Symmetry inheritance, black holes and no-hair theorems

Ivica Smolić

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Symmetries

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$$\pounds_\xi g_{ab} = \nabla_a \xi_b + \nabla_b \xi_a = 0$$

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- for example,

$$\begin{aligned} \text{2-dim Euclid} \quad & \partial_x, \partial_y \text{ and } x\partial_y - y\partial_x \\ (1+1)\text{-dim Minkowski} \quad & \partial_t, \partial_x \text{ and } x\partial_t + t\partial_x \\ \text{Kerr black hole} \quad & \partial_t \text{ and } \partial_\varphi \end{aligned}$$

Symmetry inheritance

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$$\pounds_\xi g_{ab} = 0 \quad \Rightarrow \quad \pounds_\xi \psi^{a\dots}{}_{b\dots} = 0 \;?$$

If the answer is yes, then we say that

the field $\psi^{a\dots}{}_{b\dots}$ inherits the symmetries of the metric g_{ab}

Why bother?


0
2
4
6
8
10
CARE-O-METER
J. M. HEATHER ENTERPRISES, INC.
STATE OF CALIFORNIA
SACRAMENTO COUNTY SEALER
DO NOT DETACH

math. physics

phenomenology

math. physics

phenomenology

classifications & uniqueness thms

a quest for the black hole hair


TO-DO LIST
NOTHING

Symmetry inheritance for …

⋆ fields of various spin

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- ⋆ fields of various spin
- ⋆ various types of couplings
- ⋆ various gravitational theories
- ⋆ different spacetime dimensions

Desiderata

$s = 0 \,|\, 1/2 \,|\, 1 \,|\, \cdots$

$D = 2 \,|\, 3 \,|\, 4 \,|\, 5 \,|\, \cdots$

$$\pounds_\xi \psi^{a\ldots}{}_{b\ldots} =$$

some zeros and
well-defined exceptions

+ concrete examples

Plan

Plan

overview	novel results	open questions
	1501.04967	
70s $\to$ 00s	1508.03343	?
	1609.04013	

* leave aside the related mathematical problem of collineations

$$\pounds_\xi g_{ab} = 0 \;\; \text{vs} \;\; \pounds_\xi R_{abcd} = 0 \;\; \text{vs} \;\; \pounds_\xi R_{ab} = 0 \;\; \text{vs} \;\; \pounds_\xi R = 0 \;\; \text{vs} \;\ldots$$

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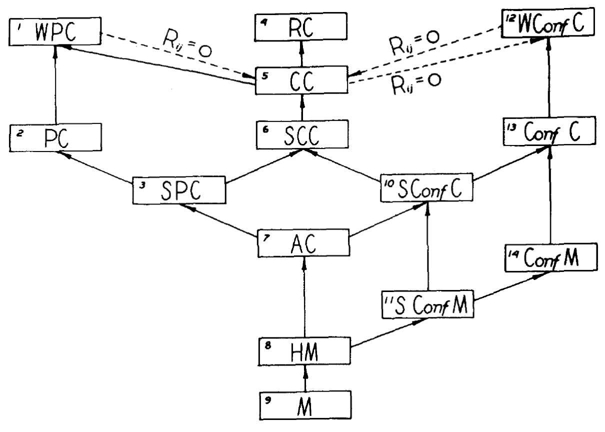
Katzin, Levine and Davis: J.Math.Phys. **10** (1969) 617



Long time ago …

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By the end of the Golden era of GR people began to wonder what are the symmetry inheritance properties of the various physical fields.

A thorough search was conducted …

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Can we be even more general?

A general strategy

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- Thus, $\pounds_\xi E_{ab} = 0$ and the problem is reduced to

$$\pounds_\xi T_{ab} = 0$$

Electromagnetic Field

EM Symm.Inh. in a Nutshell

$(1+1)$	$\pounds_\xi F_{ab} = 0$	
$(1+2)$	$\pounds_\xi F_{ab} = 0$	CDPS ’16
$(1+3)$	$\pounds_\xi F_{ab} = f \ast F_{ab}$	MW ’75 / WY ’76

$D \geq 5$	$\pounds_\xi F_{ab} = ???$

A warm up: (1+1)-dim EM field

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$$\pounds_\xi F_{ab} = (\pounds_\xi f)\,\epsilon_{ab} + f\,(\pounds_\xi \epsilon_{ab}) = 0$$

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- simplified proof via spinors [Tod 2007]

$$F_{ABA'B'} = \phi_{AB}\,\epsilon_{A'B'} + \overline{\phi}_{A'B'}\,\epsilon_{AB}\,, \quad T_{ABA'B'} = \frac{1}{2\pi}\,\phi_{AB}\,\overline{\phi}_{A'B'}$$

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$$F_{ABA'B'} = \phi_{AB}\,\epsilon_{A'B'} + \overline{\phi}_{A'B'}\,\epsilon_{AB}\,, \qquad T_{ABA'B'} = \frac{1}{2\pi}\,\phi_{AB}\,\overline{\phi}_{A'B'}$$

- "master equation" $\pounds_\xi T_{ABA'B'} = 0$ implies $\pounds_\xi \phi_{AB} = ia\,\phi_{AB}$ for some real function a and

$$\pounds_\xi F_{ab} = f *F_{ab}$$

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⋆ an example of symmetry noninheriting EM field
[Tariq and Tupper 1975; Michalski and Wainwright 1975]

$$\mathrm{d}s^2 = \frac{1}{(2r)^2}\,(\mathrm{d}r^2 + \mathrm{d}z^2) + r^2\mathrm{d}\varphi^2 - (\mathrm{d}t - 2z\,\mathrm{d}\varphi)^2$$

$$F = -\sqrt{8}\left(\frac{\cos\alpha}{r}\,\mathrm{d}r \wedge (\mathrm{d}t - 2z\,\mathrm{d}\varphi) + \sin\alpha\,\mathrm{d}z \wedge \mathrm{d}\varphi\right)$$

$$\alpha = -2\ln r + \alpha_0$$

$$\xi = r\,\frac{\partial}{\partial r} + z\,\frac{\partial}{\partial z} - \varphi\,\frac{\partial}{\partial \varphi}\,, \quad £_\xi F = -2\,{*F}$$

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[M. Cvitan, P. Dominis Prester and I.Sm.: CQG **33** (2016) 077001]

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- the key observation

$$\begin{array}{r|l} 8\pi T_{ab}\xi^a\xi^b = E_a E^a + B^2 & E^a \, \pounds_\xi E_a + B \, \pounds_\xi B = 0 \\ 4\pi * (\xi \wedge T(\xi))_a = -B E_a & B \, \pounds_\xi E_a + (\pounds_\xi B) E_a = 0 \end{array}$$

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$$\pounds_\xi F_{ab} = 0$$

Fermions

Spin-1/2 fields

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(a) if $\ell^a = \nu^A \nu^{A'}$ is collinear with one of the principal null directions of the Weyl tensor

$$\pounds_\xi \nu^A = is\, \nu^A$$

with real constant s

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with real constant s

(b) …otherwise

$$\pounds_\xi \nu^A = f \nu^A$$

Scalar Fields

Real scalar field

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- minimally coupled, canonical
  [Hoenselaers 1978; I.Sm. 2015]

$$T_{ab} = (\nabla_a \phi)(\nabla_b \phi) + (X - V(\phi))g_{ab} \,, \quad X \equiv -\frac{1}{2}(\nabla^c \phi)(\nabla_c \phi)$$

$$\Rightarrow \quad 0 = \pounds_\xi V(\phi) = V'(\phi)\, \pounds_\xi \phi$$

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$$\begin{array}{r|l} V'(\phi) \neq 0 & \pounds_\xi \phi = 0 \\ V'(\phi) = 0 & \pounds_\xi \phi = a = \text{const.} \end{array}$$

and if ξ^a has compact orbits then $a = 0$.

- an example of time dependent real scalar field in a stationary spacetime: M. Wyman, Phys. Rev. D **24** (1981) 839

$$\mathrm{d}s^2 = -e^{\nu(r)}\mathrm{d}t^2 + e^{\lambda(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)$$

$$\phi(t) = \gamma\, t\,, \quad \gamma = \mathrm{const.}$$

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$$\phi(t) = \gamma\, t\,, \quad \gamma = \mathrm{const.}$$

- two solutions: a simpler one with $e^{\nu} = 8\pi\gamma^2 r^2$ and $e^{\lambda} = 2$, and the second one in a form a Taylor series.

- “k-essence” theories
  a generic model for the inflationary evolution

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  $Xp_{,X} \neq 0 \;\Rightarrow\; £_\xi \phi$ is a solution to

$$p_{,\phi}(£_\xi \phi)^2 + 2Xp_{,X}\,£_\xi £_\xi \phi = 0$$

  which is either identically zero or doesn’t have any zeros along the orbit of ξ^a

Addendum: Ideal fluid

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- [Hoenselaers 1978]

$$T_{ab} = (\rho + p)\, u_a u_b + p\, g_{ab}$$

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$$T_{ab} = (\rho + p)\, u_a u_b + p\, g_{ab}$$

$$\pounds_\xi T_{ab} = 0 \;\Rightarrow$$

$$\pounds_\xi \rho = \pounds_\xi p = 0 = \pounds_\xi u^a$$

Complex scalar field

Complex scalar field

- energy-momentum tensor

$$T_{ab} = \nabla_{(a}\phi\nabla_{b)}\phi^* - \frac{1}{2}\left(\nabla^c\phi\nabla_c\phi^* + V(\phi^*\phi)\right)g_{ab}$$

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- e.g. in polar form $\phi = Ae^{i\alpha}$:

$$T_{ab} = \nabla_a A\,\nabla_b A + A^2\,\nabla_a\alpha\,\nabla_b\alpha + \frac{T + V(A^2)}{D-2}\,g_{ab}$$

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⋆ for $V = \mu^2 A^2$, the only symmetry noninheriting amplitude A which is bounded or periodic along the orbits of ξ^a is

$$A \sim \sin(\sqrt{\kappa}(x - x_0))$$

but $N = \text{const.} > 0$ and ξ^a is hypersurface orthogonal

Black Hole Hair

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- roughly, a broad definition:

  any non-gravitational field in a black hole spacetime

- more refined definition:

  any non-gravitational field in a black hole spacetime contributing to the conserved "charges" associated to the black hole, apart from the total mass M, the angular momentum J, the electric charge Q and the magnetic charge P

  (see also: primary/secondary hair distinction)

No-hair theorems

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- Bekenstein, PRL **28** (1971) 452

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(a) a choice of the scalar field *coupling* to gravity,

(b) an *energy condition*,

(c) details about the "asymptotics"

(d) the assumption that the scalar field ϕ inherits the spacetime symmetries

Symmetry noninheriting scalar black hole hair

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- Herdeiro and Radu, PRL **112** (2014) 221101

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  numerical stationary axially symmetric solution of the Einstein-Klein-Gordon EOM, with the complex scalar field

$$\phi = A(r,\theta)\, e^{i(m\varphi - \omega t)} \qquad \text{with} \quad \omega = \Omega_{\mathrm{H}} m$$

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- Are there any other hairy black hole solutions based on symmetry noninheritance?

  What are the constraints on the existence of the sni scalar black hole hair?

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⋆ complex scalar field with symmetry inheriting amplitude: a constraint for $H[\chi]$ with $\chi^a = k^a + \Omega_{\mathrm{H}} m^a$

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⋆ complex scalar field with symmetry inheriting phase: no sni BH hair (via Vishveshwara-Carter tm)

Hair constraints beyond Einstein

[I.Sm. arXiv:1609.04013]

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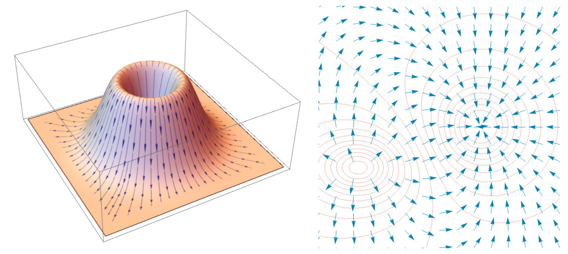
- idea: use the Frobenius' theorem (diff. geom.)



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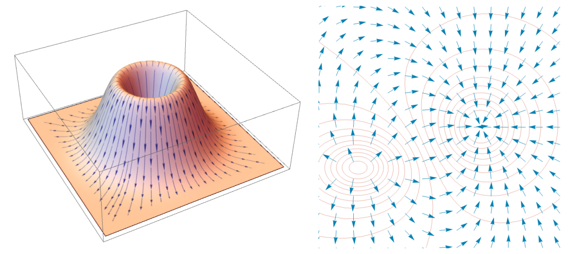

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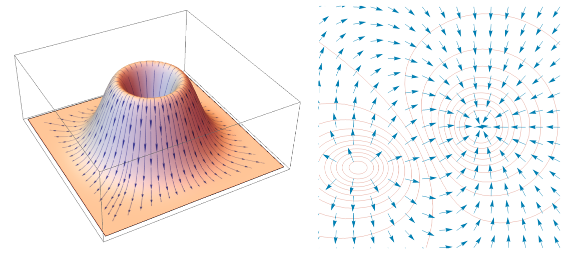

⋆ integrable iff involute, $[X_{(i)}, X_{(j)}]^a \in \Delta$

⋆ orthogonally-transitive iff $X^{(1)} \wedge \ldots \wedge X^{(n)} \wedge \mathrm{d}X^{(i)} = 0$

static	$k \wedge \mathrm{d}k = 0$	Schwarzschild
circular	$k \wedge m \wedge \mathrm{d}k =$	
	$= k \wedge m \wedge \mathrm{d}m = 0$	Kerr

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- static $\rightarrow$ Ricci static,

$$k \wedge R(k) = 0\,, \quad R_{ti} = 0$$

static	$k \wedge \mathrm{d}k = 0$	Schwarzschild
circular	$k \wedge m \wedge \mathrm{d}k =$	
	$= k \wedge m \wedge \mathrm{d}m = 0$	Kerr

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$$\dots \;\Rightarrow\; T(\chi, \chi) = 0 \quad \text{on} \quad H[\chi]$$

Open Questions

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- Wainwright and Yaremovicz [Gen.Rel.Grav. **7** (1976) 345–359 and 595–608] treat the EM field + charged ideal fluid

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- we need a systematic approach …

Thank you for your attention!