Dipartimento di Fisica
Università di Cagliari
INFN, Sezione di Cagliari

High Energy Physics Colloquia

Exploring the Nature of Spacetime: the Emergent Gravity Approach

Matteo Tuveri

Università di Cagliari & INFN Cagliari

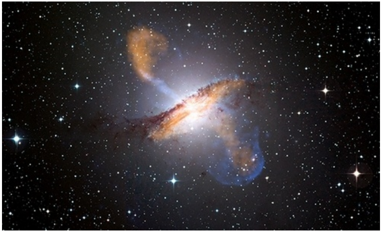


Figure : Huge jets of particles are shot out of the black holes at close to the speed of light.

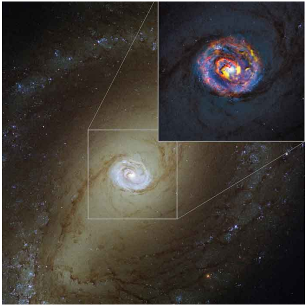


Figure : Powerful jets coming out from the center of the galaxy NGC 1433.

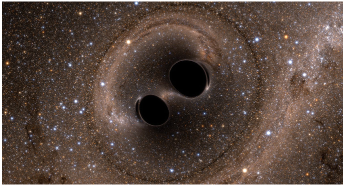


Figure : Realistic representation of merging of two black holes.

Quantum Gravity (Now)

Figure : String Theory

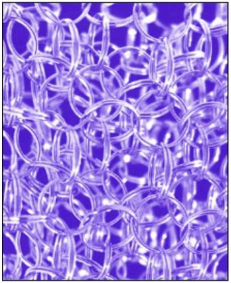
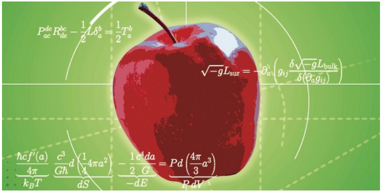
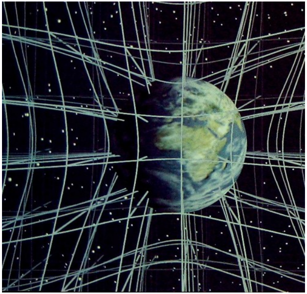


Figure : Loop Quantum Gravity

Emergent Gravity→thermodynamical approach to Quantum Gravity



Outline



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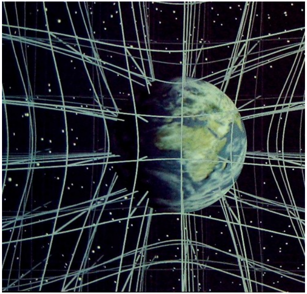

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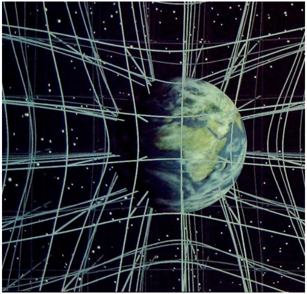
  - The Concept of Horizon ($\mathcal{H}$)
  - Black Hole Thermodynamics



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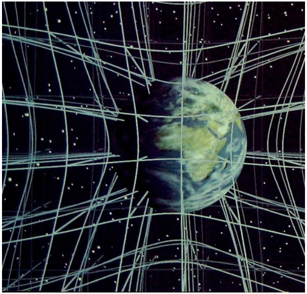

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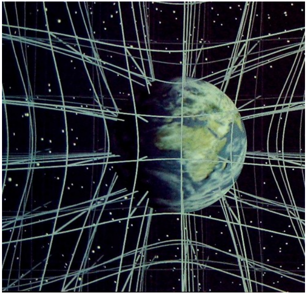

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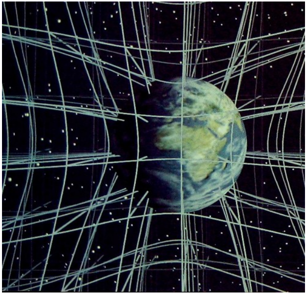
- Fluid Dynamics of Spacetime



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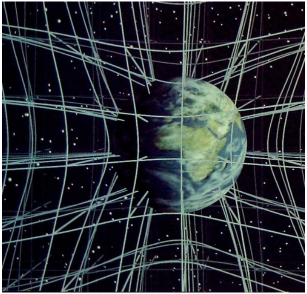

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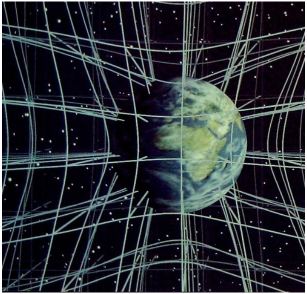
- Summary and Outlook



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- We set $-g = det \mid g_{ab} \mid$
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- We define $P^{abcd} = \frac{\partial R}{\partial R_{abcd}} = \frac{1}{2}(g^{ac}g^{bd} - g^{ad}g^{bc})$

The Concept of Horizon ($\mathcal{H}$)

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- Metric signature $(-,+++)$, $G = c = 1$, $a = 0, ..., 3$, $\alpha = 1, ..., 3$.
- Schwarzschild's solution of Einstein's gravity field equations

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 \left(d\theta^2 + sin^2\theta d\phi^2\right)$$

(Apparent) Pathology of the metric in $r = 2m$: $g_{tt} \to 0$, g_{rr} diverges on $\mathcal{H}$

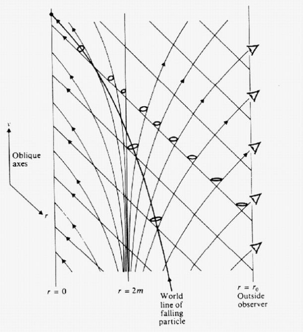

Figure : Schwarzschild Spacetime

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$(T, X) \rightarrow (t, l)$

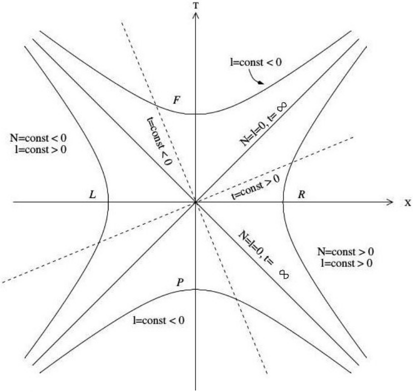

Figure : Rindler Frame

$$\textbf{\textit{Local Rindler Frame}}: ds^2 = -2\kappa l dt^2 + \frac{dl^2}{2\kappa l} + dL_\perp^2$$

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$(T, X) \to (t, l)$

For $|X| > |T|$

$$\kappa T = \sqrt{2\kappa l}\sinh(\kappa t)$$

$$\kappa X = \pm\sqrt{2\kappa l}\cosh(\kappa t)$$

For $|X| < |T|$

$$\kappa T = \pm\sqrt{-2\kappa l}\cosh(\kappa t)$$

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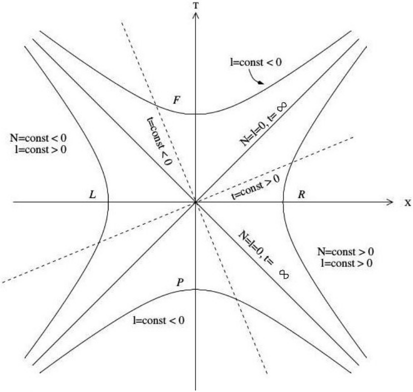

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The concept of Horizon: The Rindler Metric

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Let us set $l = \frac{1}{2}\kappa x^2$, $N^2(x^\alpha) = \sqrt{2\kappa|l|} = \kappa^2 x^2$

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Static spacetimes with the following properties:

1. Location of Horizon: $g_{00} = N^2(\vec{x}) \to 0$ on $\mathcal{H}$
2. Timelike Killing Vector field: $\xi^a = (1,0)_R = \kappa(X,T)_I$, $\xi_a \xi^a = -N^2 \xrightarrow{on\,\mathcal{H}} 0$

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The thermal effects arise from the ***Euclid Sector***. Setting

$$T_E = iT, t_E = it$$
$$N^2 = \kappa^2 x^2$$

we get the Rindler Metric in the Euclid sector:

$$-ds_E^2 = N^2 dt_E^2 + dx^2 = dT_E^2 - dX^2$$

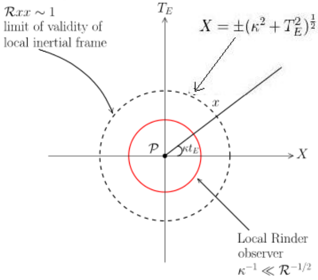

Figure : Euclid Sector

Black Hole Thermodynamics

1971 :Hawking proves that the sum of the areas of a black hole horizon ($\mathcal{H}$) cannot decrease

$$S = \frac{1}{4} A_{\mathcal{H}}.$$

1972 − 1974 :Bekenstein → black hole horizon has an entropy proportional to its area.

$$\delta M = \frac{\kappa}{8\pi} = \frac{\kappa}{2\pi} \delta \left(\frac{A}{4} \right)$$

where $\kappa = 1/(4M)$.

If $A \propto S$, $\kappa \propto T$, $M \propto E$, the previous equation becomes

$$\delta E = T \delta S.$$

1975 : Hawking discovers that the Black Holes can radiate.

1976 : Unruh effect: an accelerated observer perceives the vacuum as a thermal state with a temperature given by

$$T = \frac{\hbar}{c} \frac{\kappa}{2\pi}.$$

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  - Generalize Newton's law $\nabla^2\phi \propto \rho$ in a observer dependent way:

  One can show that $\rho = T_{ab}u^a u^b$, $\phi \propto R^{ij}_{ij} \equiv \mathcal{R}^{-2}$ to obtain

$$G_{ab}u^a u^b = 8\pi G T_{ab}u^a u^b \xrightarrow{\text{for all observers}} \mathcal{R}^{-2} = 8\pi G\rho$$

  where $\mathcal{R}$ is the radius of the curvature of the space.

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Variational Principle:

$$\delta S[n^a] = -2\int_V d^4x\sqrt{-g}\left[4P^{cd}_{ab}\nabla_c n^a \nabla_d \delta n^b - T_{ab}n^a\delta n^b + \lambda(x)n_a\delta n^b\right].$$

The maximazation must holds for all null vectors $n^a \Rightarrow$ condition on g_{ab}. The equations of Motion (EOM) are

$$G_{ab} = \frac{1}{2}T_{ab} + \Lambda g_{ab}.$$

Emergent Gravity: Functional Approach (2)

- What do we want from our theory?

  - Gravitational field equations must be invariant under the symmetry transformation of matter sector:

$$L_{matter} \to L_{matter} + constant \Rightarrow T^a_b \to T^a_b + (constant)\delta^a_b;$$

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  - Fundamental physical principle to determine the value of the cosmological constant.

- What does the emergent gravity approach give us?

  - Thermodynamical functional associated with all null vectors in a given spacetime.
  - Validity of the maximization of the functional for all the null vectors simultaneously (general covariance) leads to

$$G_{ab} = \frac{1}{2} T_{ab} + \Lambda g_{ab}$$

  where Λ is a (cosmological) constant naturally arising from the above procedure.

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Consider a Rindler observer moving with velocity $u^a = \xi^a/N$:

1. Amount of matter entropy transfer to $\mathcal{H}$

$$\delta S_{matt} = \beta_{loc}\delta E = \beta_{loc} u^a \xi^b T_{ab} dV_{prop} = \beta \xi^a \xi^b T_{ab} dV_{prop}.$$

2. Gravitational entropy measured by the Rindler observer near $\mathcal{H}$

$$\delta S_{grav} = \beta_{loc} u_a J^a dV_{prop} \rightarrow \beta[\xi_b \xi_a (2G^{ab}) + R\xi_a \xi^a] dV_{prop}$$

where J^a is the Noether current for ξ_a and $\beta_{loc}^{-1} = N\frac{\kappa}{2\pi}$.

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When $\xi_a \to k_a$

$$(2G_{ab} - T_{ab})k^a k^b = 0$$

The Bianchi identities and the conservation of T_{ab} lead to $2G^{ab} - T^{ab} = \lambda(x) g^{ab}$, $\lambda(x) = constant$.

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a) Matter falling towards the Black Hole

$r^a \rightarrow$ spacelike unit normal to the stretched horizon Σ
Matter entropy flux: $\beta_{loc} T_{ab} \xi^a r^b$
Gravitational entropy flux: $\beta_{loc} r_a J^a$
When $N \rightarrow 0$, $N r^a \rightarrow \xi^a \rightarrow k^a$, then equating the two fluxes we get

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b) Virtual, infinitesimal displacement of $\mathcal{H}$, ϵ

Proper volume $dV_{prop} = \epsilon \sqrt{\sigma} d^2 x$
Local matter entropy: $\beta_{loc} T^a_b \xi^b r_a dV_{prop}$
Local gravitational entropy: $\beta_{loc} r^a J^a dV_{prop}$
$\delta S_{grav} = \beta_{loc} \xi_a J^a dV_{prop} = \beta_{loc} T_{ab} \xi^a \xi^b dV_{prop} = \delta S_{matt}$
When $\xi^a \to \kappa \lambda k^a$:

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  - Putting together this two results, one obtains

$$\frac{1}{2}TA = 2TS$$

  where A is the surface area, $T = constant$ on the surface and $S = A/4$.

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- Free energy (no pressure): $\beta F = \beta U - S = -S + \beta \int_V d^3x \sqrt{\gamma} N(T_{ab} u^a u^b)$
- Using S_{grav}, the EFE, the relation $R = -8\pi T$ and $\beta = \int_0^{2\pi/\kappa} dt$, we get

$$\beta F = \frac{1}{16\pi} \int_V d^4x \sqrt{-g} R$$

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- *Holographic equipartition*: comoving observers in any static spacetime will find $\boldsymbol{N_{bulk} = N_{surf}}$

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  - the geometrical quantities like tensors and scalars emerge from the dynamics of null hypersurfaces (Rindler spacetime).

- Basically we have that

$$'' H \propto J^a + T^{ab''} \leftrightarrow H = PV + U$$

thus the variation of the enthalpy, viz. the Hilbert-Einstein action, leads to

$$\delta_\lambda H \rightarrow T \delta_\lambda S = \delta_\lambda E + P \delta_\lambda V.$$

The minimization of the enthalpy leads to thermodynamical equation equivalent to the Einstein's field equations $R_{ab} = 8\pi L_P^2 T_{ab}$.

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- By using the geometrical dictionary of null hypersurfaces foliating spacetime and the properties of null vectors defined on them, we can suitably rewrite this functional as

$$S[n^a] = -\int_V d^4x\sqrt{-g}\left(\Theta_{ab}\Theta^{ab} - \kappa^2 + (\theta+\kappa)^2 - T_{ab}n^a n^b\right)$$

where Θ_{ab} is related to $\nabla_a n_b$, κ is the surface gravity and $\theta = \Theta^a_a$.

Fluid Dynamics of Spacetime (2)

- We project on to the null surface the EOM arising from the extrimization of the previous functional (performed w.r.t. the null vectors n^a) and we find that

$$q_a^m \pounds_l \Omega_m + \theta \Omega_a - D_a \left(\kappa + \frac{\theta}{2} \right) + D_m \sigma_a^m = T_{mn} l^m q_a^n$$

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- Further algebra allow us to write the time-variation of the fluid-gravitational action as

$$\frac{1}{\sqrt{-g}}\frac{dS}{dt} = \frac{\mathcal{L}_{grav} A d\lambda}{8\pi} = -dE + TdS_H + Pd\delta A.$$

Fluid Dynamics of Spacetime (3)

- By choosing an adapted coordinate system , Θ can be written as

$$\Theta_{AB} = \frac{1}{2}\left(D_A v_b + D_B v_A + \frac{\partial q_{AB}}{\partial t} \right) \xrightarrow{q_{AB} \ does \ not \ depend \ on \ t} \sigma_{AB}$$

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- Spacetime fluid dynamics implies *dissipation without dissipation*!

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- New geometrical quantity: *nonlocal* symmetric biscalar, $q_{ab}(p, P)$, called q-metric:

$$q_{ab}(p, P; L_0^2) = A g_{ab} - \left(A - \frac{1}{A} \right) n_a n_b$$

where g_{ab} is the classical metric tensor, $\sigma^2 = \sigma^2(p, P)$ is the corresponding classical geodesic interval and

$$A[\sigma; L_0] = 1 + \frac{L_0^2}{\sigma^2}, \qquad n_a = \frac{\nabla_a \sigma^2}{2\sqrt{\sigma^2}}.$$

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  do not coincide. According to the regularization techniques we have to take the first limit as the wright one.
- Putting together this argument with the thermodynamical ones previous seen, one finds that the density of atoms of spacetime (distribution function) is given by

$$f(x^i, n_a) \equiv 1 - \frac{1}{6} L_0^2 R_{ab} n^a n^b.$$

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- The entropy ansatz finds its mathematical foundation in a mesoscopic regime in which one can define non-local geometrical quantities having quantum properties.
- We can also find how this "atoms" are distributed in a renormalized spacetime with a zero-point lenght.

Thanks for the Attention!

Black Hole Entropy from Conserved Quantities

1. Given a covariant theory, the dynamical black hole gravitational entropy is:

$$S_{grav} = \int_{\mathcal{H}} J^{ab}\epsilon_{ab}.$$

   The Noether current $J^a = \nabla_b J^{ab}$ defines the Noether Charge J^{ab}.

2. In GR, the Noether current is

$$J^a = [2G^{ab}\xi_b + R\xi^a + v^a_\xi]$$

3. No matter: on-shell, the gravitational entropy is

$$S_{grav} = -\int_V d^4x\sqrt{-g}\left[4P^{cd}_{ab}\nabla_c\xi^a\nabla_d\xi^b\right].$$

4. With matter: $2G_{ab} = T_{ab}$. When ξ is a Killing vector field, the matter entropy is

$$S_{mat} = \int_V d^4x\sqrt{-g}\,T_{ab}\xi^a\xi^b.$$

5. ***Specif*** result: $\xi^a \to$ Killing vector field, $\mathcal{H} \to$ bifurcate Killing horizon

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Invariance for diffeomorphism: $x^a \to x^a + \xi^a \Rightarrow$ ***Noether Current***, $\boldsymbol{J^a}$

$$J^a = [2G^{ab}\xi_b + R\xi^a + v_\xi^a] = 2R^{ab}\xi_b + v_\xi^a$$

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a) ON-SHELL: $2G^{ab} = 0 \Rightarrow 2R^{ab}\xi_b = R\xi^a$

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b) No surface term $v^a_\xi = 0$, but with $2G^{ab} = T^{ab}$

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c) ***First Law of Black Hole Thermodynamics***

$$S_{grav} = \frac{\beta}{2}\int_{\mathcal{H}} d\Sigma_{ab}J^{ab} = \frac{1}{4}\int_{\mathcal{H}} d^2x\sqrt{\sigma} = \frac{1}{4}A_{\mathcal{H}}$$

Black Hole Entropy from Conserved Quantities (3)

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- Matter entropy $S_{matt} = \beta \int_V d^3x \sqrt{h} k_a T^{ab} \xi_b$
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$$S_{grav} = -4\int_V d^4x\sqrt{-g}[P^{cd}_{ab}\nabla_c n^a\nabla_d n^b - \nabla_c(4P^{cd}_{ab}n^a\nabla_d n^b)]$$

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When $\xi^a \to k^a$ it's necessary to introduce $\beta_{loc} = \beta N$.

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