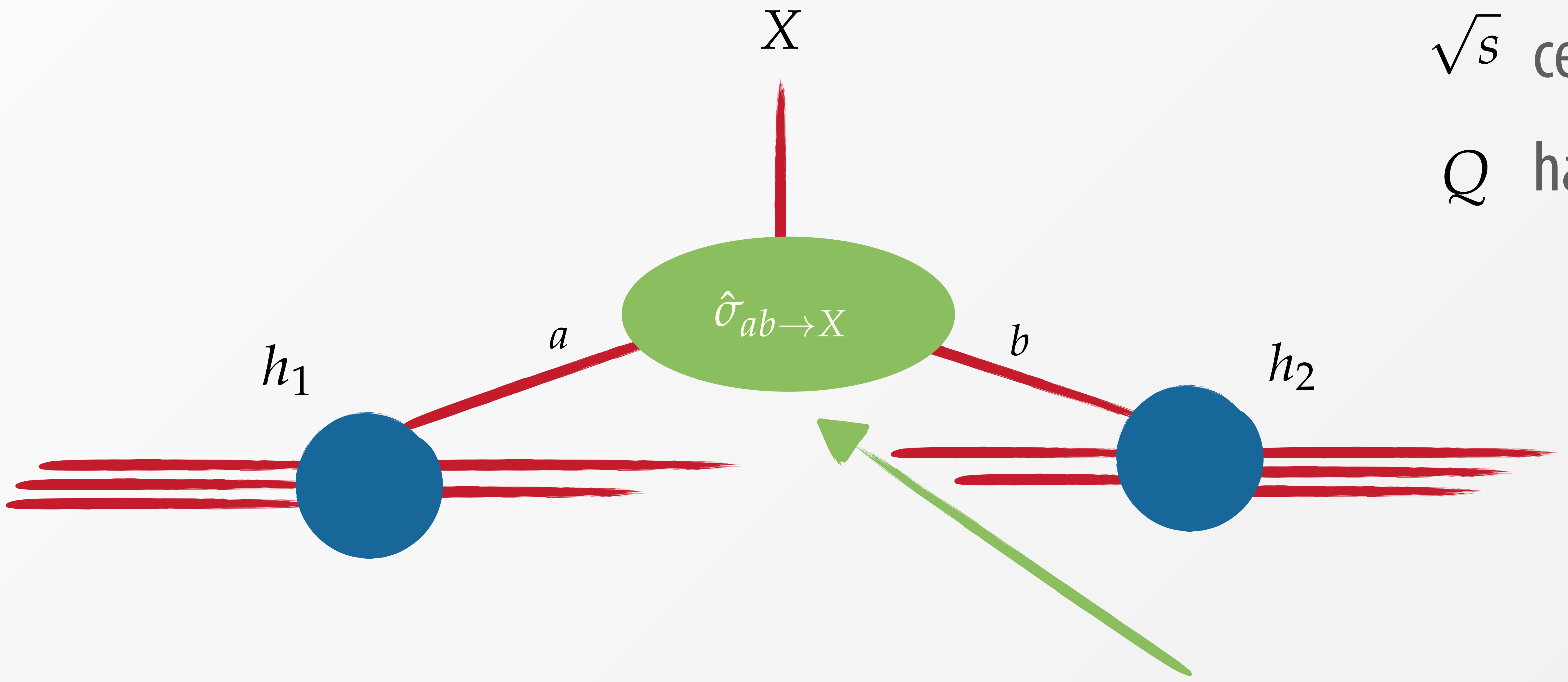
partonic cross-section

short-distance: **perturbative**

Factorization



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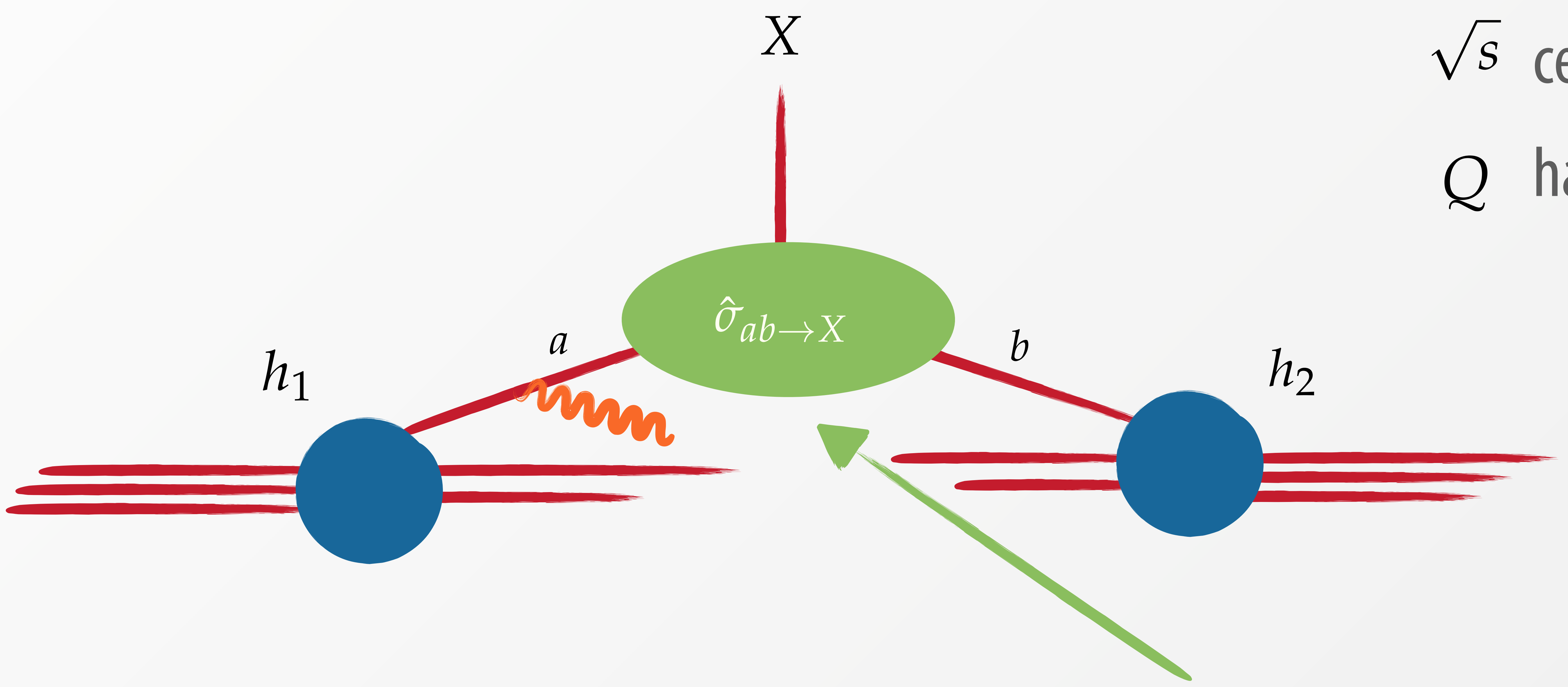
short-distance: **perturbative**

QCD at short distance is perturbative (**asymptotic freedom**)

$$\hat{\sigma} = \hat{\sigma}_0(1 + \ldots)$$

LO

Factorization



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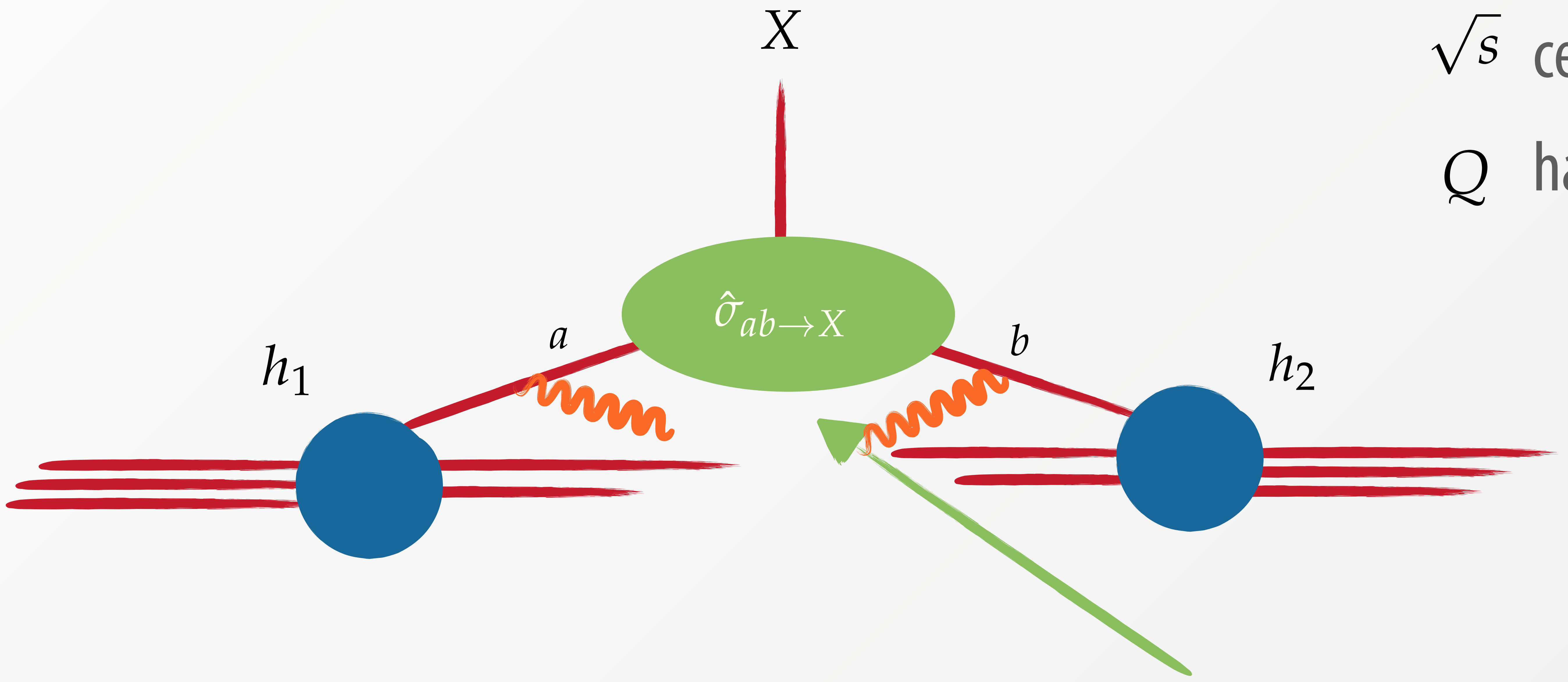
$$\hat{\sigma} = \hat{\sigma}_0 (1 + \alpha_s c_1 + \alpha_s^2 c_2 + \ldots)$$

NLO

Factorization



$\sqrt{s}$ centre-of-mass energy

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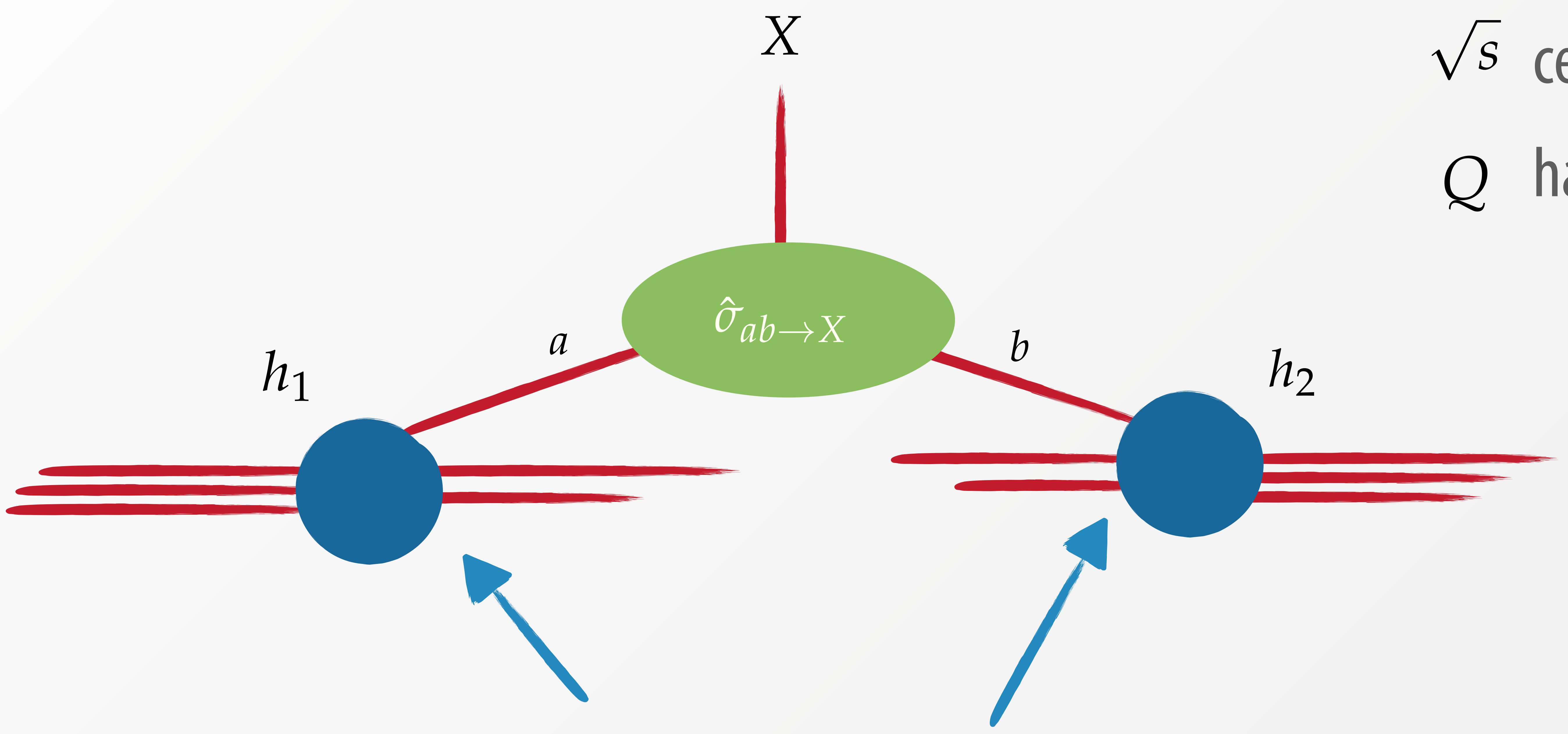

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Parton Distribution Functions (PDFs)

long-distance: **non-perturbative**

Parton distribution functions (PDFs) are **universal** objects which encode information on the substructure of the proton and which describe the dynamics of **quarks** and **gluons** (**partons**)

PDFs are currently **extracted from experiments**

Parton Distribution Functions

PDFs depend on two kinematic variables $f(x, Q^2)$

fraction of the momentum of the proton

Parton Distribution Functions

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Scale of the process

Parton Distribution Functions

PDFs depend on two kinematic variables $f(x, Q^2)$ and are parametrized at an initial scale Q_0

Parton Distribution Functions

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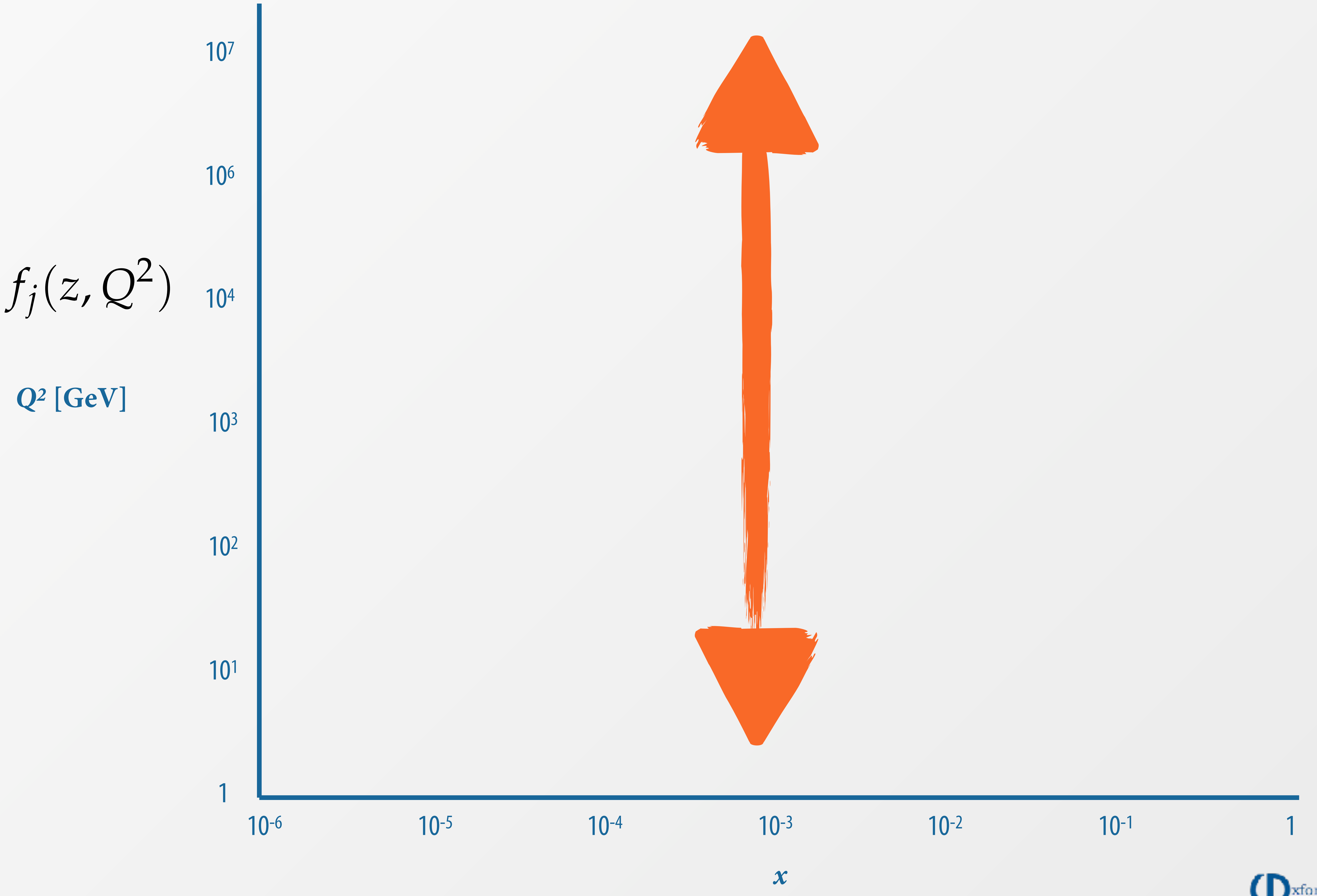
Evolution in Q^2 is encoded in **DGLAP equation**

$$Q^2 \frac{\partial}{\partial Q^2} f_i(x, Q^2) = \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}, \alpha_s(Q^2)\right) f_j(z, Q^2)$$



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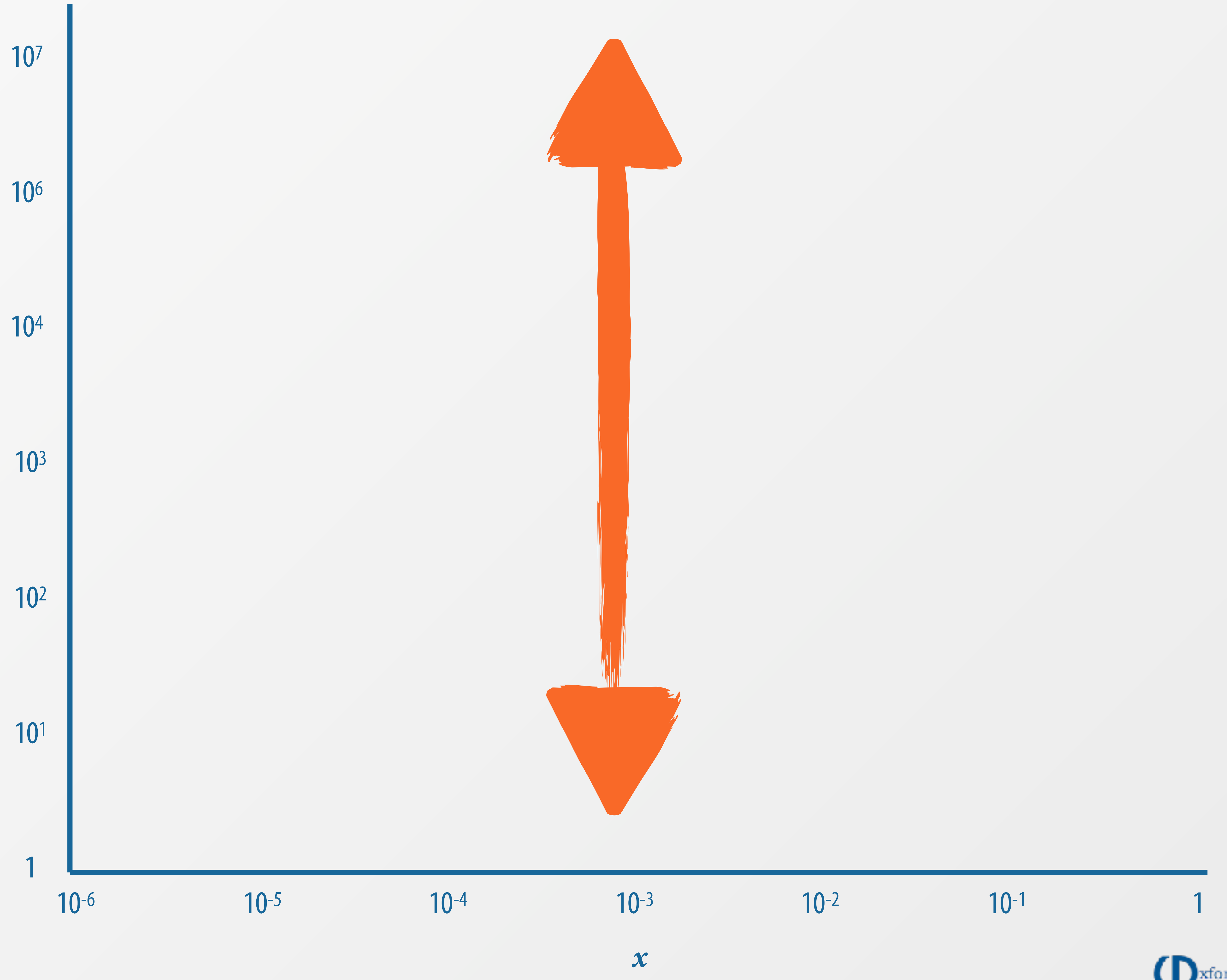
splitting functions



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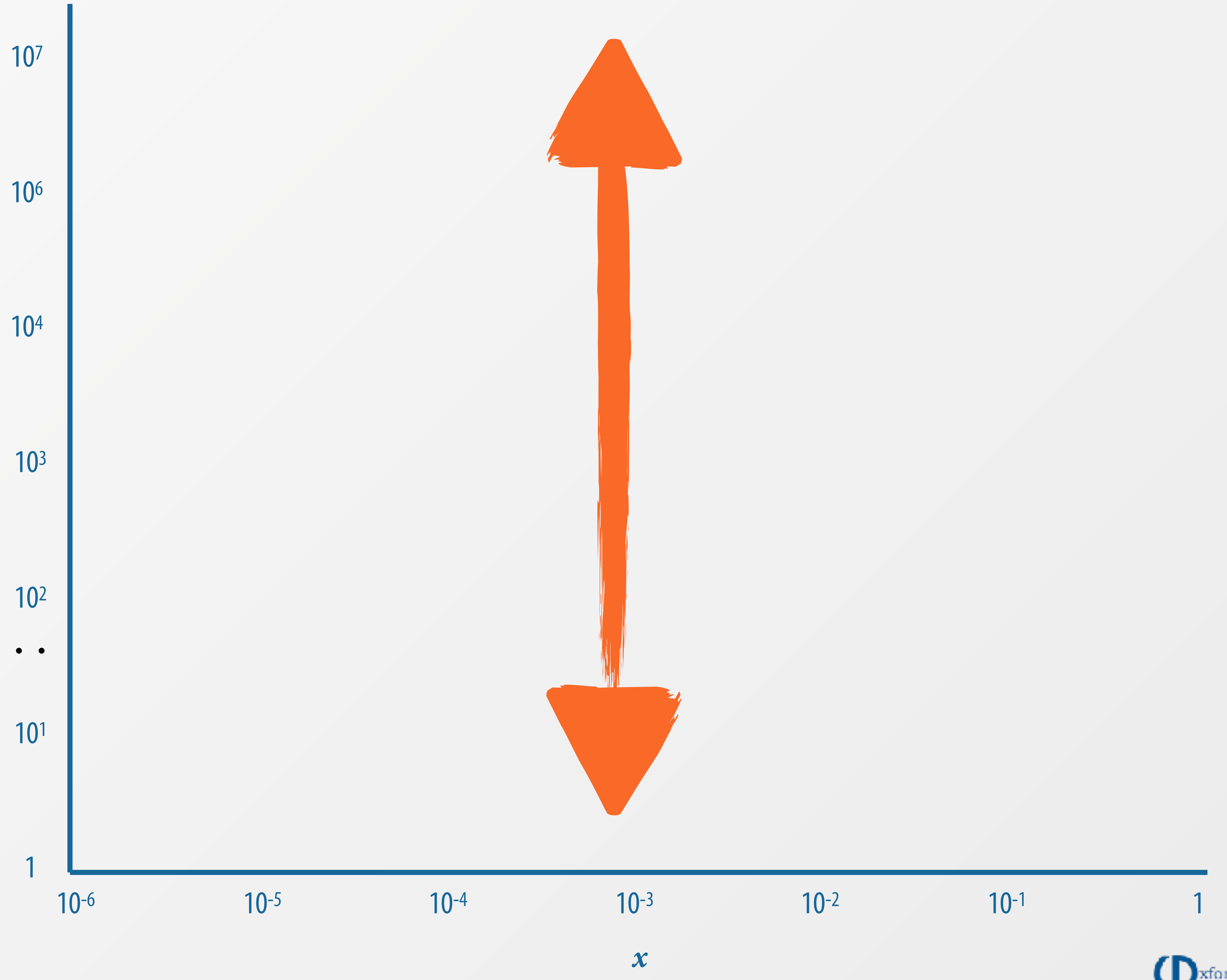
splitting functions

$$P_{ij}\left(x, \alpha_s(Q^2)\right) = P_{ij}^{(0)}(x) + \alpha_s P_{ij}^{(1)}(x) + \alpha_s^2 P_{ij}^{(2)}(x) + \ldots$$



DGLAP equation

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$2n_f + 1$ **coupled** differential equation

number of (active) flavours

However, strong interactions do not tell apart quarks and antiquarks (**charge conjugation** and $SU(n_f)$ **flavour symmetry**)

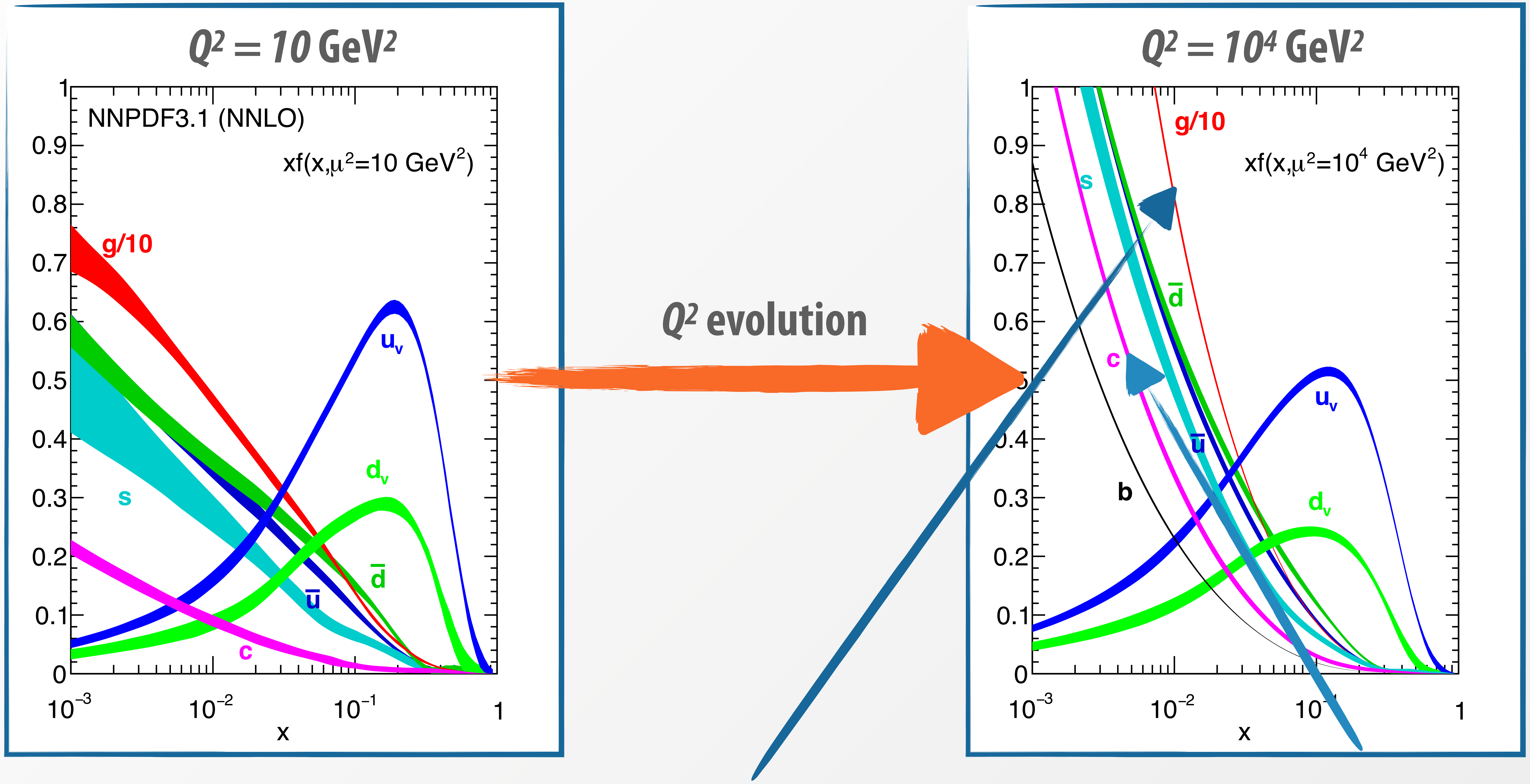
$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j}, \qquad P_{q_i \bar{q}_j} = P_{\bar{q}_i q_j}, \qquad P_{q_i g} = P_{\bar{q}_i g} \equiv P_{qg}, \qquad P_{g q_i} = P_{g \bar{q}_i} \equiv P_{gq}$$

Only **singlet combination** couples to **gluon**

$$\Sigma(x, Q^2) = \sum_i [q_i(x,t) + \bar{q}_i(x,t)]$$

$$Q^2 \frac{\partial}{\partial \ln Q^2} \begin{pmatrix} \Sigma \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix}$$

DGLAP equation



growth of small-*x* gluon

Parton lose momentum and shifts at smaller values of *x*

Parton Distribution Function Fits

$$\sigma_X(Q^2,s) = \sum_{a,b} f_{a/h_1}(Q^2) \otimes f_{b/h_2}(Q^2) \otimes \hat{\sigma}_{ab\to X}(Q^2,s)$$

$$Q^2 \frac{d}{dQ^2} f_i(Q^2) = P_{ij}(\alpha_s(Q^2)) \otimes f_j(Q^2)$$

theoretical prediction (to be compared with data)

theoretical input

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theoretical prediction (to be compared with data) → $\sigma_X(Q^2,s)$

theoretical input → $\hat{\sigma}_{ab\to X}(Q^2,s)$, $P_{ij}(\alpha_s(Q^2))$

PDF fits are typically based on **fixed-order** theory…

$$\hat{\sigma} = \hat{\sigma}_0(1 + \alpha_s c_1 + \alpha_s^2 c_2 + \ldots)$$

$$P_{ij}\left(x, \alpha_s(Q^2)\right) = P_{ij}^{(0)}(x) + \alpha_s P_{ij}^{(1)}(x) + \alpha_s^2 P_{ij}^{(2)}(x) + \ldots$$

…but is fixed-order theory always good enough?

Large logarithms

Single (double) logarithmic enhancement

$$\alpha_s^k \ln^j \qquad 0 \leq j \leq (2)k$$

Perturbative convergence is spoiled when

$$\alpha_s \ln^{(2)} \sim 1$$

e.g. small-x behaviour of splitting functions

Finite in the limit $x \to 0$

$$xP(x,\alpha_s) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n \left[\sum_{m=1}^{n} A_{m-1}^{(n)} \ln^{m-1}\frac{1}{x} + x\bar{P}^{(n)}(x)\right]$$

Instability at small-*x*

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Instability at small-*x*

All-order **resummation** of the logarithmically enhanced terms

($n \geq 0$, m=n) leading-logarithm (LL*x*), ($n \geq 0$, m=n,n-1) next-to-leading-logarithm (NLL*x*), etc.

Resummation in PDF fits

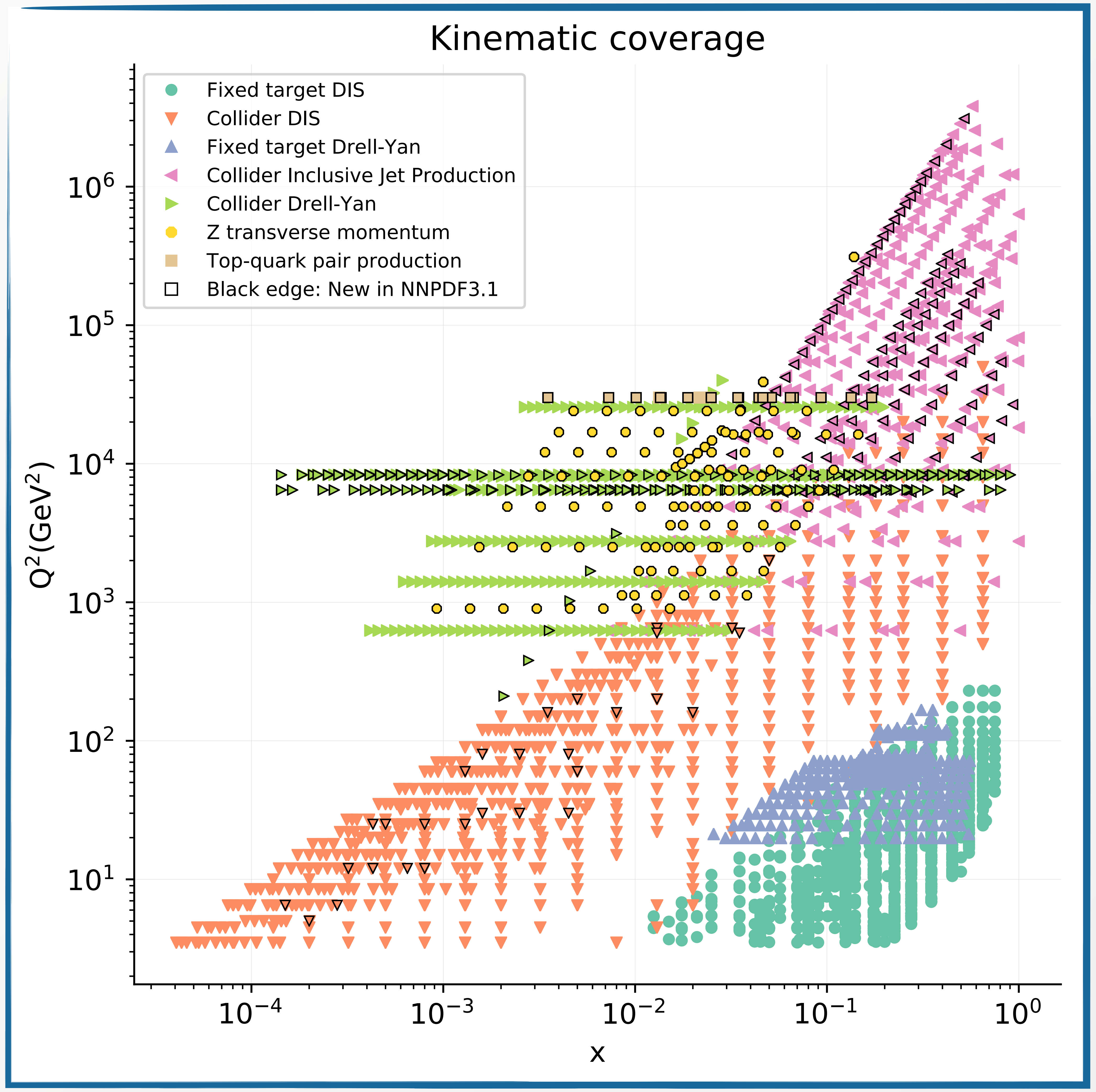
Including resummation in PDF fits:

- provides **consistent predictions** when resummed computations are used
- improves the **quality** of the PDF fits
- helps in investigating the impact of **missing higher orders**

. . . it brings us closer to 'all-order' PDFs

Global PDF fits

Processes used in global PDF fits



Global PDF fits

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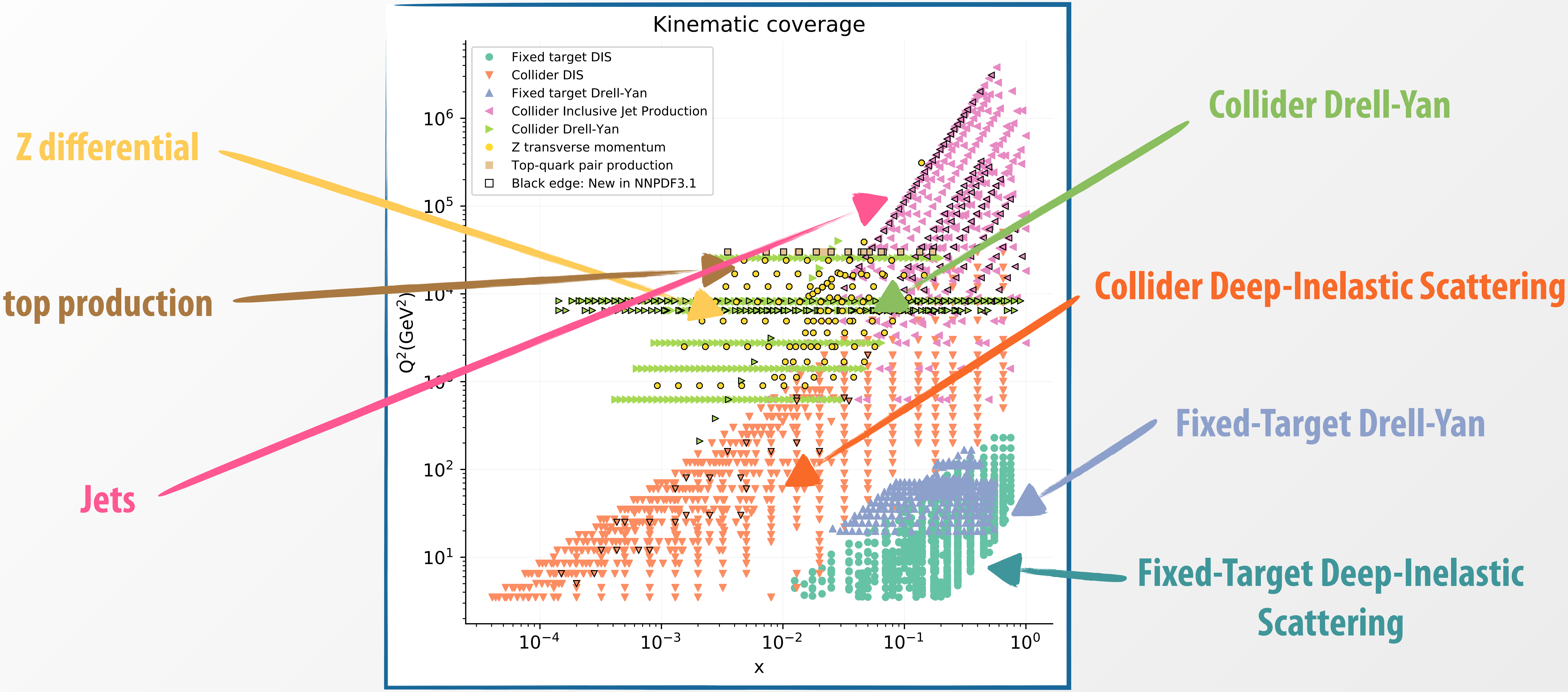
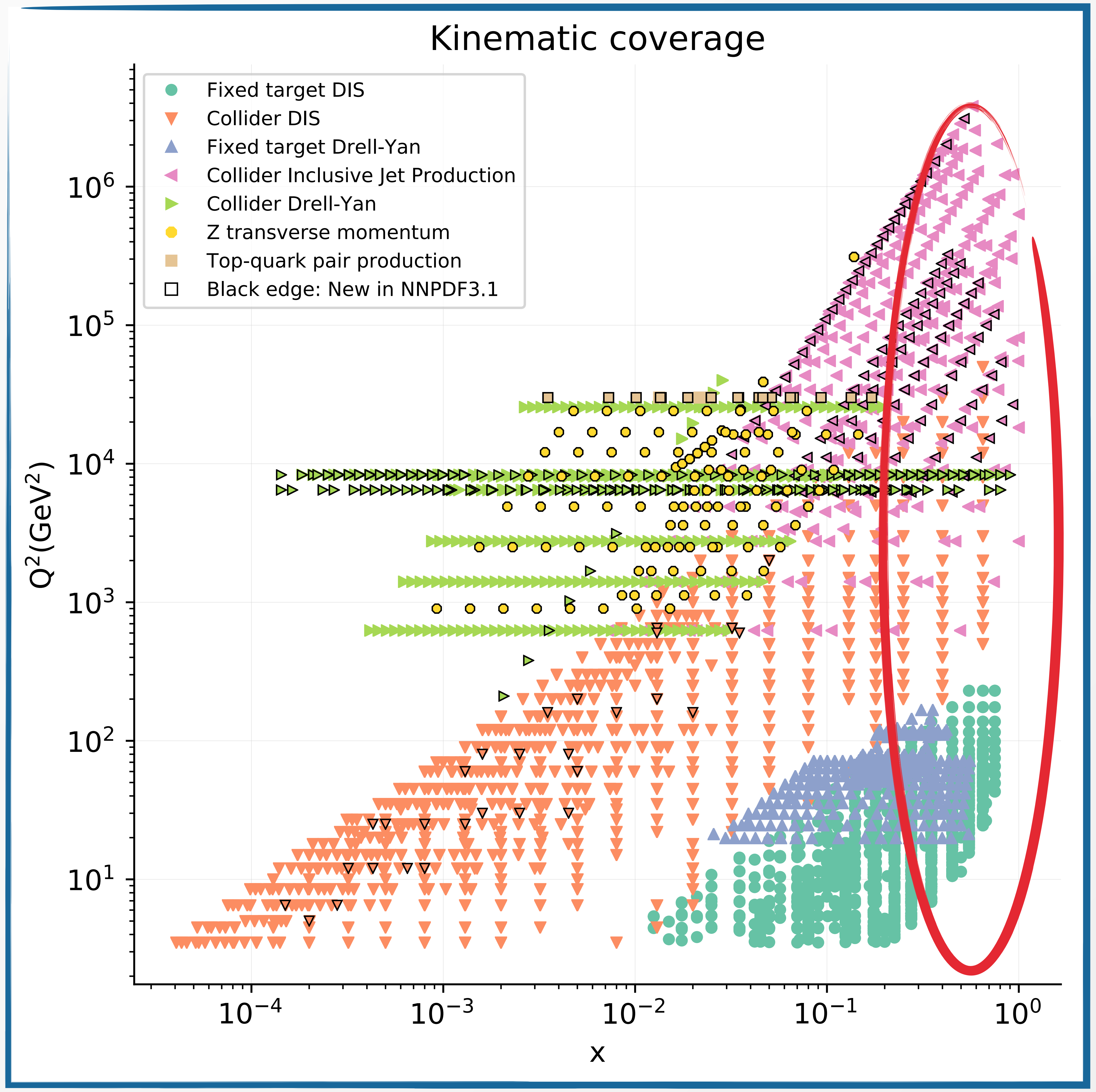


Resummation in global PDF fits



Large *x*: **threshold** resummation

double logs due to **soft** gluon emission

$$\left(\frac{\ln^k(1-x)}{(1-x)}\right)_+$$

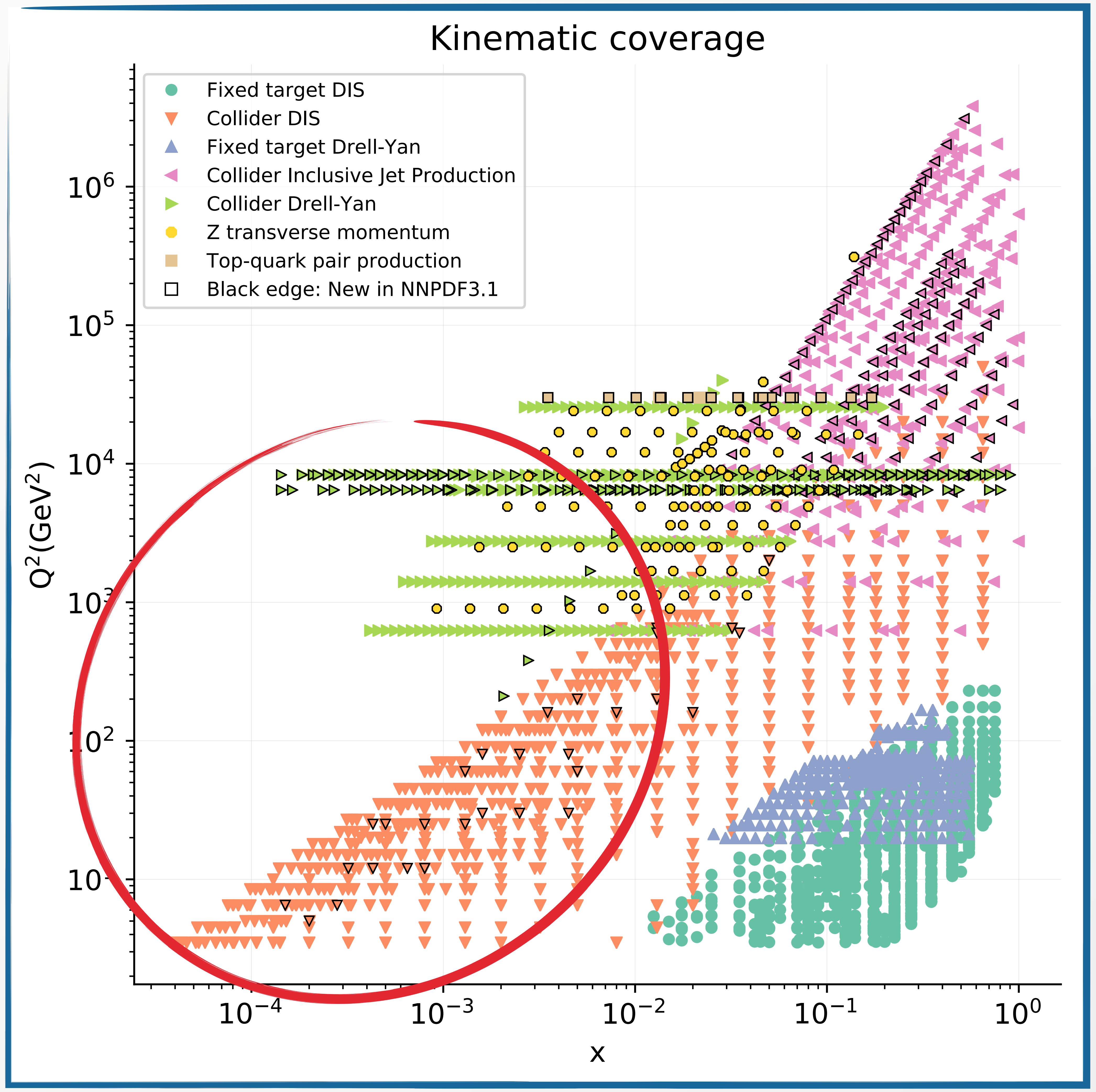
[Bonvini,Marzani,Rojo,LR,Ubiali,Ball,Bertone, Carrazza,Hartland 1507.01006]

Resummation in global PDF fits

Small x: **high-energy** resummation

single logs due to high-energy gluon emission

$$\frac{1}{x}\ln^k x$$



[Ball,Bertone,Bonvini,Marzani,Rojo,LR 1710.05935]

Resummation in global PDF fits

Resummation affects:

Observable (coefficient functions) $\sigma = \sigma_0 C(\alpha_s(\mu)) \otimes f(\mu) \, [\otimes f(\mu)]$

Evolution (splitting functions) $\mu^2 \frac{d}{d\mu^2} f(\mu) = P(\alpha_s(\mu)) \otimes f(\mu)$

	observable (coefficient function)	evolution (splitting function)
small x	**NLL*x****	**NLL*x***
large x	**(N)NNLL**	—

*starts at NLL*x*

PDFs with large-x resummation

[Bonvini,Marzani,Rojo,LR,Ubiali,Ball,Bertone, Carrazza,Hartland 1507.01006]

PDFs with large-x resummation: NNPDF3.0res

[Bonvini,Marzani,Rojo,LR,Ubiali,Ball,Bertone, Carrazza,Hartland 1507.01006]

Datasets considered in NNPDF3.0res

process	observable	included?
DIS	**$d\sigma/(dx dQ^2)$ (NC, CC, F2c…)**	✔
DY Z/γ	**$d\sigma/(dy dM^2)$**	✔
DY W	**differential in lepton kinematics**	✘ no public code available yet
$t\bar{t}$	**total σ**	✔
jets	**inclusive $d\sigma/(dy dp_T)$**	✘ NLL known to be poor

Accuracy is **competitive** with global fit, except for large-x gluon (jets not included)

Resummation is included supplementing fixed-order computations with **K-factors**

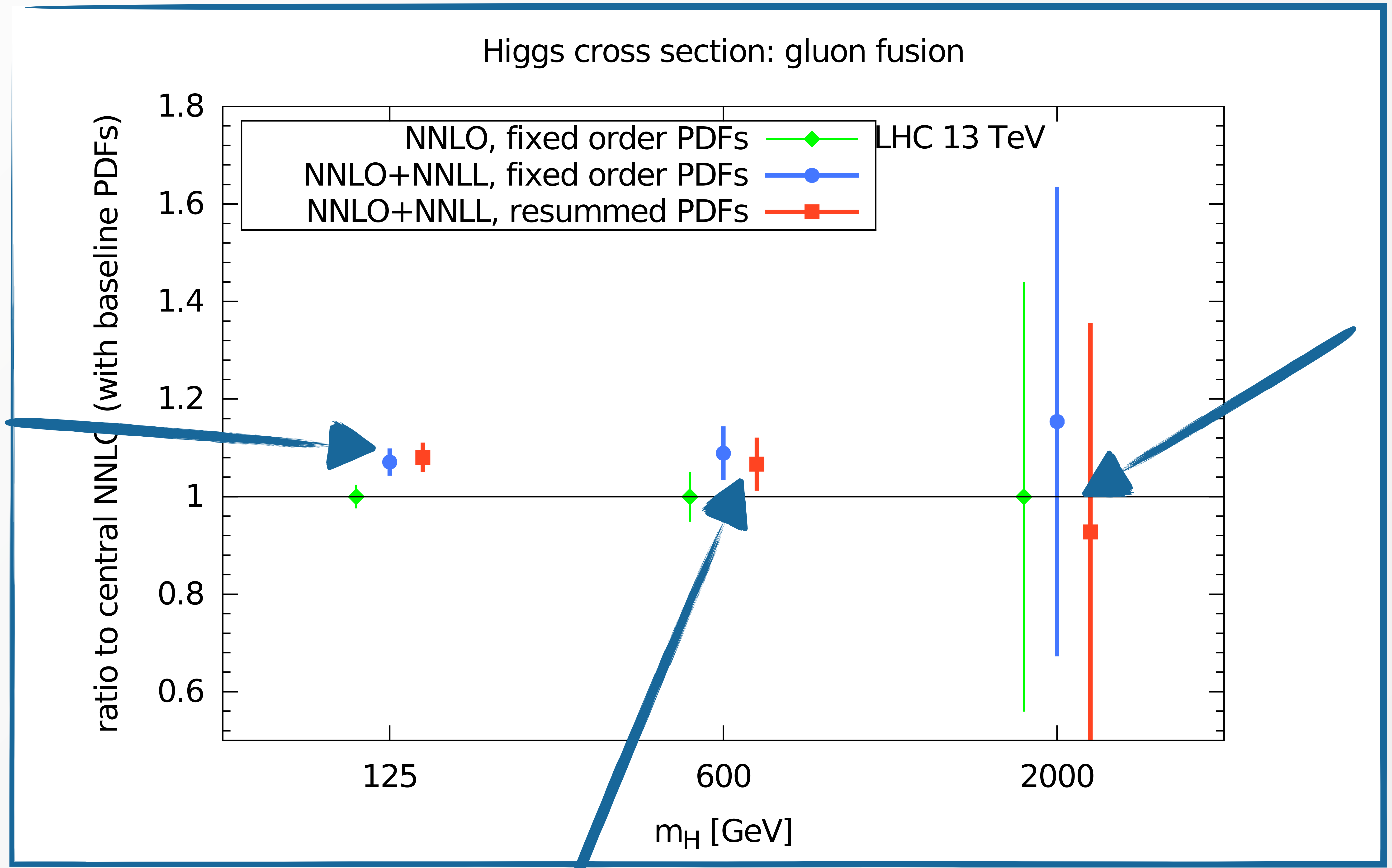
$$K^{\mathrm{N^kLO+N^kLL}} = \frac{\sigma^{\mathrm{N^kLO+N^kLL}}}{\sigma^{\mathrm{N^kLO}}}$$

public code **TROLL**

www.ge.infn.it/~bonvini/troll

NNPDF3.0res: Impact on phenomenology

Higgs Production



SM Higgs is not affected by resummation of PDFs

m_H~2 TeV NNLO+NNLL with resummed PDFs is similar to FO PDFs (larger uncertainty)

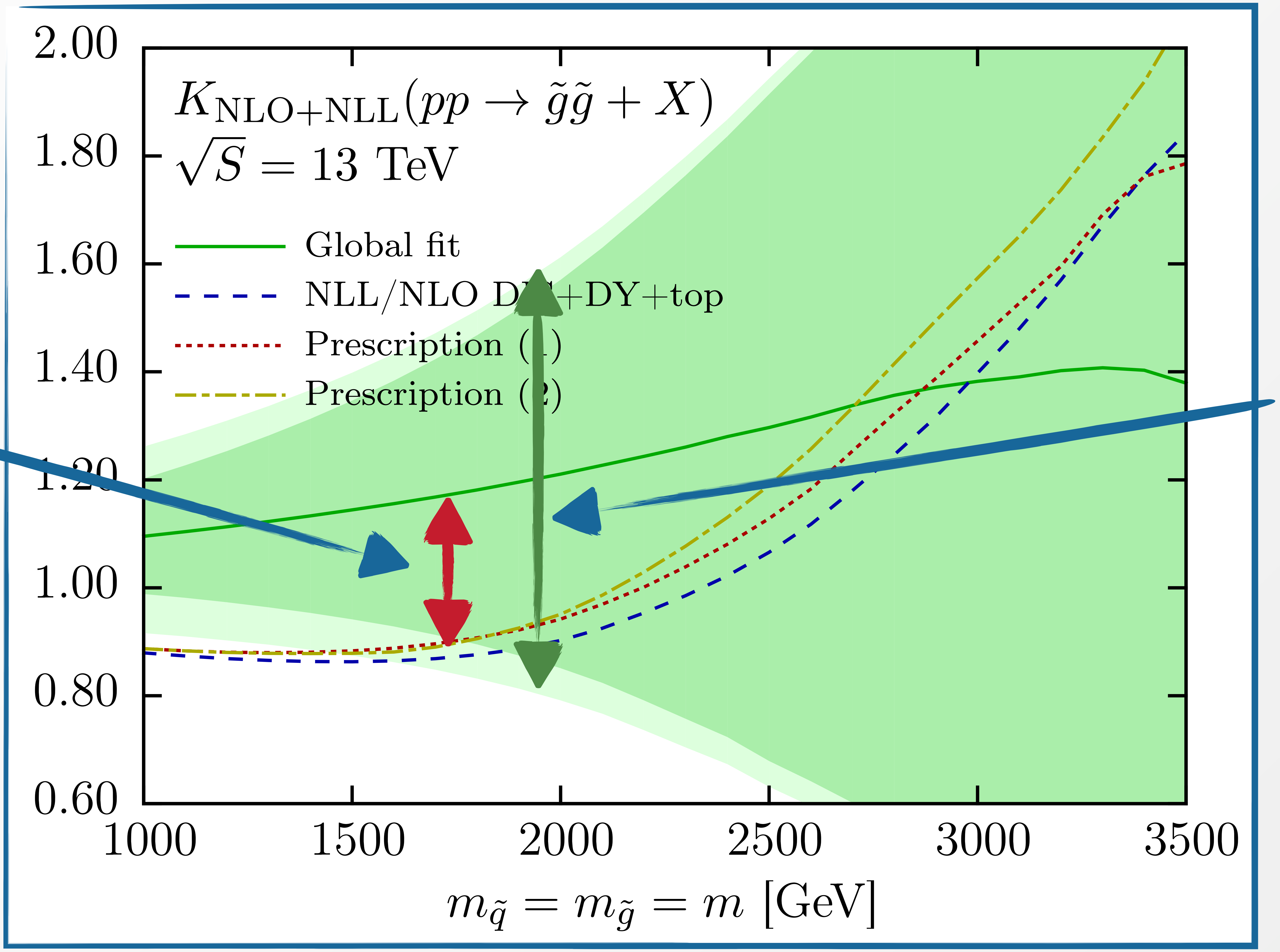
m_H ~600 GeV cancellation of 1/2 of the enhancement

NNPDF3.0res: Impact on phenomenology

Susy particles

[Beenakker,Borschensky,Krämer,Kulesza,Laenen,Marzani,Rojo 1510.00375]

Predictions for MSSM particles are modified when using resummed PDFs



However, PDF errors are very large

Outlook

- First ever **global** fit of PDFs with **threshold resummation**
- PDFs are **suppressed in the large-*x* region**; at intermediate values of *x* quark PDFs are slightly enhanced (sum rule); negligible effects at $x<0.01$
- Inclusion of resummation **compensates the enhancement** from resummation in partonic cross sections
- Consistent resummed calculations might be closer to fixed order results

Limitations: larger uncertainties due to **reduced dataset**.

Methodology enables to have truly global resummed PDFs when calculations for missing processes will be available.

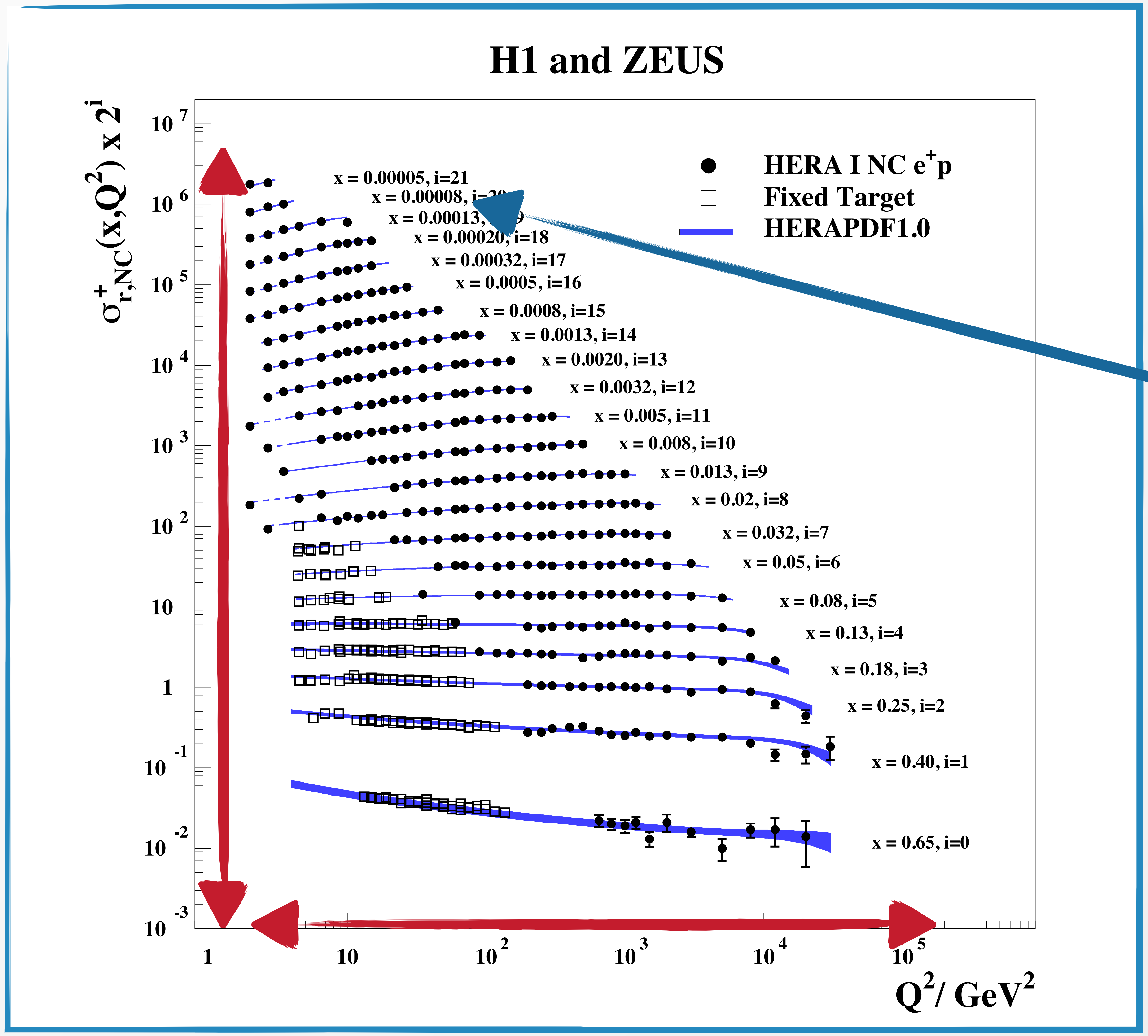
New processes to be included: DY Z/γ (Zp_T), $t\bar{t}$ (differential)…

PDFs with small-x resummation

[Ball,Bertone,Bonvini,Marzani,Rojo,LR 1710.05935]

Need for small-*x* resummation?

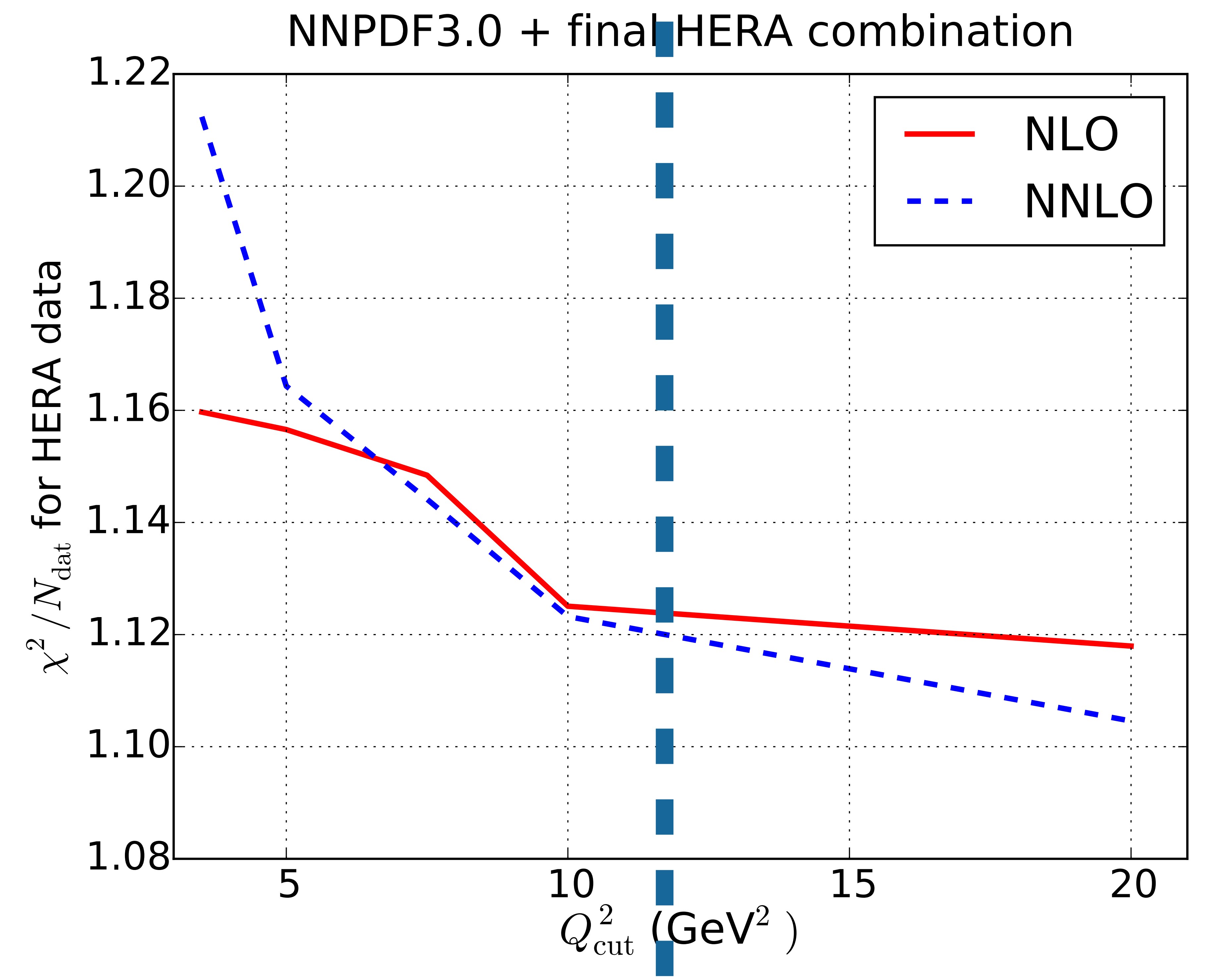
Deep Inelastic Scattering HERA dataset



data collected down to very small *x*

Very good agreement over vast range of *x* and Q^2

Need for small-*x* resummation?



more points at small *x* included

Courtesy of Juan Rojo

Description of HERA data poorer when data points at smaller values of *x* are included and fixed-order theory is used

Fixed order theory could be not sufficient to describe data points at small *x* and/or small Q^2

Effect is more pronounced if NNLO theory is used

This may indicate the need for **small-*x* resummation**

(very brief) Overview of small-x resummation

Small-x resummation based on kt-factorization and BFKL. Developed mostly in the 90s-00s [Catani,Ciafaloni,Colferai,Hautmann,Salam, Stasto][Altarelli,Ball,Forte] [Thorne,White]

Affects both **evolution** (LLx, NLLx) and **coefficient functions** (NLLx, lowest logarithmic order) in the singlet sector

Splitting functions are resummed using **ABF** (Altarelli,Ball,Forte) procedure

New formalism for **coefficient function** [Bonvini,Marzani,Peraro 1607.02153] and further improvements on the ABF formalism [Bonvini,Marzani,Muselli,Peraro 1708.07510]

Resummed splitting functions and coefficient functions available through public code **HELL** www.ge.infn.it/~bonvini/hell

Use in PDF fits possible thanks to the interface with **APFEL** apfel.hepforge.org

Towards a global small-x resummed fit

All ingredients for a PDF fit to **DIS data** are now available

In principle, one should add additional processes:

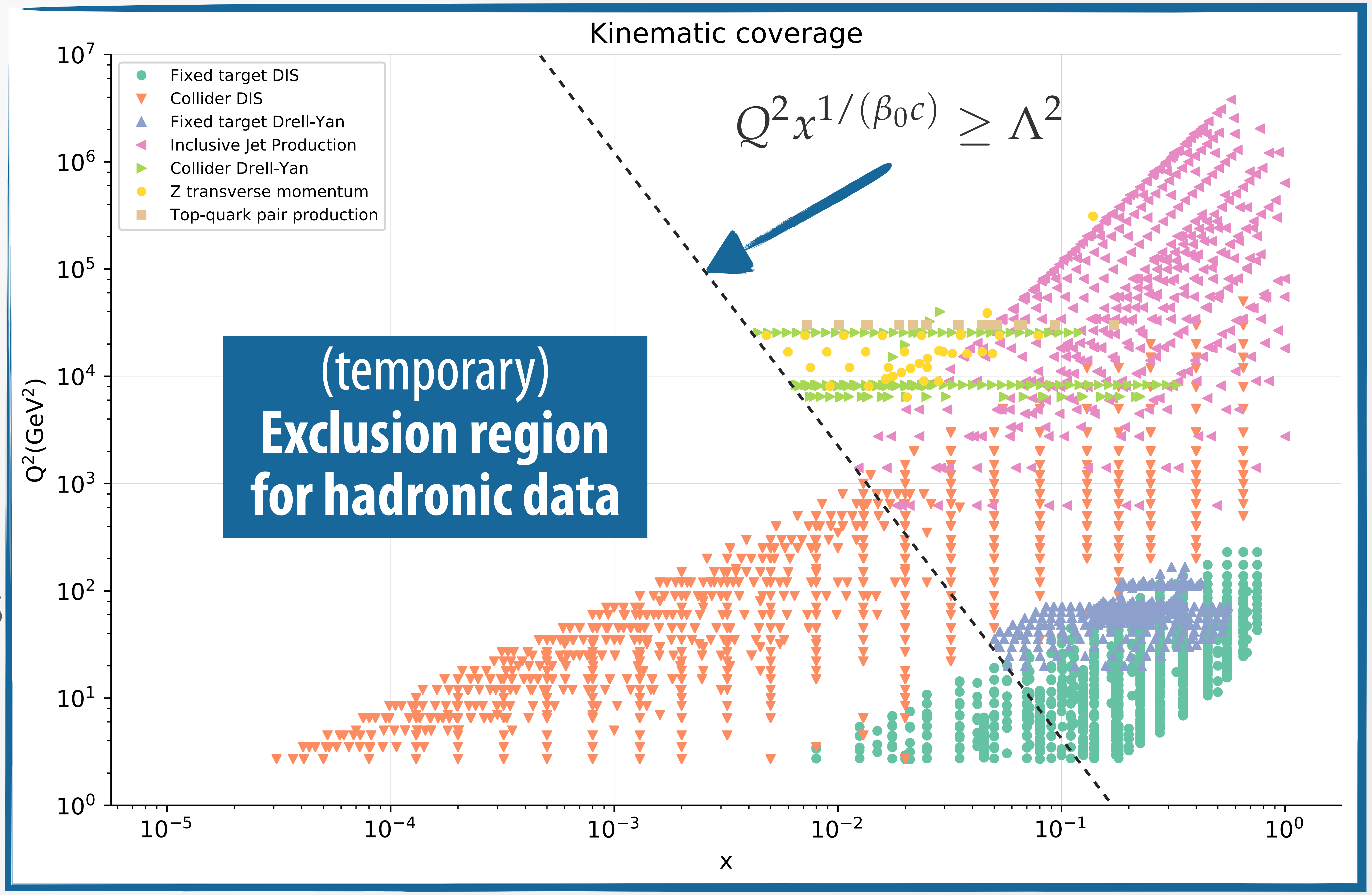
- DY
- Jets
- top
- …

Ongoing work in this direction

However, a global fit is possible if **conservatives cuts** on hadronic data are applied and points which may feature small-x enhancement are excluded

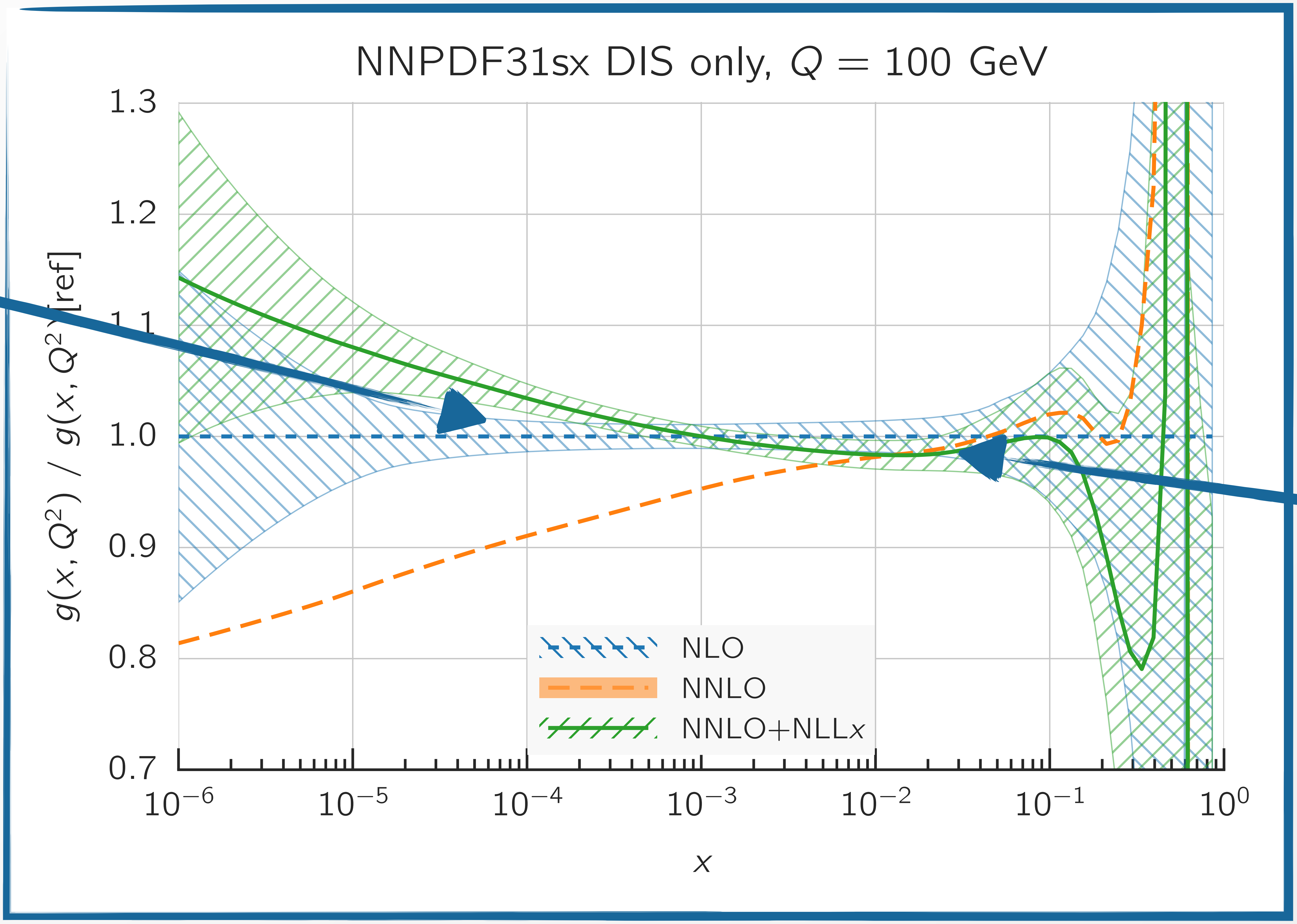
$$\alpha_s(Q^2)\log\frac{1}{x} \geq c \sim 1$$

Value of c (slope of the line) selects the exclusion region



NNPDF31sx: impact on PDFs

stabilization of the gluon with respect to the perturbative order



PDFs compatible within error at medium and large *x*

NNPDF31sx: fit quality

	$\chi^2/N_{\rm dat}$ NLO	$\chi^2/N_{\rm dat}$ NLO+NLLx	$\Delta\chi^2$	$\chi^2/N_{\rm dat}$ NNLO	$\chi^2/N_{\rm dat}$ NNLO+NLLx	$\Delta\chi^2$
NMC	1.35	1.35	+1	1.30	1.33	+9
SLAC	1.16	1.14	−1	0.92	0.95	+2
BCDMS	1.13	1.15	+12	1.18	1.18	+3
CHORUS	1.07	1.10	+20	1.07	1.07	−2
NuTeV dimuon	0.90	0.84	−5	0.97	0.88	−7
HERA I+II incl. NC	1.12	1.12	-2	1.17	1.11	−62
HERA I+II incl. CC	1.24	1.24	-	1.25	1.24	−1
HERA $\sigma_c^{\rm NC}$	1.21	1.19	−1	2.33	1.14	−56
HERA F_2^b	1.07	1.16	+3	1.11	1.17	+2
DY E866 $\sigma^d_{\rm DY}/\sigma^p_{\rm DY}$	0.37	0.37	-	0.32	0.30	-
DY E886 σ^p	1.06	1.10	+3	1.31	1.32	-
DY E605 σ^p	0.89	0.92	+3	1.10	1.10	-
CDF Z rap	1.28	1.30	-	1.24	1.23	-
CDF Run II k_t jets	0.89	0.87	−2	0.85	0.80	−4
D0 Z rap	0.54	0.53	-	0.54	0.53	-
D0 $W \to e\nu$ asy	1.45	1.47	-	3.00	3.10	+1
D0 $W \to \mu\nu$ asy	1.46	1.42	-	1.59	1.56	-
ATLAS total	1.18	1.16	−7	0.99	0.98	−2
ATLAS W, Z 7 TeV 2010	1.52	1.47	-	1.36	1.21	−1
ATLAS HM DY 7 TeV	2.02	1.99	-	1.70	1.70	-
ATLAS W, Z 7 TeV 2011	3.80	3.73	−1	1.43	1.29	−1
ATLAS jets 2010 7 TeV	0.92	0.87	−4	0.86	0.83	−2
ATLAS jets 2.76 TeV	1.07	0.96	−6	0.96	0.96	-
ATLAS jets 2011 7 TeV	1.17	1.18	-	1.10	1.09	−1
ATLAS Z p_T 8 TeV (p_T^{ll}, M_{ll})	1.21	1.24	+2	0.94	0.98	+2
ATLAS Z p_T 8 TeV (p_T^{ll}, y_{ll})	3.89	4.26	+2	0.79	1.07	+2
ATLAS σ_{tt}^{tot}	2.11	2.79	+2	0.85	1.15	+1
ATLAS $t\bar{t}$ rap	1.48	1.49	-	1.61	1.64	-
CMS total	0.97	0.92	−13	0.86	0.85	−3
CMS Drell-Yan 2D 2011	0.77	0.77	-	0.58	0.57	-
CMS jets 7 TeV 2011	0.88	0.82	−9	0.84	0.81	−3
CMS jets 2.76 TeV	1.07	0.98	−7	1.00	1.00	-
CMS Z p_T 8 TeV (p_T^{ll}, y_{ll})	1.49	1.57	+1	0.73	0.77	-
CMS σ_{tt}^{tot}	0.74	1.28	+2	0.23	0.24	-
CMS $t\bar{t}$ rap	1.16	1.19	-	1.08	1.10	-
Total	**1.117**	**1.120**	**+11**	**1.130**	**1.100**	**−121**

$\chi^2_{\rm NNLO+NLL\mathit{x}}$ smallest

1.10 vs ~**1.12** (NLO & NLO+NLLx), **1.13** (NNLO)

NNPDF31sx: fit quality

	$\chi^2/N_{\rm dat}$ NLO	NLO+NLLx	$\Delta\chi^2$	$\chi^2/N_{\rm dat}$ NNLO	NNLO+NLLx	$\Delta\chi^2$
NMC	1.35	1.35	+1	1.30	1.33	+9
SLAC	1.16	1.14	−1	0.92	0.95	+2
BCDMS	1.13	1.15	+12	1.18	1.18	+3
CHORUS	1.07	1.10	+20	1.07	1.07	−2
NuTeV dimuon	0.90	0.84	−5	0.97	0.88	−7
HERA I+II incl. NC	1.12	1.12	-2	1.17	1.11	−62
HERA I+II incl. CC	1.24	1.24	-	1.25	1.24	−1
HERA $\sigma_c^{\rm NC}$	1.21	1.19	−1	2.33	1.14	−56
HERA F_2^b	1.07	1.16	+3	1.11	1.17	+2
DY E866 $\sigma^d_{\rm DY}/\sigma^p_{\rm DY}$	0.37	0.37	-	0.32	0.30	-
DY E886 σ^p	1.06	1.10	+3	1.31	1.32	-
DY E605 σ^p	0.89	0.92	+3	1.10	1.10	-
CDF Z rap	1.28	1.30	-	1.24	1.23	-
CDF Run II k_t jets	0.89	0.87	−2	0.85	0.80	−4
D0 Z rap	0.54	0.53	-	0.54	0.53	-
D0 $W \to e\nu$ asy	1.45	1.47	-	3.00	3.10	+1
D0 $W \to \mu\nu$ asy	1.46	1.42	-	1.59	1.56	-
ATLAS total	1.18	1.16	−7	0.99	0.98	−2
ATLAS W, Z 7 TeV 2010	1.52	1.47	-	1.36	1.21	−1
ATLAS HM DY 7 TeV	2.02	1.99	-	1.70	1.70	-
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sensible improvement in the χ^2…

$(\chi^2_{\rm NNLO}-\chi^2_{\rm NNLO+NLLx})=$ **-121**

NNPDF31sx: fit quality

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Total	**1.117**	**1.120**	**+11**	**1.130**	**1.100**	**−121**

sensible improvement in the χ^2…
mostly driven by **HERA DIS data**

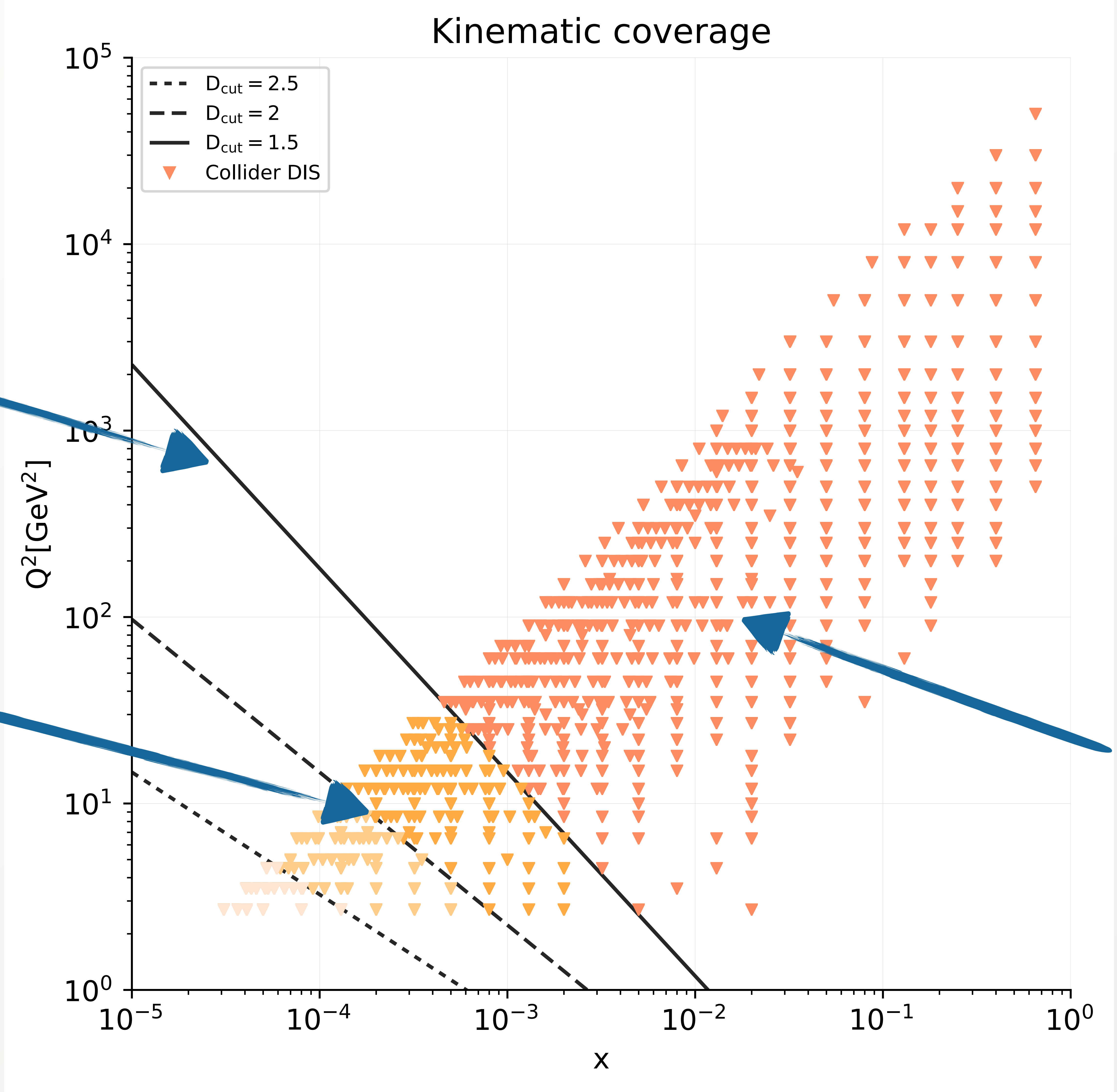
Small-x resummation and HERA data

Compute the χ^2 removing data points in the region where resummation effects are expected

cuts on DIS data

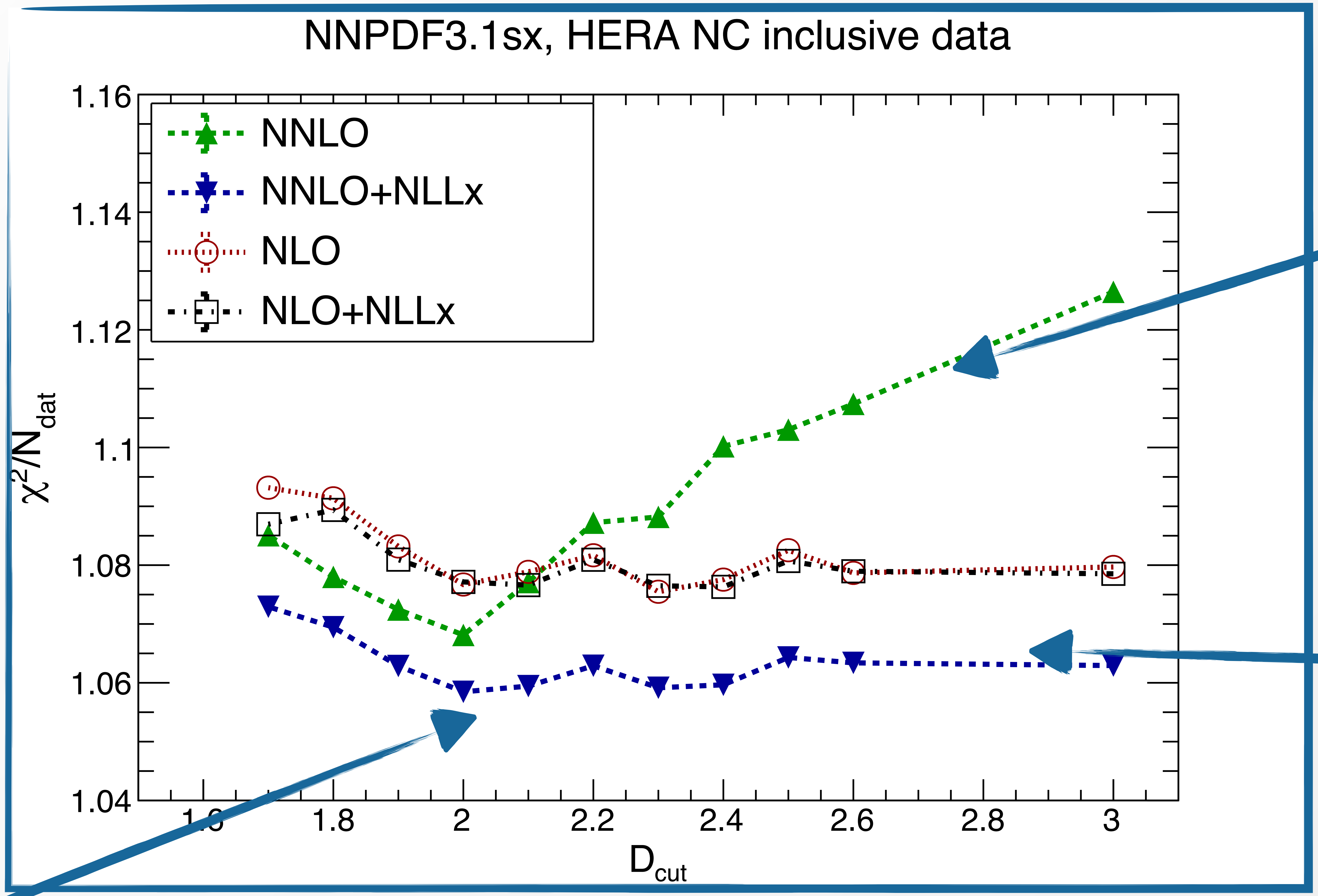
$$\alpha_s(Q^2)\ln\left(\frac{1}{x}\right) \geq D_{\rm cut}$$

resummation effects might be important here



fixed-order description should be good here

Small-x resummation and HERA data

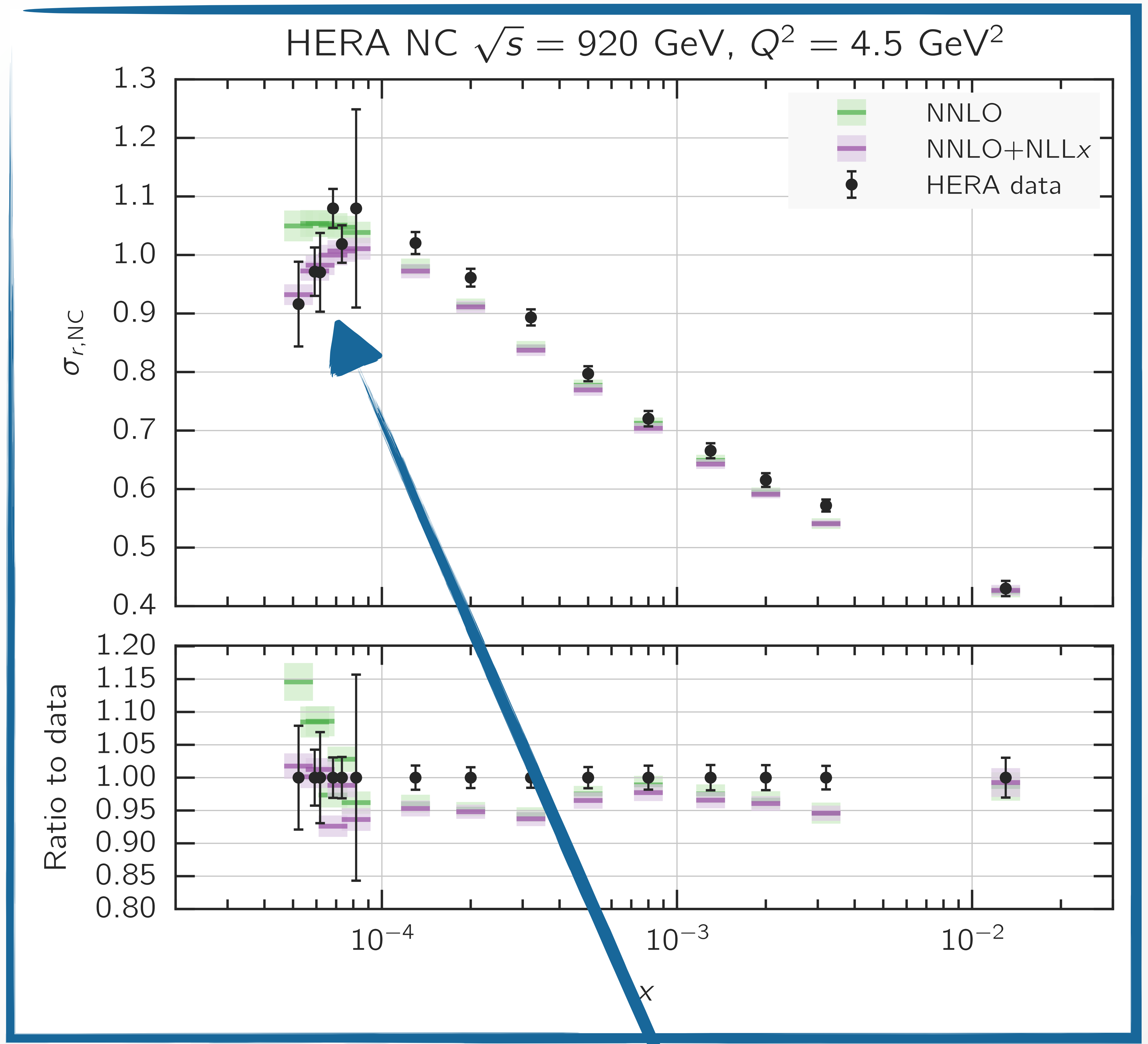


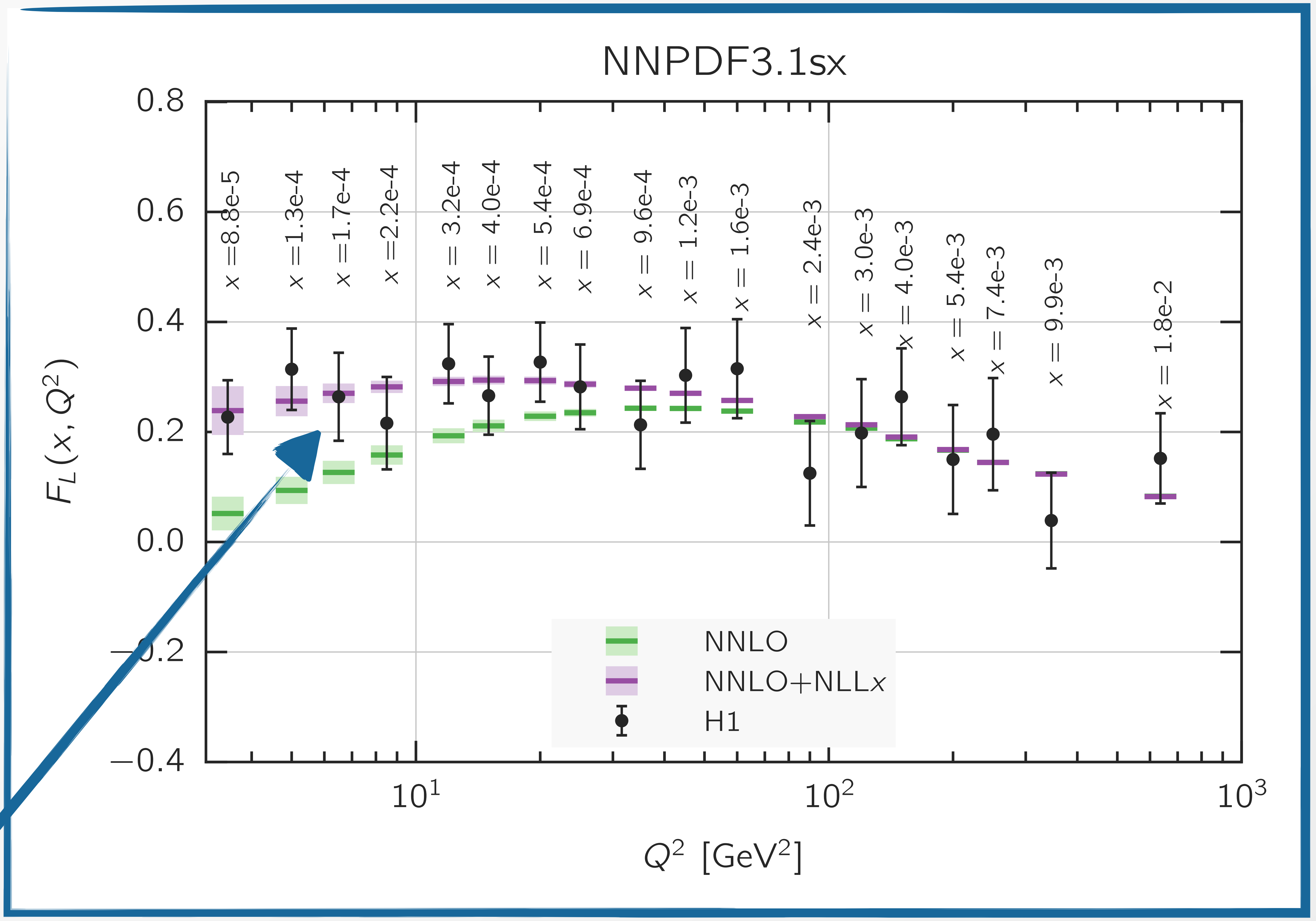
NNLO worsens if small-x data are included

NNLO+NLLx χ^2 flattens at larger values of D_{cut}

NNLO+NLLx offers the best description

Small-x resummation and HERA data





improved description of data at small-x and their slope

Small-x resummation and collider processes

Need to include small-x resummation in hadronic cross section (especially DY)

First estimate of impact of resummation can be obtained by computing approximate results with resummation **only in PDF evolution**

$$\sigma_X(Q^2,s) = \sum_{a,b} f_{a/h_1}(Q^2) \otimes f_{b/h_2}(Q^2) \otimes \hat{\sigma}_{ab\to X}(Q^2,s)$$

$$Q^2 \frac{d}{dQ^2} f_i(Q^2) = \boxed{P_{ij}(\alpha_s(Q^2))} \otimes f_j(Q^2)$$

Small-x resummation and collider processes

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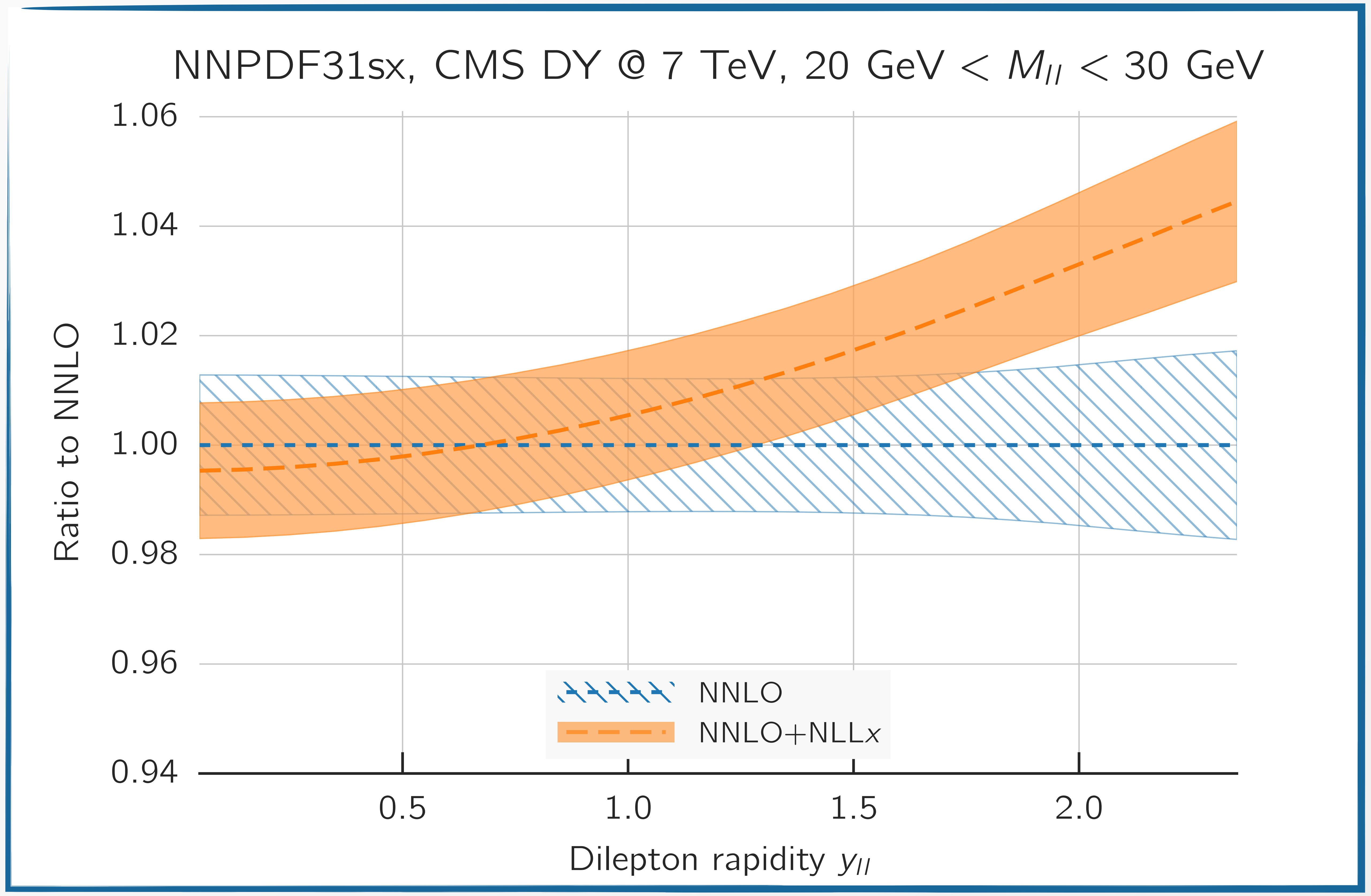


precision LHC phenomenology in extreme kinematic regions would require small-x resummed PDFs

Conclusions & outlook

- **First global fit** with small-x resummation in the NNPDF framework
- Evidence that **NNLO+NLLx improves** with respect to NNLO
- Description of the data at small x/small Q^2 significantly improves when resummation effects are included
- Potential for reducing uncertainties for processes not necessarily related to small-x physics

- Computation of small-x resummation for other processes needed
- Motivation to explore further probes of small-x dynamics at the LHC, such as low-mass DY at LHCb
- PDF sets with **joint** (large-x + small-x) resummation?

Backup

Threshold resummation in a nutshell

$$\sigma(x,Q^2) = x\int_x^1 \frac{dz}{z}\mathcal{L}\left(\frac{x}{z},Q^2\right)\frac{\hat{\sigma}(z,Q^2)}{z}$$

Convolution integral diagonalise in **Mellin space**

$$\sigma(N,Q^2) = \mathcal{L}(N,Q^2)\sigma_0(N,Q^2)C(N)$$

Double logarithmic enhancement due to soft gluon emission

$$C(N) = 1+\sum_{n=1}^{\infty}\alpha_s\sum_{k=0}^{2n}c_{nk}\ln^k N + \mathcal{O}(1/N)$$

N-soft

Exponentiation

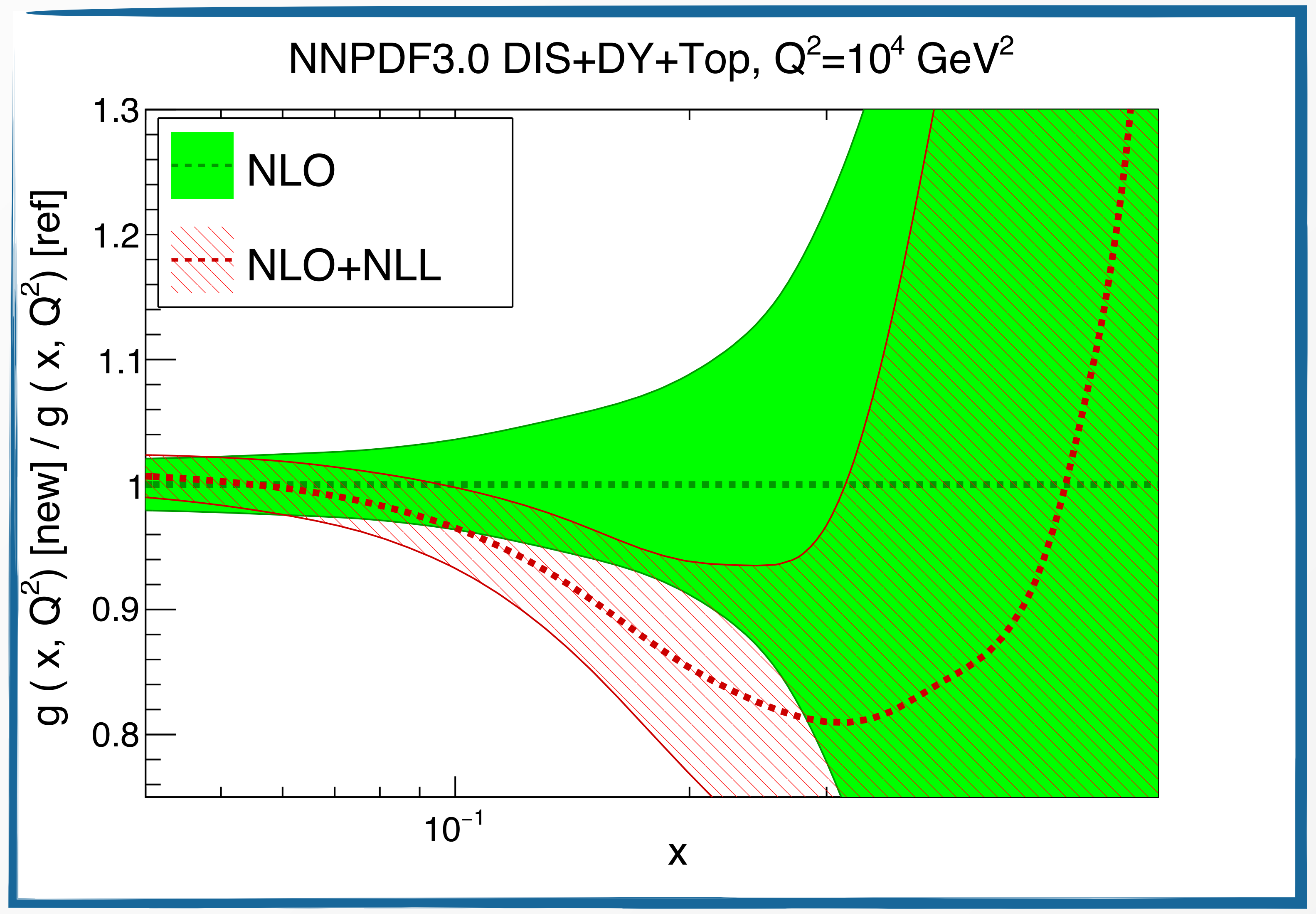
$$C(N) = g_0(\alpha_s)\exp\left[\frac{1}{\alpha_s}g_1(\alpha_s\ln N) + g_2(\alpha_s\ln N) + \alpha_s g_3(\alpha_s\ln N) + \dots\right]$$

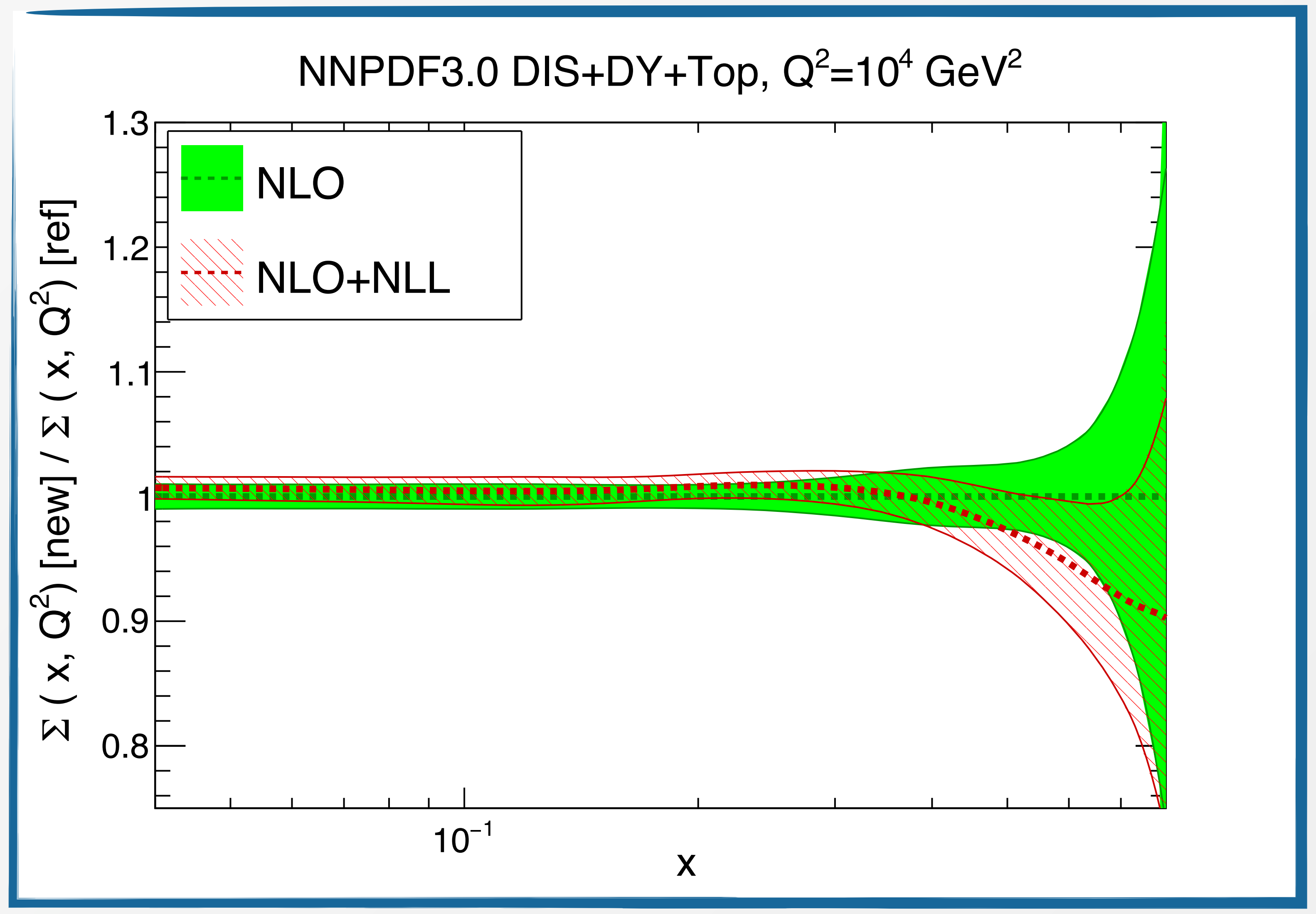
LL **NLL** **NNLL**

The functions g_i resum $\alpha_s^k\ln^k N$ to all orders

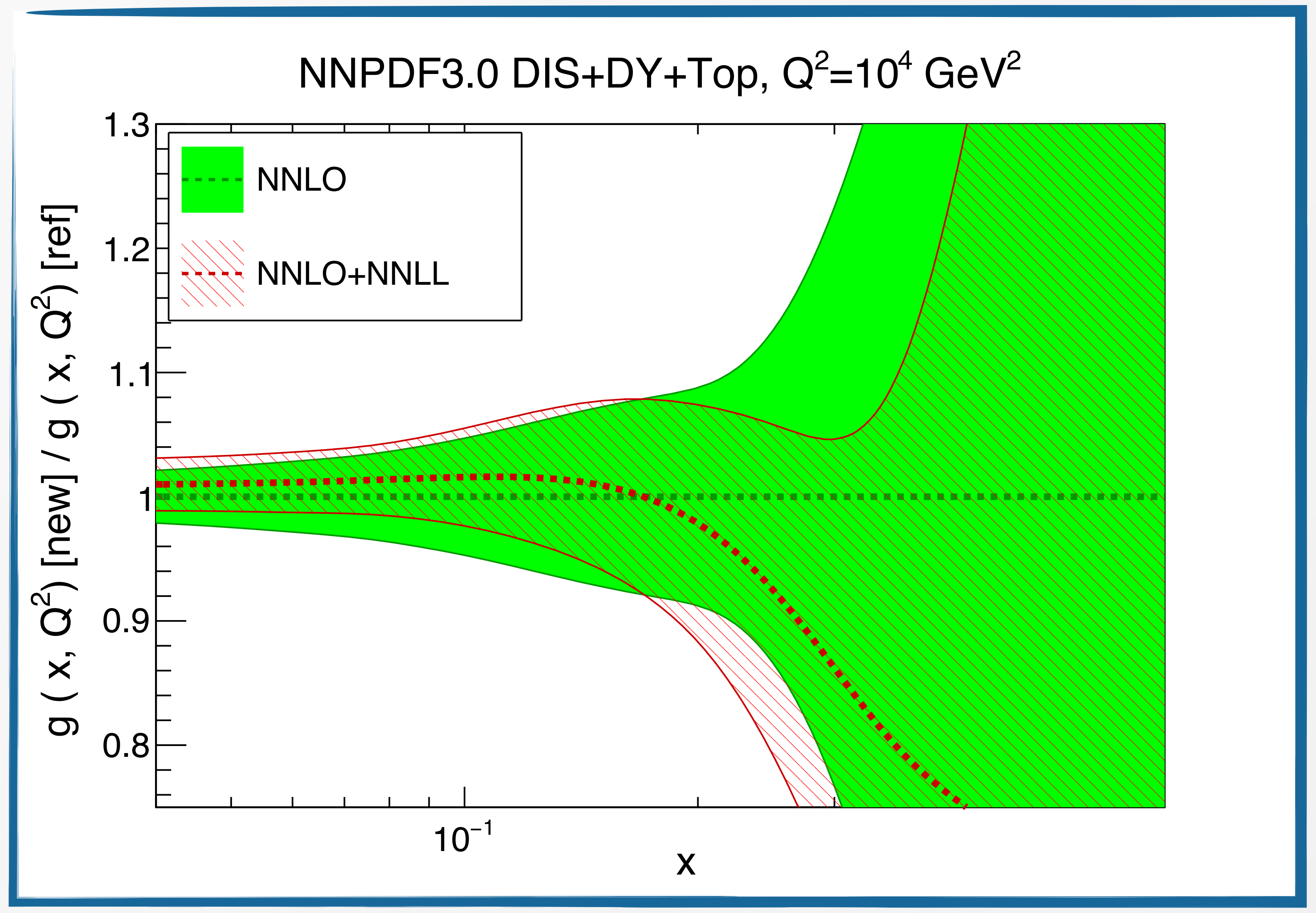
Impact on PDFs

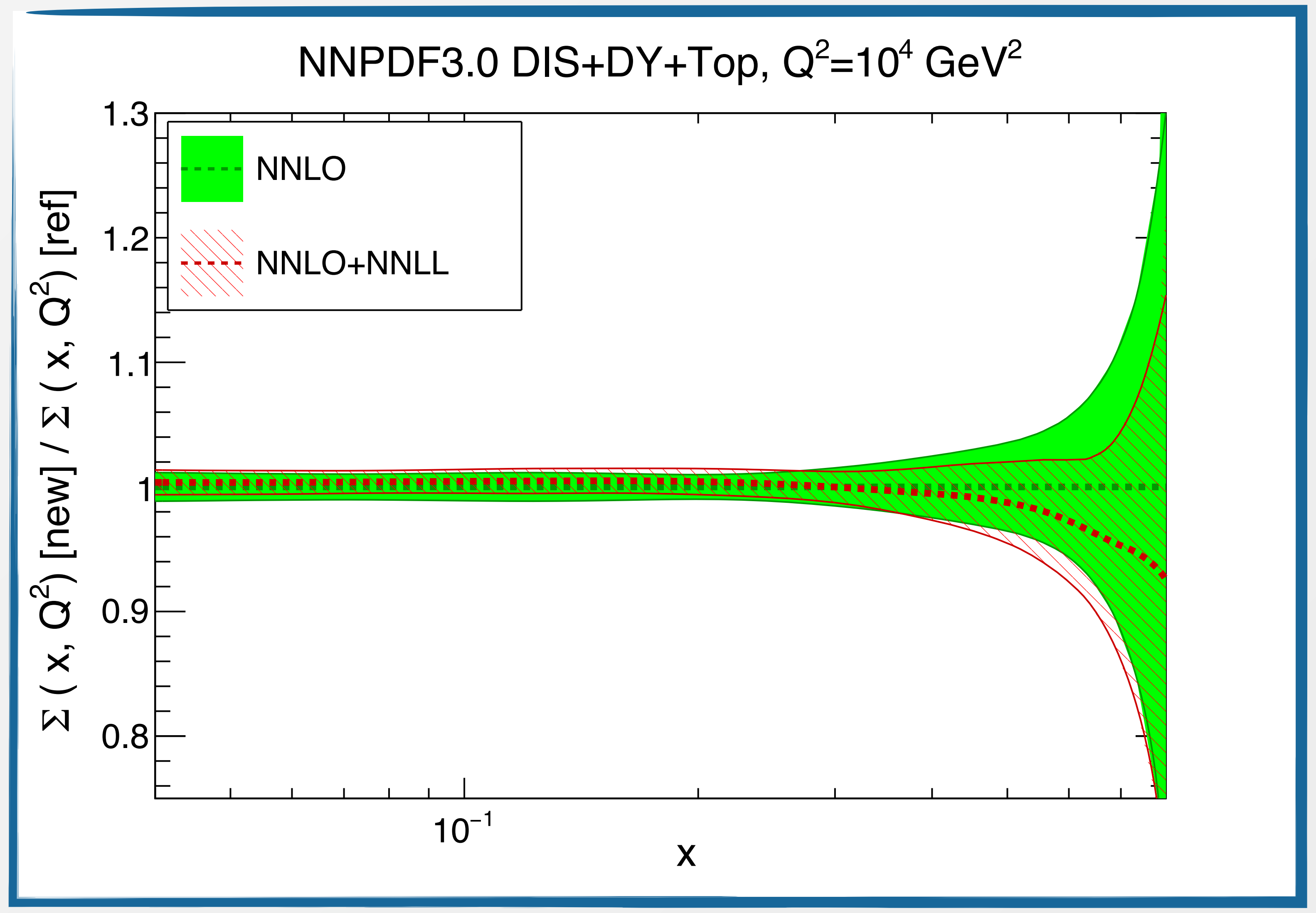
NLO+NLL





NNLO+NNLL



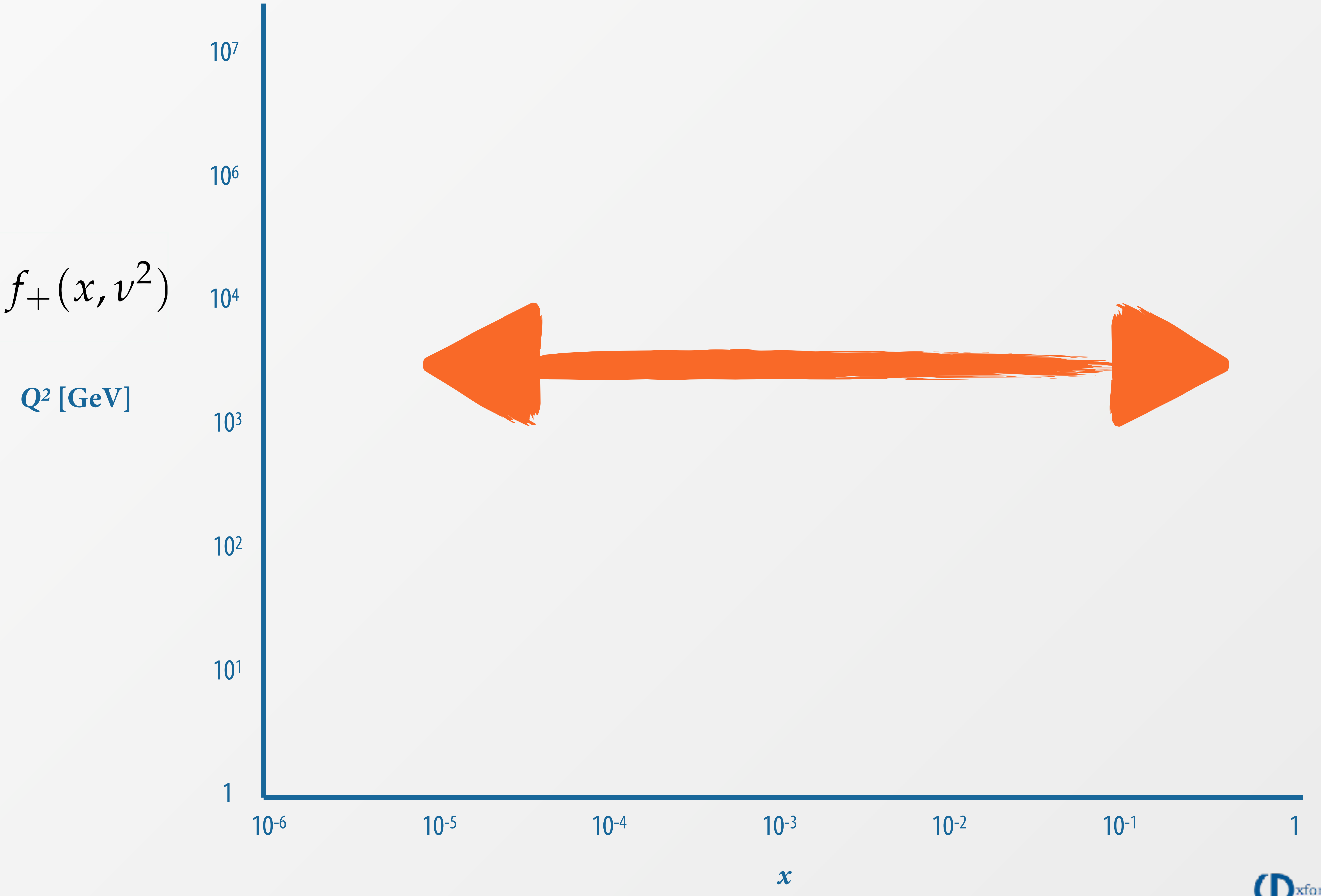


Parton Distribution Functions

PDFs depend on two kinematic variables $f(x, Q^2)$

Evolution in x^2 is encoded in **BFKL equation**

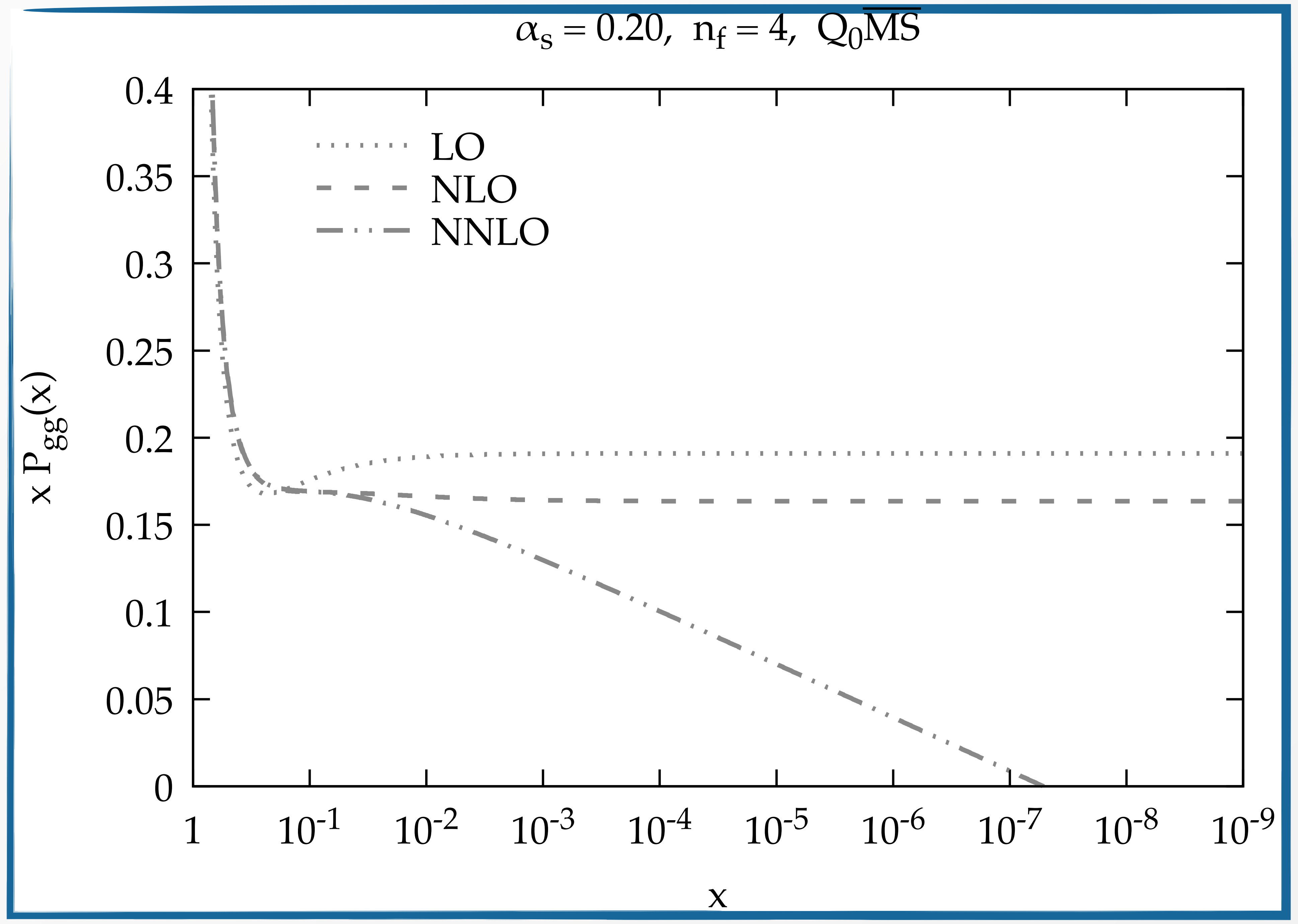
$$-x\frac{\partial}{\partial x}f_+(x,Q^2) = \int_0^\infty \frac{d\nu^2}{\nu}K\left(\frac{\mu^2}{\nu^2},\alpha_s(Q^2)\right) f_+(x,\nu^2)$$

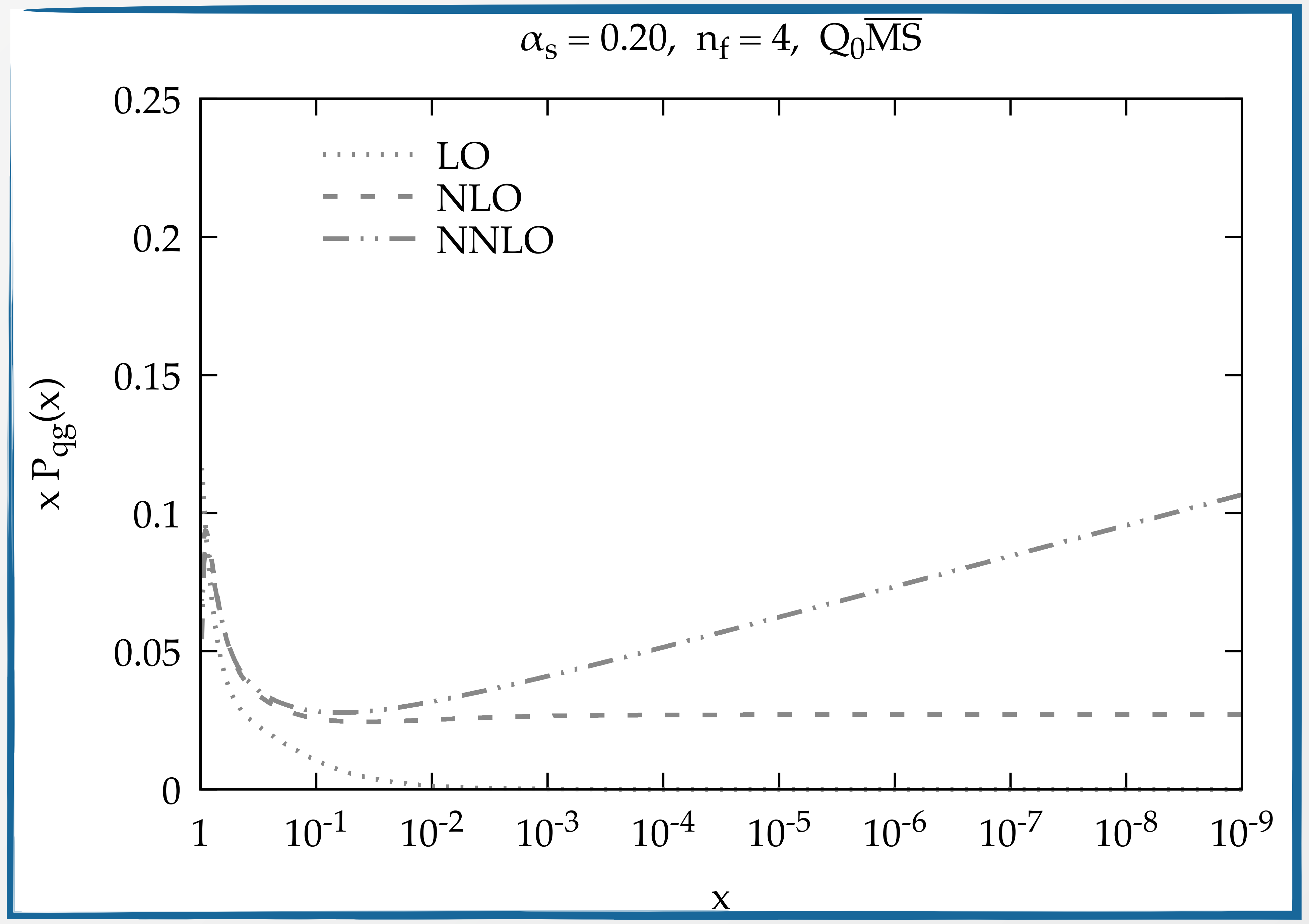


Small-x resummation of DGLAP evolution

ABF procedure based on

- **duality** with BFKL evolution
- **symmetry** of the BFKL kernel
- **momentum conservation**
- resummation of (subleading, but fundamental) **running coupling effects**





Courtesy of Marco Bonvini

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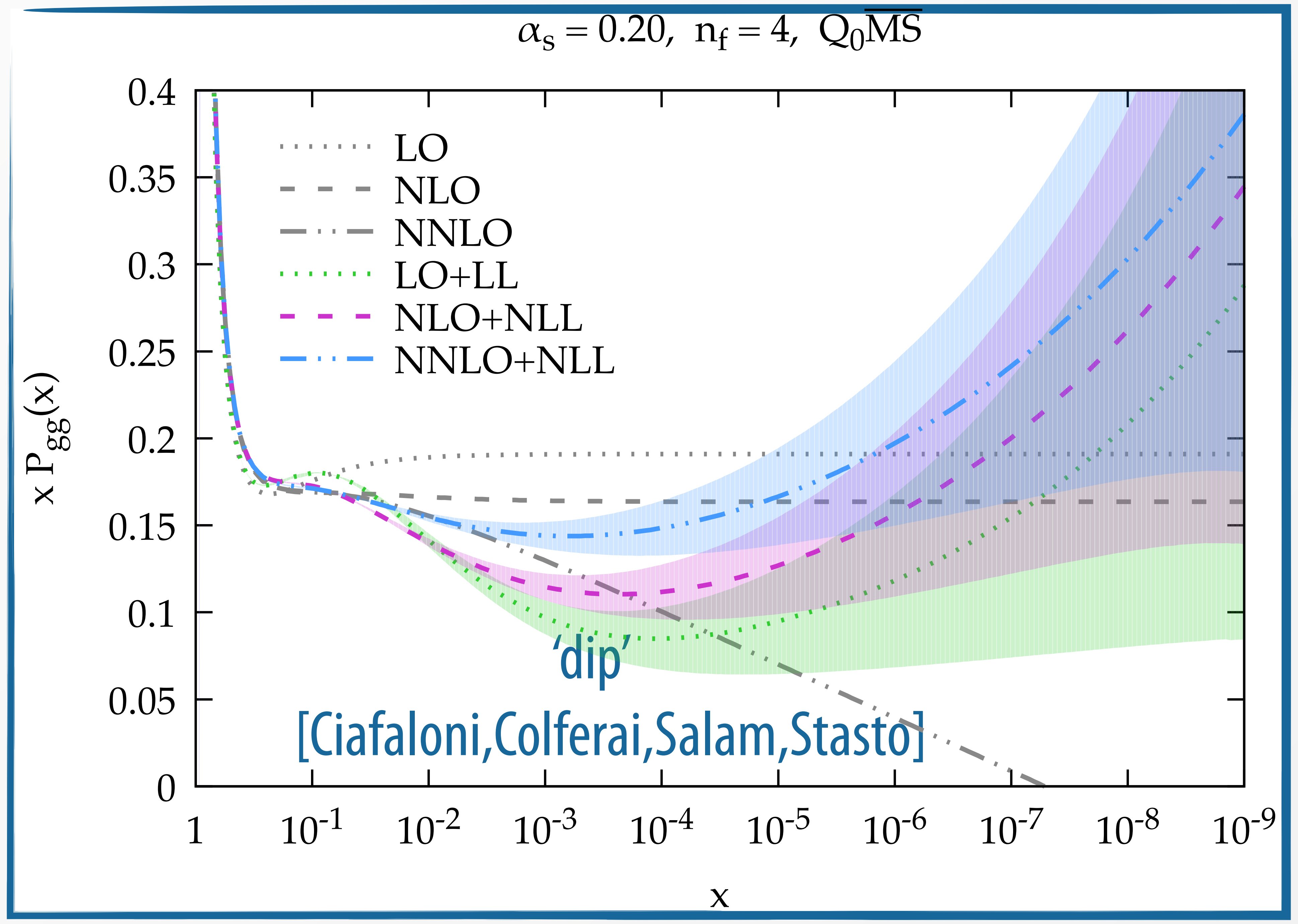
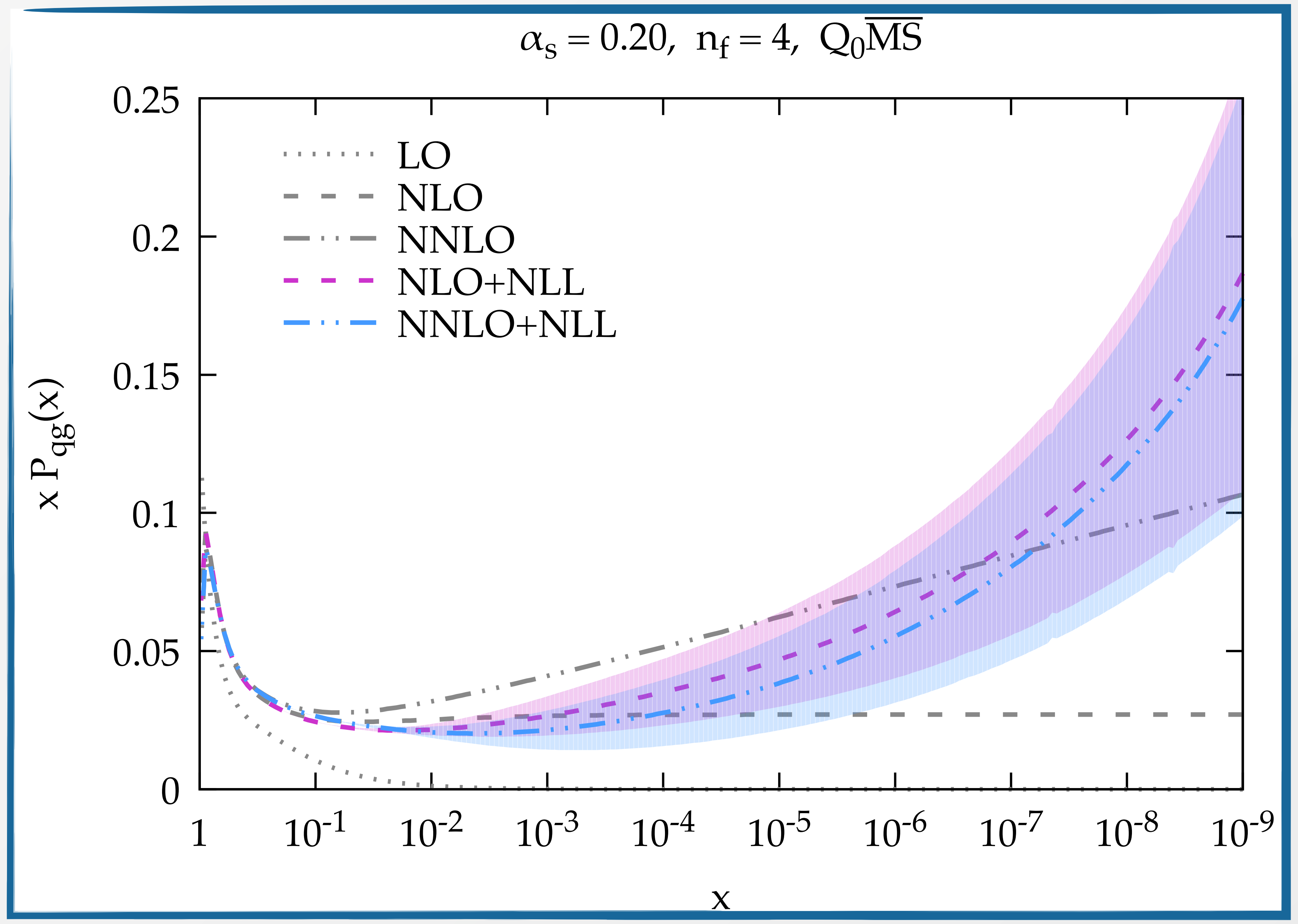
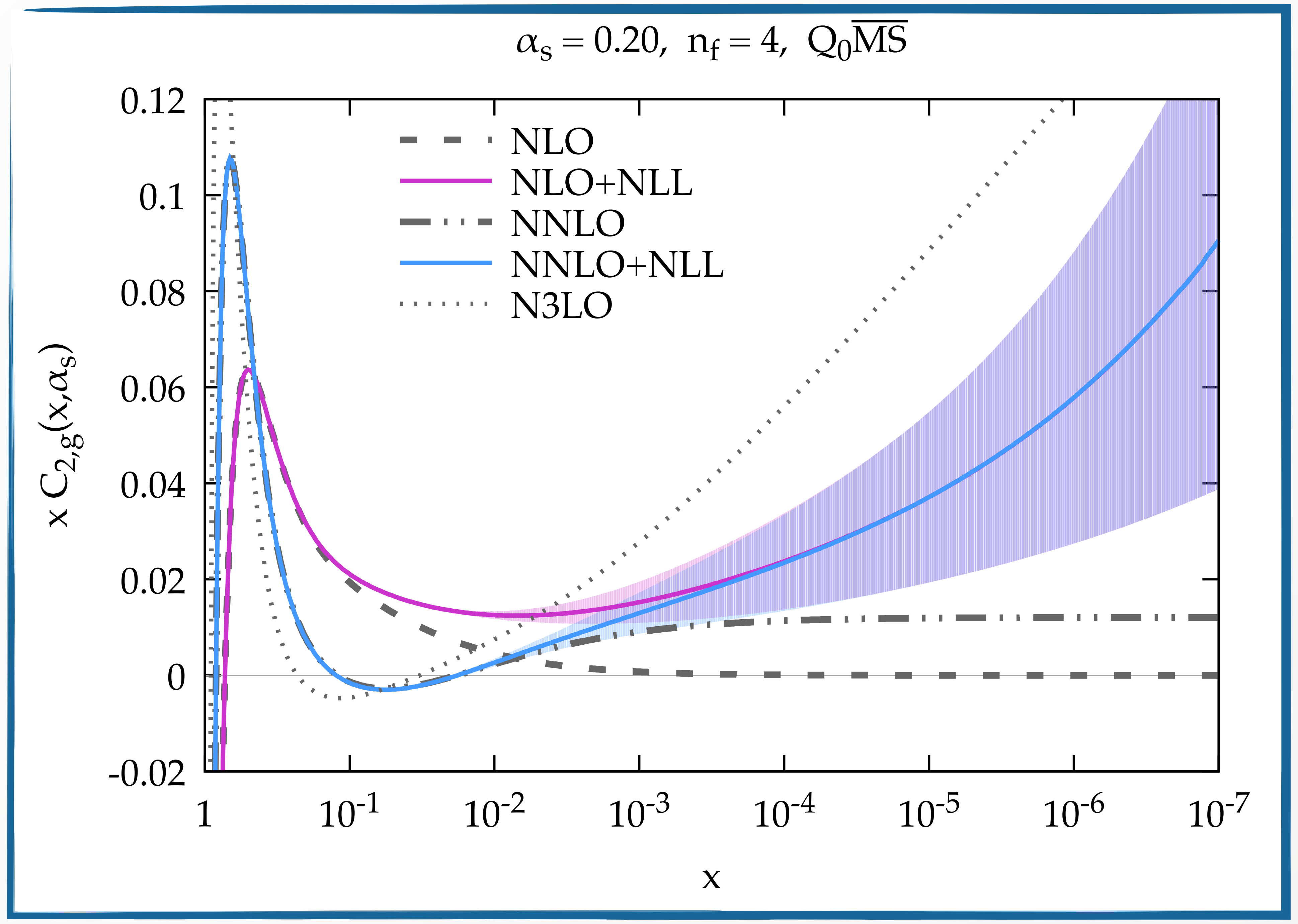
Now matching at NNLO available!

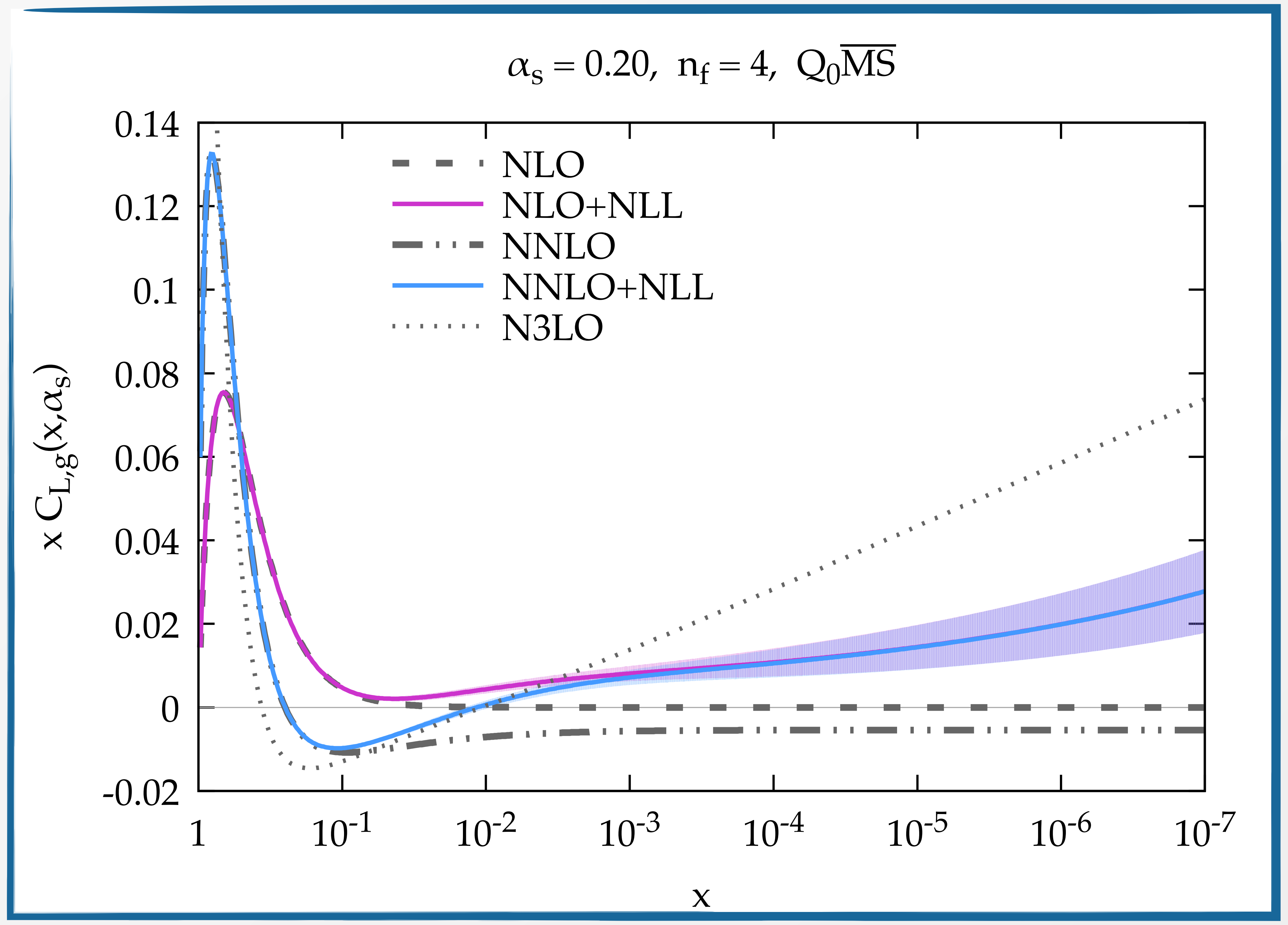




Courtesy of Marco Bonvini

Small-x resummation of coefficient functions





Courtesy of Marco Bonvini

- **massive** DIS coefficient functions available and implemented in **HELL**
- **VFNS** (FONLL = S-ACOT) implementation
- resummed **matching conditions** in **HELL**

$$C_{L,g}^{[n_f+1]}(m) = C_{L,g}^{[n_f]}(m), \qquad C_{2,g}^{[n_f+1]}(m) = C_{2,g}^{[n_f]}(m) - K_{hg}(m)$$

$$f_i^{[n_f+1]}(m) = \sum_{j=g,q_i\ldots q_{n_f}} K_{ij}(m) f_j^{[n_f]}, \qquad i = g, q, \ldots q_{n_f+1}$$

See also [Thorne,White]