Playing with multi-critical systems:

RG and CFT approaches in the ε-expansion in d>2

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Based on

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- JHEP 1704 (2017) 127 [arXiv:1703.04830]
- [arXiv:1705.05558]
- Phys. Rev. D98 (2017) 081701 [arXiv:1706.06887]

M. Safari, G.P.V.

- [arXiv:1708.09795]

Outline

- Introduction: universal data of critical theories from
    - CFT
    - Functional Perturbative RG
- Multicritical theories
- A new non trivial example: higher derivative multicritical theories
- Conclusions

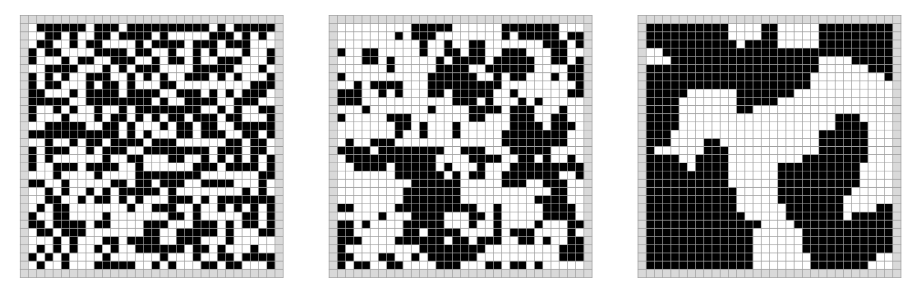
Introduction

Physical systems, very different at microscopic level, can show phases characterized by the same Universal behavior when the correlation length diverges (2nd order phase transition).

Critical phenomena are conveniently described by Quantum and Statistical Field Theories.

$$S = -J \sum_{\langle ij \rangle} s_i s_j + B \sum_i s_i$$

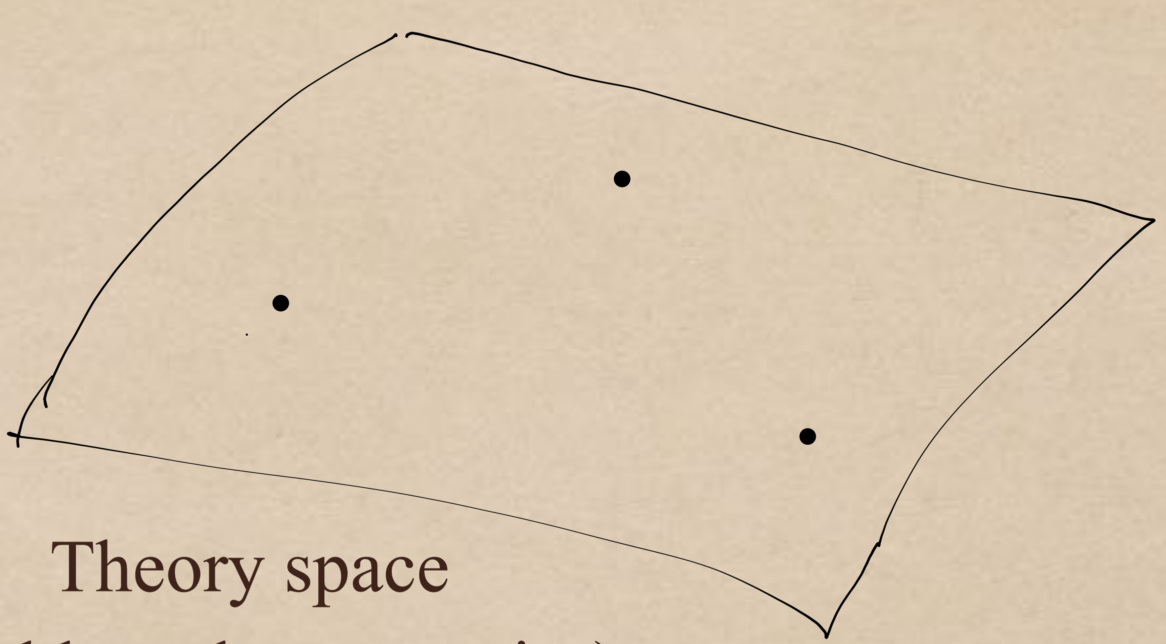
$$s_i = \pm 1$$



Most famous example:

3D Ising universality class (Magnetic systems, Water) in a Landau-Ginzburg description as a scalar QFT,

Critical theories



Theory space
(fields and symmetries)

The critical theories are
points in a suitable theory space
characterized by scale invariance.
If there is Poincare' invariance it is
often lifted to conformal invariance

RG

In a Renormalization Group description critical field theories are associated to fixed points of the flow, where scale invariance is realized.

- These fixed points may control the IR behavior of the theories. (example: Wilson-Fisher fixed point) Wilson (1971), Wilson and Fisher (1972)

- Fundamental physics in a QFT description require renormalizability conditions which in the most general case goes under the name of Asymptotic Safety: existence of a fixed point with a finite number of UV attractive directions. Asymptotic freedom is a particular case with a gaussian fixed point.

Formulations:

- Perturbation theory in presence of small parameters, e.g. ε-expansion below the critical dimension

- Wilsonian non perturbative, exact equations but not solvable in practice. (Polchinski and Wetterich/Morris equations)

CFT

Critical theories often show an enhanced conformal symmetry

In d=2 it is a infinite dimensional Virasoro symmetry, but also in d>2 one can take advantage of the SO(d+1,1) symmetry group.

Conformal data: a CFT is fixed by the scaling dimensions of the primary operators and by the structure constants defining their 3 point correlators.

$$\langle O_a(x) O_b(y) \rangle = \frac{c_a\, \delta_{ab}}{|x-y|^{2\Delta_a}}$$

$$\langle O_a(x) O_b(y) O_c(z) \rangle = \frac{C_{abc}}{|x-y|^{\Delta_a+\Delta_b-\Delta_c} |y-z|^{\Delta_b+\Delta_c-\Delta_a} |z-x|^{\Delta_c+\Delta_a-\Delta_b}}$$

Recently the old proposal of Polyakov was pushed forward in what is called Conformal Bootstrap, based on the consistency of conformal block expansions of the 4 point correlators (in s,t channels)

El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin and Vichi (2012)

Also in CFT the perturbative ε-expansion is very useful and several different aproaches are available.

Lagrangian description

The main constraints are given by the field content and the symmetries, but this leaves still too many possible theories for a generic dimension d.

It is therefore useful to start from some kind of Landau-Ginzburg description to single out some possible solutions.

$$S = \int \mathrm{d}^d x \sum_i g_i O_i(\phi)$$

- This is the starting point for an RG analysis.

- In a CFT this leads to include the Schwinger-Dyson Equations (SDE) which force a recombination in multiplet of composite operators (in particular changing the nature from primary to descendant).

$$\left\langle \frac{\delta S}{\delta \phi}(x)\, O_1(y)\, O_2(z) \ldots \right\rangle = 0$$

Ignore contact terms

Rychkov, Tan (2015)

Basu, Krishnan (2015)

Nii (2016)

Hasegawa and Nakayama (2017)

Codello, Safari, G.P.V., Zanusso (2017)

RG and CFT: pro et contra at criticality

RG is generally affected by scheme dependence but it is very powerful

(Functional) perturbative RG: systematic expansion but resummation needed

Functional nonperturbative RG:
very powerful but no fully systematic way to organize corrections available.

CFT is not scheme dependent!

CFT: using the full machinery at analytic level is in general very complicated.

Conformal bootstrap is hard to apply for more complicated models

Perturbative approaches share the convergence problems with RG

Can we obtain in some approximation the same results in the two approaches?

Universal data and RG

How to get in an RG framework informations on the critical theory?
If conformal, the so called conformal data?

- It can be partially done in the perturbative ε-expansion approximation using the universal beta function coefficients, e.g. in a massless $\overline{\mathrm{MS}}$ scheme

Critical quantities are encoded in the expansion coefficients describing the flow around the scale invariant point: $\beta^i(g_*) = 0$

$$\beta^k(g_* + \delta g) = \sum_i M^k{}_i\, \delta g^i + \sum_{i,j} N^k{}_{ij}\, \delta g^i\, \delta g^j + O(\delta g^3)$$

$$M^i{}_j \equiv \left.\frac{\partial \beta^i}{\partial g^j}\right|_* \qquad\qquad N^i{}_{jk} \equiv \frac{1}{2} \left.\frac{\partial^2 \beta^i}{\partial g^j \partial g^k}\right|_*$$

Moving to a diagonal basis in the linear sector

$$\sum_{i,j} \mathcal{S}^a{}_i\, M^i{}_j\, (\mathcal{S}^{-1})^j{}_b = -\theta_a \delta^a{}_b$$

Universal data and RG

$$\theta_a = d - \Delta_a$$

$$\tilde{C}^a{}_{bc} = \sum_{i,j,k} \mathcal{S}^a{}_i \, N^i{}_{jk} \, (\mathcal{S}^{-1})^j{}_b \, (\mathcal{S}^{-1})^k{}_c$$

RG flow seen along the eigendirections around the fixed point up to second order

$$S = S_* + \sum_a \mu^{\theta_a} \lambda^a \int \mathrm{d}^d x \, \mathcal{O}_a(x) + O(\lambda^2) \, .$$

Take home message

$$\beta^a = -(d - \Delta_a)\lambda^a + \sum_{b,c} \tilde{C}^a{}_{bc} \, \lambda^b \lambda^c + O(\lambda^3) \, .$$

one can extract not only the scaling dimensions, but also, reversing an argument from Cardy for a CFT, some OPE coefficients (structure constants) at order O(ε)

$$\langle \mathcal{O}_a(x) \, \mathcal{O}_b(y) \cdots \rangle = \sum_c \frac{1}{|x-y|^{\Delta_a + \Delta_b - \Delta_c}} \, C^c{}_{ab} \, \langle \mathcal{O}_c(x) \cdots \rangle$$

Scheme dependence

RG scheme changes correspond to a coupling reparameterization $\bar{g}^i = \bar{g}^i(g)$

$$\bar{\beta}^i(\bar{g}) = \frac{\partial \bar{g}^i}{\partial g^j}\,\beta^j(g)$$

Linear term coefficients transform homogeneously

$$\bar{M}^i{}_j = \frac{\partial \bar{g}^i}{\partial g^l}\,M^l{}_k\,\frac{\partial g^k}{\partial \bar{g}^j} \qquad \Rightarrow \qquad \bar{\theta}_a = \theta_a$$

Quadratic term coefficients transform inhomogeneously

$$\bar{N}^i{}_{jk} = \frac{\partial \bar{g}^i}{\partial g^c}\Big\{N^c{}_{ab} + \frac{1}{2}M^c{}_d\frac{\partial^2 g^d}{\partial \bar{g}^l \partial \bar{g}^m}\frac{\partial \bar{g}^l}{\partial g^a}\frac{\partial \bar{g}^m}{\partial g^b} - \frac{1}{2}M^d{}_a\frac{\partial^2 g^c}{\partial \bar{g}^l \partial \bar{g}^m}\frac{\partial \bar{g}^l}{\partial g^b}\frac{\partial \bar{g}^m}{\partial g^d} - \frac{1}{2}M^d{}_b\frac{\partial^2 g^c}{\partial \bar{g}^l \partial \bar{g}^m}\frac{\partial \bar{g}^l}{\partial g^a}\frac{\partial \bar{g}^m}{\partial g^d}\Big\}\frac{\partial g^a}{\partial \bar{g}^k}\frac{\partial g^b}{\partial \bar{g}^j}\,.$$

Take home message

$$\Rightarrow \qquad \bar{\tilde{C}}^c{}_{ab} = \tilde{C}^c{}_{ab} + \frac{1}{2}\left(\theta_c - \theta_a - \theta_b\right)\frac{\partial^2 g^c}{\partial \bar{g}^l \partial \bar{g}^m}\frac{\partial \bar{g}^l}{\partial g^a}\frac{\partial \bar{g}^m}{\partial g^b}$$

Scheme dependence

Invariance condition

$$\tilde{\tilde{C}}^{c}{}_{ab} = \tilde{C}^{c}{}_{ab} \quad \Leftrightarrow \quad (\theta_c - \theta_a - \theta_b)\,\frac{\partial^2 g^c}{\partial \bar{g}^l \partial \bar{g}^m}\frac{\partial \bar{g}^l}{\partial g^a}\frac{\partial \bar{g}^m}{\partial g^b} = 0$$

Condition for scheme independence

$$\theta_c - \theta_a - \theta_b = 0$$

In general not fulfilled. But it can be at the critical dimension.

Employing the ε-expansion and $\overline{\mathrm{MS}}$ scheme
dimensionless OPE coefficients are less sensitive to scheme changes

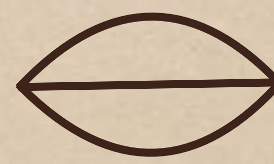
Ising Universality Class

ε-expansion below d=4 for the LG critical model $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + g\phi^4$

Leading counterterms in perturbation theory at order g^2 , dim reg $\overline{\text{MS}}$

$d = 4 - \epsilon$

$$\mathcal{L}_{c.t.} = \frac{1}{\epsilon}\frac{1}{2(4\pi)^2}(12g)^2\phi^4 - \frac{1}{\epsilon}\frac{1}{6(4\pi)^4}(4!g)^2(\partial\phi)^2$$

$12g\phi^2$  $12g\phi^2$ $\quad$ $4!\, g\phi$  $4!\, g\phi$

Rescaling the coupling: $g \to (4\pi)^2 g$

beta function: $\beta_g = -\epsilon g + 72 g^2$

Two fixed points:

$g_* = 0$ — UV gaussian

$g_* = \frac{\epsilon}{72}$ — IR Wilson-Fisher

Anomalous dimension: $\eta = 2\tilde{\gamma}_1 = 96 g^2$ $\qquad$ $\eta = \frac{\epsilon}{54}$

η is a universal quantity, independent from any coupling reparameterization!

Ising Universality Class

Anomalous dimension from scale invariance and SDE

Interacting 2-point function at criticality: $\langle \phi_x \phi_y \rangle = \dfrac{c}{|x-y|^{2\Delta}}$ $\quad c = \dfrac{1}{4\pi^2}$

$\Delta = \delta + \gamma_1 \quad \delta = \dfrac{d}{2} - 1$

EOM: $\Box\phi = 4g\phi^3$

$d = 4 - \epsilon$

$$\Box_x \Box_y \langle \phi_x \phi_y \rangle = c \frac{2\Delta(2\Delta+2)(2\Delta+2-d)(2\Delta+4-d)}{|x-y|^{2\Delta+4}} \simeq \frac{32c\gamma_1}{|x-y|^6}$$

$$\langle \Box_x \phi_x \Box_y \phi_y \rangle = 16g^2 \langle \phi_x^3 \phi_y^3 \rangle \simeq 16g^2 3! \frac{c^3}{|x-y|^6}$$

At leading order

$$\Rightarrow \quad \gamma_1 = 3g^2c^2 + O(g^3)$$

Take home message

Rescaling the coupling as before: $g \to (4\pi)^2 g$

$$\Rightarrow \quad \gamma_1 = 48g^2 + O(g^3)$$

In agreement with the 2-loop result!

Leading CFT constraints on multicritical theories

Assuming conformal invariance and the SDE in ε-expansion

- First partial studies for Ising (also O(N)), Tricritical and Lee-Yang UC

Rychkov, Tan (2015)
Basu, Krishnan (2015)
Nii (2016)
Hasegawa and Nakayama (2017)

- Systematic full study for all single scalar field multicritical models

A. Codello, M. Safari, G.P.V., O. Zanusso JHEP 1704 (2017) 127

Landau-Ginzburg lagrangian

$d = d_m - \epsilon$

$$S[\phi] = \int d^dx \left\{ \frac{1}{2}(\partial\phi)^2 + \mu^{\left(\frac{m}{2}-1\right)\epsilon} \frac{g}{m!} \phi^m \right\}$$

Upper critical dimension

$$d_m = \frac{2m}{m-2}$$

even $m = 2n$ $d_c = 4, 3, \cdots$ (unitary: e.g. Ising, Tricritical,…)

odd $m = 2n+1$ $d_c = 6, \frac{10}{3}, \cdots$ (non unitary: e.g. Lee-Yang, Blume-Capel,…)

Leading CFT constraints on multicritical theories

EOM: $\Box\phi = \frac{g}{(m-1)!}\phi^{m-1}$

Composite operators $[\phi^i]$

↓

Primary operators ($i \neq m-1$)

Scaling dimensions $\Delta_i = i\delta + \gamma_i$ $\delta = \frac{d}{2} - 1 = \delta_m - \frac{\epsilon}{2}$ $\delta_m = \frac{2}{m-2}$

Note that for $\epsilon \neq 0$ $\Delta_{m-1} = \Delta_1 + 2 \implies \gamma_{m-1} = \gamma_1 + (m-2)\frac{\epsilon}{2}$.

Free theory

$$\langle \phi^k(x)\phi^l(y)\rangle \overset{\text{free}}{=} \delta_{kl}\, k! \frac{c^k}{|x-y|^{2k\delta_m}} \qquad c = \frac{1}{4\pi}\frac{\Gamma(\delta_m)}{\pi^{\delta_m}} = \frac{1}{(d_m-2)S_{d_m}}$$

$$\langle \phi^{n_1}(x_1)\phi^{n_2}(x_2)\phi^{n_3}(x_3)\rangle \overset{\text{free}}{=} \frac{C^{\text{free}}_{n_1,n_2,n_3}}{|x_1-x_2|^{\delta_m(n_1+n_2-n_3)}|x_2-x_3|^{\delta_m(n_2+n_3-n_1)}|x_3-x_1|^{\delta_m(n_3+n_1-n_2)}}$$

$$C^{\text{free}}_{n_1,n_2,n_3} = \frac{n_1!\, n_2!\, n_3!}{\left(\frac{n_1+n_2-n_3}{2}\right)!\left(\frac{n_2+n_3-n_1}{2}\right)!\left(\frac{n_3+n_1-n_2}{2}\right)!} c^{\frac{n_1+n_2+n_3}{2}} \quad , n_i+n_j-n_k \geq 0$$

Together with the constraints on 2 and 3-points CFT correlators one can compute the leading non trivial values for the scaling dimensions Δ_a and families of structure constants C_{abc}

Leading CFT constraints on multicritical theories

Some examples: $m = 2n$

Take home message

Anomalous dimensions

- $$\Box_x \Box_y \langle \phi(x)\phi(y) \rangle \overset{\text{LO}}{=} \frac{16n}{(n-1)^2}\gamma_1 \frac{c}{|x-y|^{4+\frac{2}{n-1}}}$$
$$\overset{\text{LO}}{=} \frac{g^2}{(2n-1)!}\frac{c^{2n-1}}{|x-y|^{4+\frac{2}{n-1}}} \longrightarrow \gamma_1 = \frac{2(n-1)^2}{(2n)!}\Gamma\left(\tfrac{1}{n-1}\right)^{2(n-1)}\frac{g^2}{(4\pi)^{2n}} + O(g^3)$$

Itzykson, Drouffe (1989) J. O'Dwyer, H. Osborn (2008)

- $\Box_x \langle \phi(x)\phi(y)\phi^2(z) \rangle \longrightarrow \gamma_2 = \frac{g}{(4\pi)^2} + O(g^2)\,, \qquad n = 2$

- $\Box_x \Box_y \langle \phi(x)\phi(y)\phi^2(z) \rangle \longrightarrow \gamma_2 = 8\frac{(n+1)(n-1)^3}{(n-2)(2n)!}\Gamma\left(\tfrac{1}{n-1}\right)^{2(n-1)}\frac{g^2}{(4\pi)^{2n}} + O(g^3)\,, \qquad n > 2$

- $\Box_x \langle \phi(x)\phi^k(y)\phi^{k+1}(z) \rangle \longrightarrow \gamma_k = \frac{2(n-1)}{n!^2}\frac{k!}{(k-n)!}\Gamma\left(\tfrac{1}{n-1}\right)^{n-1}\frac{g}{(4\pi)^n} + O(g^2)\,, \qquad k \geq n$

The last relation is valid, in particular, for $k = 2n-1$, so that

- $\gamma_{2n-1} = \gamma_1 + (n-1)\epsilon \longrightarrow g = \frac{n!^3}{(2n)!}(4\pi)^n\Gamma\left(\frac{1}{n-1}\right)^{1-n}\epsilon + O(\epsilon^2)$

Leading CFT constraints on multicritical theories

Structure constants

Take home message

- $\Box_x \langle \phi(x)\, \phi^{2k}(y)\, \phi^{2l-1}(z) \rangle \longrightarrow C_{1,2k,2l-1} = \frac{n!^3}{(2n)!} \frac{(n-1)^2}{(k-l)(k-l+1)} \frac{(2k)!(2l-1)!}{(n+l-k-1)!(k+n-l)!(k+l-n)!} \epsilon + O(\epsilon^2)$

$$k+l \geq n, 1-n \leq (l-k) \leq n, l-k \neq 0,1$$

- $\Box_x \Box_y \langle \phi(x)\phi(y)\phi^{2k}(z) \rangle \longrightarrow C_{1,1,2k} = \frac{(n-1)^4}{k(k-1)(k-n)(k-n+1)} \frac{n!^6}{(2n)!^2} \frac{(2k)!}{k!^2(2n-k-1)!} \epsilon^2 + O(\epsilon^3)$

$$k \neq n-1, n \text{ and } 2 \leq k \leq 2n-1$$

The results for $\gamma_1, \gamma_2, C_{1,k,l}, C_{1,1,2k}$ as functions of the coupling extend to the case of odd theories, for which one has also

- $\Box_x \Box_y \Box_z \langle \phi(x)\phi(y)\phi(z) \rangle \longrightarrow C_{111} = \frac{(2n-1)^6}{2^8 n(n-1)n!^3} (c_{\text{odd}}^{n-\frac{1}{2}} g)^3 + O(g^4)$ $\qquad c_{\text{odd}} = \frac{1}{4\pi} \frac{\Gamma\left(\frac{2}{2n-1}\right)}{\pi^{\frac{2}{2n-1}}}$

Functional perturbative RG example: Ising UC

How to study deformations around the Wilson-Fisher fixed point? $d = 4 - \epsilon$

Couplings: $$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + g_1\phi + g_2\phi^2 + g_3\phi^3 + g_4\phi^4$$

Dimensionful beta functions
(global rescaling as before)

$$\beta_1 = 12\, g_2 g_3 - 108\, g_3^3 - 288\, g_2 g_3 g_4 + 48\, g_1 g_4^2$$
$$\beta_2 = 24\, g_4 g_2 + 18\, g_3^2 - 1080\, g_3^2 g_4 - 480\, g_2 g_4^2$$
$$\beta_3 = 72\, g_4 g_3 - 3312\, g_3 g_4^2$$
$$\beta_4 = 72\, g_4^2 - 3264\, g_4^3$$

Functions:

$$\mathcal{L} = \frac{1}{2}Z(\phi)(\partial\phi)^2 + V(\phi)$$

1 loop 2 loop

$$\beta_V = \frac{1}{2}\eta\phi V^{(1)} + a\frac{(V^{(2)})^2}{(4\pi)^2} + b\frac{V^{(2)}(V^{(3)})^2}{(4\pi)^4} + \cdots$$

$$\beta_Z = \eta Z + \frac{1}{2}\eta\phi Z^{(1)} + c\frac{(V^{(4)})^2}{(4\pi)^4} + \cdots$$

2 loop

$$a = \frac{1}{2} \qquad b = -\frac{1}{2} \qquad c = -\frac{1}{6}$$

Take home message

Field independent Z → anomalous dimension

FPRG for multicritical models

We limit to a truncation $\mathcal{L} = \frac{1}{2} Z(\phi)(\partial\phi)^2 + V(\phi)$

RG, even O'Dwyer, Osborn (2008) Codello, Safari, G.P.V., Zanusso arXiv:1705:05558

RG, odd Codello, Safari, G.P.V., Zanusso Phys. Rev. D98 (2017) 081701

Rescaling functions and fields to dimensionless quantities $v(\varphi), z(\varphi)$

$$\beta_v = -\,d\,v(\varphi) + \frac{d-2+\eta}{2}\,\varphi\,v'(\varphi) + \frac{n-1}{n!}\frac{c^{n-1}}{4} v^{(n)}(\varphi)^2$$

$$-\frac{n-1}{48}\,c^{2n-2}\,\Gamma(\delta_n) \sum_{\substack{r+s+t=2n \\ r,s,t\neq n}} \frac{K^n_{rst}}{r!s!t!}\, v^{(r+s)}(\varphi)\, v^{(s+t)}(\varphi)\, v^{(t+r)}(\varphi)$$

$$-\frac{(n-1)^2}{16\,n!}\,c^{2n-2} \sum_{s+t=n} \frac{n-1+L^n_{st}}{s!t!}\, v^{(n)}(\varphi)\, v^{(n+s)}(\varphi)\, v^{(n+t)}(\varphi)$$

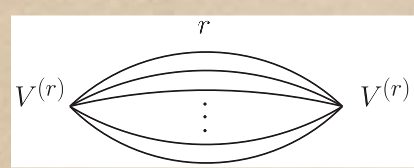
$$K^n_{rst} = \frac{\Gamma\left(\frac{n-r}{n-1}\right)\Gamma\left(\frac{n-s}{n-1}\right)\Gamma\left(\frac{n-t}{n-1}\right)}{\Gamma\left(\frac{r}{n-1}\right)\Gamma\left(\frac{s}{n-1}\right)\Gamma\left(\frac{t}{n-1}\right)}, \qquad L^n_{st} = \psi(\delta_n) - \psi(s\delta_n) - \psi(t\delta_n) + \psi(1)$$

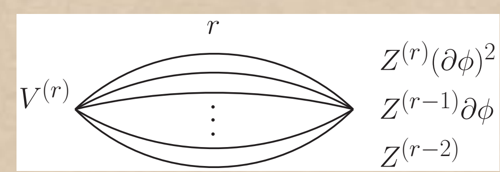
FPRG for multicritical models

$$\beta_z = \eta\, z(\varphi) + \frac{d-2+\eta}{2}\,\varphi\, z'(\varphi) - \frac{(n-1)^2}{(2n)!}\frac{c^{2n-2}}{4}\, v^{(2n)}(\varphi)^2 + \frac{n-1}{n!}\frac{c^{n-1}}{2}\left[z^{(n)}(\varphi)\, v^{(n)}(\varphi) + z^{(n-1)}(\varphi)\, v^{(n+1)}(\varphi)\right]$$

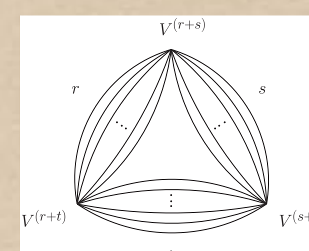
Contributions from multi-loop diagrams:

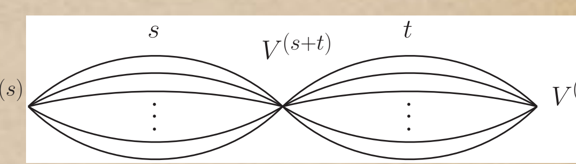
LO

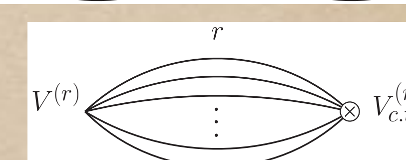




NLO







First one finds the fixed point for the critical coupling

$$\frac{(2n)!^2}{4\,n!^3}\, c^{n-1} g = \epsilon - \frac{n}{n-1}\eta + \frac{n!^4}{(2n)!}\left[\frac{1}{3}\,\Gamma(\delta_n)\; n!^2 \sum_{\substack{r+s+t=2n \\ r,s,t\neq n}} \frac{K^n_{rst}}{(r!s!t!)^2} + (n-1)\sum_{s+t=n}\frac{n-1+L^n_{st}}{s!^2 t!^2}\right]\epsilon^2$$

Then one expands in all the couplings associated to all operators

FPRG for multicritical models

General pattern of mixing

$$\begin{array}{rllllll} V: & 1\ \phi\ \cdots\ \phi^{2n-1} & \phi^{2n} & \cdots\ \phi^{4n-3} & \phi^{4n-2} & \cdots \\ Z: & & (\partial\phi)^2 & \cdots\ \phi^{2n-3}(\partial\phi)^2 & \phi^{2n-2}(\partial\phi)^2 & \cdots \\ W_1: & & & & \phi\Box^2\phi & \cdots \\ W_2: & & & & (\partial_\mu\partial_\nu\phi)^2 & \cdots \\ W_3: & & & & (\Box\phi)^2 & \cdots \end{array}$$

$$\begin{pmatrix} M^{(0)} & & & \\ & M^{(2)} & & \\ & & M^{(4)} & \\ & & & \ddots \end{pmatrix}$$

$$\tilde{\gamma}_i = \frac{2(n-1)n!}{(2n)!}\frac{i!}{(i-n)!}\epsilon \qquad \tilde{\omega}_i = \frac{2(n-1)n!}{(2n)!}\frac{(i+1)!}{(i-n+1)!}\epsilon$$

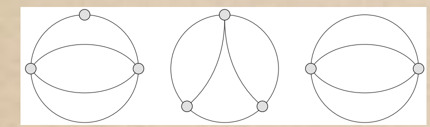
At leading order the stability matrix M is triangular.

$$\begin{aligned} \tilde{\gamma}_i &= i\frac{\eta}{2} + \frac{(n-1)i!}{(i-n)!}\frac{2\,n!}{(2n)!}\left[\epsilon - \frac{n}{n-1}\eta\right] + 2n\,\eta\,\delta_i^{2n} \\ &+ \frac{(n-1)i!n!^6}{(2n)!^2}\Gamma(\delta_n)\sum_{\substack{r+s+t=2n \\ r,s,t\neq n}}\frac{K^n_{rst}}{(r!s!t!)^2}\left[\frac{2n!}{3(i-n)!} - \frac{r!}{(i-2n+r)!}\right]\epsilon^2 \\ &+ \frac{(n-1)^2 i!n!^5}{(2n)!^2}\sum_{s+t=n}\frac{n-1+L^n_{st}}{(s!t!)^2}\left[\frac{1}{(i-n)!} - \frac{2s!}{n!(i-2n+s)!}\right]\epsilon^2. \end{aligned}$$

OPE coefficients are read off the quadratic expansion of the beta functions

The Blume-Capel or Ticritical Lee-Yang

A nontrivial (non unitary) UC in $d=3$ $\qquad \mathcal{L} = \frac{1}{2}(\partial\phi)^2 + g\phi^5 \qquad d = \frac{10}{3} - \epsilon$



$$\beta_v = -\frac{10}{3}v + \frac{2}{3}\varphi v' + \epsilon\left(v - \frac{1}{2}\varphi v'\right) + \frac{\eta}{2}\varphi v' + \frac{1}{3}v^{(2)}(v^{(4)})^2 - \frac{3}{2}(v^{(3)})^2 v^{(4)}\,,$$
$$\beta_z = \frac{2}{3}\varphi z' + \eta\left(z + \frac{1}{2}\varphi z'\right) - \frac{1}{30}(v^{(5)})^2\,.$$

Dimensionless beta functions

Fixed point: $\quad g(\epsilon) = \frac{\sqrt{-\epsilon}}{60\sqrt{102}}$

$$\theta_i = \frac{10}{3} - \frac{2i}{3} + \epsilon\left(-1 + \frac{i}{2}\right) - \tilde{\gamma}_i$$
$$\tilde{\gamma}_i = \frac{\epsilon}{153}\left(\frac{52}{5}i - \frac{139}{12}i^2 - \frac{1}{2}i^3 + \frac{19}{12}i^4 - \delta_{i,5}\right)$$

Critical exponents

$$\eta = 2\tilde{\gamma}_1 = -\frac{\epsilon}{765}$$
$$\nu = \theta_2^{-1} = \frac{1}{2} - \frac{7\epsilon}{1020}$$

OPE coefficients: overlap with CFT +SDE in

$$\frac{\tilde{C}^1{}_{15}}{\sqrt{\tilde{\gamma}_1}} = 4\sqrt{15} + O(\epsilon)\,, \quad \frac{\tilde{C}^1{}_{24}}{\sqrt{\tilde{\gamma}_1}} = 32\sqrt{15} + O(\epsilon)\,,$$
$$\frac{\tilde{C}^1{}_{33}}{\sqrt{\tilde{\gamma}_1}} = -108\sqrt{15} + O(\epsilon)\,,$$

CFT results can be completed using $g(\epsilon)$

HIgher derivative multicritical scalar theories

Higher derivative models are getting some attention in the last years.
They are non unitary, but can be physically relevant.

Motivations:

Theory of elasticity (e.g. Riva-Cardy model) — Nakayama (2016)

Physics of polymers (isotropic Lifshitz theories) — Schwahn et. al. (1999)

Quantum Gravity (possibly related in various ways)

Theoretically some correspond to new families of non unitary CFT also in higher dimensions.

Recent works

Free theories — Osborn, Stergiou (2016) Brust, Hinterbichler (2017)

Multicritical in CFT — Gliozzi, Guerrieri, Petkou, Wen (2017)

O(N) quartic in RG — Gracey (2017)

ε-expansion RG analysis

We want to compare to (and go beyond) recent CFT results.

We consider the Z_2 symmetric theories, with a kinetic term $S = \int \mathrm{d}^d x \, \frac{1}{2}\phi \, (-\Box)^k \phi$

At the critical dimension d_c the theory is free

$$d_c = \frac{2nk}{n-1}$$

Field dimension $[\phi] = \delta = \frac{d}{2} - k$

Free propagator $G_0(x) = \frac{c}{|x|^{2\delta}}$ $\qquad c = \frac{1}{(4\pi)^k \Gamma(k)} \frac{\Gamma(\delta)}{\pi^\delta}$

The critical theory at $d = d_c - \epsilon$ can have marginal interactions characterized by Z_2 symmetric operators with $2l$ derivatives and α fields if (dimensional analysis)

$$\alpha \frac{k}{n-1} + 2l = \frac{2nk}{n-1}$$

with l integer and α an even integer.

Examples:

$$k = 1 \to (l = 0, \alpha = 2n) \qquad \phi^{2n}$$

$$k = 2 \to (l = 0, \alpha = 2n) \qquad \phi^{2n}$$

$$k = 2 \to (l = 1, \alpha = n+1) \to n = 1 + 2m \qquad \phi^{n-1}(\partial\phi)^2$$

Improving the present picture

Therefore in a perturbative framework there is a pattern of mixing of operators with a different number of derivatives (relevant, marginal or irrelevant)

We find that a recent analysis (F. Gliozzi, A. Guerrieri, A. C. Petkou and C. Wen (2017)) done in a CFT framework is valid only for a subset of the theories such that the pair of integers $(k, n-1)$ are coprime, so that they correspond to the case of a pure ϕ^{2n} interaction operator at the fixed point, with one critical coupling (theories of first kind)

- We reproduce these results obtaining some higher order results

In the other cases (theories of second kind) the pattern increases of complexity with k since the critical theory can be characterized by several couplings.

- We shall address here only the family of theories with $(k=2, n=1+2m)$ which are characterized by two critical couplings. Novel pattern.

M. Safari and G.P.V.

$(k, n-1)$ coprime (first kind)

The relevant diagrams are similar to the case k=1. We go up to 2(n-1) loops.

Beta functions constructed looking at the $1/\epsilon$ poles in $\overline{\mathrm{MS}}$ scheme

Working with dimensionless quantitites with convenient rescalings $\quad v \to v\,[(4\pi)^k\Gamma(k)]^n\,\Gamma(n\delta)/\Gamma(\delta)^n$

beta functional

$$\beta_v = -\,dv + \frac{d-2k+\eta}{2}\phi\, v' + \frac{1}{n!}\, v^{(n)\,2}$$
$$-\Gamma(n\delta_n)\,\frac{1}{3}\sum_{\substack{r+s+t=2n\\ r,s,t\neq n}} \frac{K^{n,k}_{rst}}{r!s!t!}\, v^{(r+s)}v^{(s+t)}v^{(t+r)} - \frac{1}{n!}\sum_{s+t=n}\frac{J^{n,k}_{st}}{s!t!}\, v^{(n)}v^{(n+s)}v^{(n+t)}$$

with $\quad K^{n,k}_{rst} \equiv \dfrac{\Gamma\left((n-r)\delta\right)\Gamma\left((n-s)\delta\right)\Gamma\left((n-t)\delta\right)}{\Gamma\left(r\delta\right)\Gamma\left(s\delta\right)\Gamma\left(t\delta\right)} \qquad J^{n,k}_{st} \equiv \psi(n\delta)-\psi(s\delta)-\psi(t\delta)+\psi(1)$

Fixed point relation: $\quad \dfrac{(2n)!^2}{n!^3}g = (n-1)\epsilon - n\,\eta + \dfrac{n!^4(n-1)^2}{(2n)!}\left[\dfrac{1}{3}\Gamma(n\delta_n)\, n!^2 \displaystyle\sum_{\substack{r+s+t=2n\\ r,s,t\neq n}}\dfrac{K^{n,k}_{rst}}{(r!s!t!)^2} + \sum_{s+t=n}\dfrac{J^{n,k}_{st}}{s!^2t!^2}\right]\epsilon^2$

Anomalous dimension: $\quad \gamma_\phi = \dfrac{\eta}{2} = (-1)^{k+1}\,\dfrac{n(\delta_n)_k}{k(\delta_n+k)_k}\,\dfrac{2(n-1)^2 n!^6}{(2n)!^3}\,\epsilon^2$

Examples (first kind)

$k=2, n=2$

$d_c = 8$

$$\beta_v = -dv + \frac{d-4+\eta}{2}\varphi v^{(1)} + \frac{1}{2}\left(v^{(2)}\right)^2 + \frac{1}{12}v^{(2)}\left(v^{(3)}\right)^2$$

$k=3, n=2$

$d_c = 12$

$$\beta_v = -dv + \frac{d-6+\eta}{2}\varphi v^{(1)} + \frac{1}{2}\left(v^{(2)}\right)^2 + \frac{43}{120}v^{(2)}\left(v^{(3)}\right)^2$$

$k=3, n=3$

$d_c = 9$

$$\beta_v = -dv + \frac{d-6+\eta}{2}\varphi v^{(1)} + \frac{1}{6}\left(v^{(3)}\right)^2 - \frac{35\pi^2}{8192}\left(v^{(4)}\right)^3 + \frac{31}{1260}v^{(3)}v^{(4)}v^{(5)} - \frac{7}{144}v^{(2)}\left(v^{(5)}\right)^2$$

Results

Anomalous dimensions for composite operators

$$\tilde{\gamma}_i = \frac{\eta}{2} i + \frac{(n-1)i!}{(i-n)!}\frac{2n!}{(2n)!}\left[\epsilon - \frac{n}{n-1}\eta\right]$$
$$+\frac{(n-1)^2 i! n!^6}{(2n)!^2}\Gamma(n\delta_n)\sum_{\substack{r+s+t=2n\\ r,s,t\neq n}}\frac{K^{n,k}_{rst}}{(r!s!t!)^2}\left[\frac{2n!}{3(i-n)!}-\frac{r!}{(i-2n+r)!}\right]\epsilon^2$$
$$+\frac{(n-1)^2 i! n!^5}{(2n)!^2}\sum_{s+t=n}\frac{J^{n,k}_{st}}{s!^2 t!^2}\left[\frac{1}{(i-n)!}-\frac{2s!}{n!(i-2n+s)!}\right]\epsilon^2$$

From CFT known only up to order O(ε)!

F. Gliozzi, A. Guerrieri, A. C. Petkou and C. Wen (2017)

OPE coefficients

$$\tilde{C}^k_{ij} = \frac{1}{n!}\frac{i!}{(i-n)!}\frac{j!}{(j-n)!} - \Gamma(n\delta_n)\frac{(n-1)n!^3}{(2n)!}\sum_{\substack{r+s+t=2n\\ r,s,t\neq n}}\frac{K^{n,k}_{rst}}{r!s!t!^2}\frac{j!}{(j-s-t)!}\frac{i!}{(i+s-2n)!}\epsilon$$
$$-\frac{(n-1)n!^2}{(2n)!}\sum_{s+t=n}\frac{J^{n,k}_{st}}{s!t!}\left[\frac{1}{n!}\frac{j!}{(j-n-s)!}\frac{i!}{(i-n-t)!}+\frac{1}{s!}\frac{i!}{(i-n)!}\frac{j!}{(j-n-s)!}\right.$$
$$\left.+\frac{1}{s!}\frac{j!}{(j-n)!}\frac{i!}{(i-n-s)!}\right]\epsilon$$

Agreement with CFT looking at correlators of composite operators ϕ^i .

CFT: derivation (without computing loops)

For this critical theory the SDE are given $\Box^k \phi = (-1)^{k-1} \frac{g}{(2n-1)!} \phi^{2n-1}$

Scaling dimensions of ϕ^i : $\Delta_i = \left(\frac{d}{2} - k\right) i + \gamma_i$

- Anomalous dimension γ_1 starting from $\langle \phi_x \phi_y \rangle = \frac{c}{|x-y|^{2\Delta_1}}$

$$\Box_x^k \Box_y^k \langle \phi_x \phi_y \rangle = \frac{g^2}{(2n-1)!^2} \langle \phi_x^{2n-1} \phi_y^{2n-1} \rangle \longrightarrow (-1)^{k-1} 2^{4k} k \Gamma(k)^2 \left(\frac{k}{n-1}\right)_{2k} c\,\gamma_1 = \frac{g^2}{(2n-1)!} c^{2n-1}$$

Evaluate r.h.s. at tree level

- Recurrence relation for $\gamma_{m+1} - \gamma_m$ from $\Box_x^k \langle \phi_x \phi_y^m \phi_z^{m+1} \rangle = \frac{g(-1)^{k-1}}{(2n-1)!} \langle \phi_x^{2n-1} \phi_y^m \phi_z^{m+1} \rangle$

- Fixed point coupling $g(\epsilon)$ imposing descendant condition on γ_{2n-1}

- One can find the structure constants $C_{1,2m,2l-1}$ starting from

$$\Box_x^k \langle \phi_x \phi_y^{2m} \phi_z^{2l-1} \rangle = \frac{g(-1)^{k-1}}{(2n-1)!} \langle \phi_x^{2n-1} \phi_y^{2m} \phi_z^{2l-1} \rangle$$

All the methods give the same perturbative results.

The case $(k=2, n=1+2m)$ (second kind)

Critical theory $$\mathcal{L}_{FP} = \frac{1}{2}\phi\Box^2\phi + \frac{1}{2}h\phi^{2m}(\partial\phi)^2 + g\phi^{2(2m+1)}$$

Including some deformations $$\mathcal{L} = \tfrac{1}{2}\phi\Box^2\phi + V + \tfrac{1}{2}Z(\partial\phi)^2 + \tfrac{1}{2}W_1\phi\Box^2\phi + \tfrac{1}{2}W_2(\partial\phi)^2\Box\phi + \tfrac{1}{2}W_3(\partial\phi)^4$$

Several diagrams contribute to quadratic and even more to cubic terms in the potentials (couplings). In perturbation theory we compute the counter-terms as $1/\epsilon$ poles in the $\overline{\mathrm{MS}}$ scheme, at functional level.

$$\delta_c = \frac{k}{n-1} = \frac{1}{m}, \qquad d_c = \frac{2nk}{n-1} = \frac{4m+2}{m} = 4 + \frac{2}{m},$$

For example

V^2 counterterms $$U_{l,\text{c.t.}} = \frac{1}{2\,r!}\frac{1}{(4\pi)^{(r-1)(k+\delta_c)}\Gamma^r(k)}\frac{\Gamma^r(\delta_c)}{\Gamma(r\delta_c)}\frac{(-1)^l}{l!}\frac{2}{(r-1)\epsilon}V^{(r)}(\phi)(-\Box)^l V^{(r)}(\phi)$$

Similarly at quadratic order one has VZ and Z^2 counterterms

Cubic c.t. can also be computed. Non local divergent terms are present in separate diagrams but cancel in the sum.

$$(k=2, n=1+2m)$$

Beta functionals at quadratic order (dimensionless) from melon diagrams

m 2m -loops

$$\beta_v = -dv + \frac{d-4+\eta}{2}\varphi v^{(1)} + \frac{v^{(m+1)}z^{(m-1)}}{(m+1)!} + \frac{(v^{(2m+1)})^2}{(2m+1)!}$$

2m m m 3m -loops

$$\beta_z = -2z + \frac{d-4+\eta}{2}\varphi z^{(1)} + 2\frac{v^{(2m+1)}z^{(2m+1)}}{(2m+1)!} + \frac{z^{(m+1)}z^{(m-1)}}{(m+1)!} + \frac{3m+2}{2(2m+1)}\frac{(z^{(m)})^2}{(m+1)!} - \frac{2(m+1)}{(2m+1)}\frac{(v^{(3m+2)})^2}{(3m+1)!}$$

4m 3m 2m -loops

$$\beta_{w_1} = \eta w_1 + \frac{d-4+\eta}{2}\varphi w_1^{(1)} + \frac{(m+1)^2\Gamma(\delta_c)}{m^4\Gamma(4+\delta_c)}\frac{(v^{(4m+2)})^2}{(4m+1)!} - \frac{(m+1)\Gamma(\delta_c)}{m^3\Gamma(3+\delta_c)}\frac{z^{(3m)}v^{(3m+2)}}{(3m+1)!} - \frac{\Gamma(\delta_c)}{m^2\Gamma(2+\delta_c)}\frac{(z^{(2m)})^2}{2(3m+1)(2m)!}$$

β_{w_1} defines the anomalous dimension $\eta = -\beta_{w_1}(\varphi=0)$ at FP

$\beta_{w_{2,3}}$ needed if looking for properties of some irrelevant operators.

$$(k=2, n=1+2m)$$

Pattern of mixing

$1\quad \phi$	$\cdots$	ϕ^{2m+1}	$\phi^{2(m+1)}$	$\cdots$	ϕ^{4m+1}	$\phi^{2(2m+1)}$	$\cdots$	ϕ^{6m+1}	$\cdots$	
			1	$\cdots$	ϕ^{2m-1}	ϕ^{2m}	$\cdots$	ϕ^{4m-1}	$\cdots$	$\times(\partial\phi)^2$
						1	$\cdots$	ϕ^{2m-1}	$\cdots$	$\times(\partial^2\phi)^2$

Fixed points

$$2m\epsilon g = \frac{(2(2m+1))!(2m)!}{(3m+1)!(m+1)!^2}\, g\, h + \frac{(2(2m+1))!^2}{(2m+1)!^3}\, g^2$$

$$m\epsilon h = \left[\frac{3m+2}{2(2m+1)} + \frac{m}{m+1}\right] \frac{(2m)!^2}{(m+1)!m!^2}\, h^2 - \frac{2(m+1)}{2m+1}\, \frac{(2(2m+1))!^2}{(3m+1)!m!^2}\, g^2$$

Anomalous dimensions (from triangular part of the stability matrix)

$$\tilde{\gamma}_1 = \frac{\Gamma(\delta_c)}{m^2\Gamma(2+\delta_c)}\, \frac{(2m)!}{4(3m+1)} h^2 - \frac{\Gamma(\delta_c)}{m^4\Gamma(4+\delta_c)} 2(m+1)^2(2m+1)^2(4m+1)!\, g^2$$

$$\tilde{\gamma}_i = \frac{i!}{(i-m-1)!}\, \frac{(2m)!}{(m+1)!^2}\, h + \frac{i!}{(i-2m-1)!}\, \frac{2(2(2m+1))!}{(2m+1)!^2}\, g, \qquad i = m+1, \cdots, 3m+1$$

$$\tilde{\omega}_{m-1} = \frac{(2m)!}{(m+1)!} h$$ (first non zero)

Others have non trivial mixing giving complicated expressions.

$$(k=2, n=1{+}2m)$$

One fixed point is simple:
pure derivative interaction

$$h = \frac{2m(m+1)(2m+1)}{2+7m(m+1)} \frac{(m+1)!m!^2}{(2m)!^2}\epsilon \qquad g=0$$

Anomalous dimensions

$$\eta = \frac{\Gamma(\delta_c)}{\Gamma(2{+}\delta_c)} \frac{2(m{+}1)^2(2m{+}1)^2}{(3m{+}1)(2{+}7m(m{+}1))^2} \frac{(m{+}1)!^2 m!^4}{(2m)!^3}\epsilon^2$$

$$\tilde{\gamma}_i = \frac{i!}{(i{-}m{-}1)!} \frac{2m(2m+1)}{2+7m(m+1)} \frac{m!}{(2m)!}\epsilon \qquad i>m$$

$$\tilde{\omega}_i = (i{-}2m{-}2)!\left[\frac{3m+2}{2m+1}\frac{m+1}{(i{-}3m{-}2)!} + \frac{1}{(i{-}3m{-}3)!} + \frac{m(m+1)}{(i-3m-1)!}\right] \frac{2m(2m+1)}{2+7m(m+1)} \frac{m!}{(2m)!}\epsilon \qquad i>3m$$

At order $O(\epsilon)$ they are associated to all operators in V and Z terms.

Some explicit results for k=2, n=3

$$\mathcal{L}_{FP} = \frac{1}{2}\phi\,\Box^2\phi + \frac{1}{2}h\phi^2(\partial\phi)^2 + g\phi^6 \qquad\qquad d_c = 6$$

$$\tilde{\gamma}_1 = -90\,g^2 + \frac{1}{16}\,h^2$$

$$\tilde{\gamma}_i = 20\,i(i-1)(m-2)\,g + \frac{1}{2}i(i-1)\,h\, + \mathcal{O}(\text{coup}^2), \qquad i = 2,3,4$$

$$\tilde{\omega}_0 = h$$

Scaling relation $\quad \tilde{\gamma}_1 = \frac{\eta}{2}, \qquad \tilde{\gamma}_{2n-1} = (n-1)\epsilon - \frac{\eta}{2}$

There are 3 non trivial fixed points

$g = 0$	$g = \frac{(3\sqrt{138}-13)\,\epsilon}{11100}$	$g = -\frac{(13+3\sqrt{138})\,\epsilon}{11100}$
$h = \frac{3\epsilon}{8}$	$h = \frac{1}{185}\left(42-4\sqrt{138}\right)\epsilon$	$h = \frac{2}{185}\left(21+2\sqrt{138}\right)\epsilon$

Anomalous dimensions

Anomalous dimensions $\tilde{\gamma}$

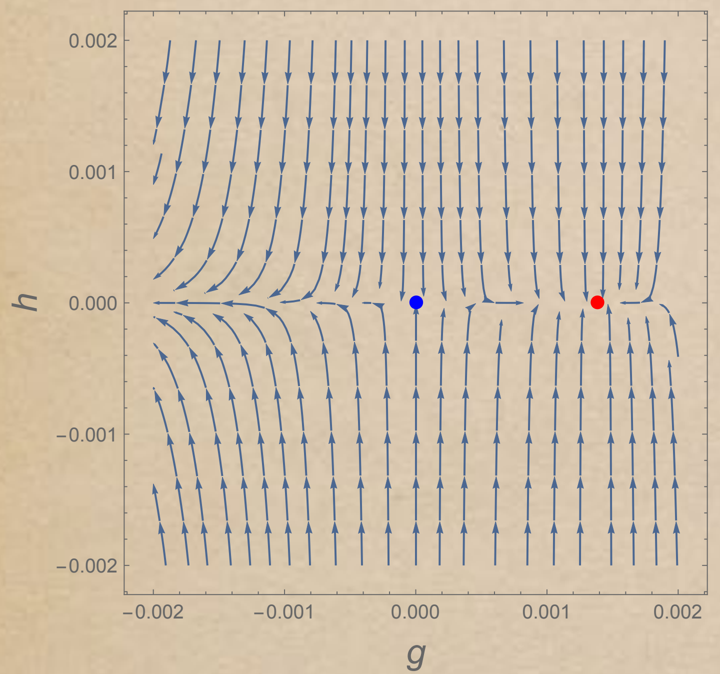
$\tilde{\gamma}_1 = \frac{9\epsilon^2}{1024}$	$\tilde{\gamma}_1 = \frac{(8519-762\sqrt{138})\,\epsilon^2}{1369000}$	$\tilde{\gamma}_1 = \frac{(8519+762\sqrt{138})\,\epsilon^2}{1369000}$
$\tilde{\gamma}_2 = \frac{3\epsilon}{8}$	$\tilde{\gamma}_2 = \frac{1}{185}\left(42-4\sqrt{138}\right)\epsilon$	$\tilde{\gamma}_2 = \frac{2}{185}\left(21+2\sqrt{138}\right)\epsilon$
$\tilde{\gamma}_3 = \frac{9\epsilon}{8}$	$\tilde{\gamma}_3 = \frac{2}{185}\left(50-3\sqrt{138}\right)\epsilon$	$\tilde{\gamma}_3 = \frac{2}{185}\left(50+3\sqrt{138}\right)\epsilon$
$\tilde{\gamma}_4 = \frac{9\epsilon}{4}$	$\tilde{\gamma}_4 = \frac{4\epsilon}{5}$	$\tilde{\gamma}_4 = \frac{4\epsilon}{5}$
$\vdots$	$\tilde{\gamma}_5 = 2\epsilon - \tilde{\gamma}_1$	

Pattern at k=2

Two kind of perturbative phase diagrams

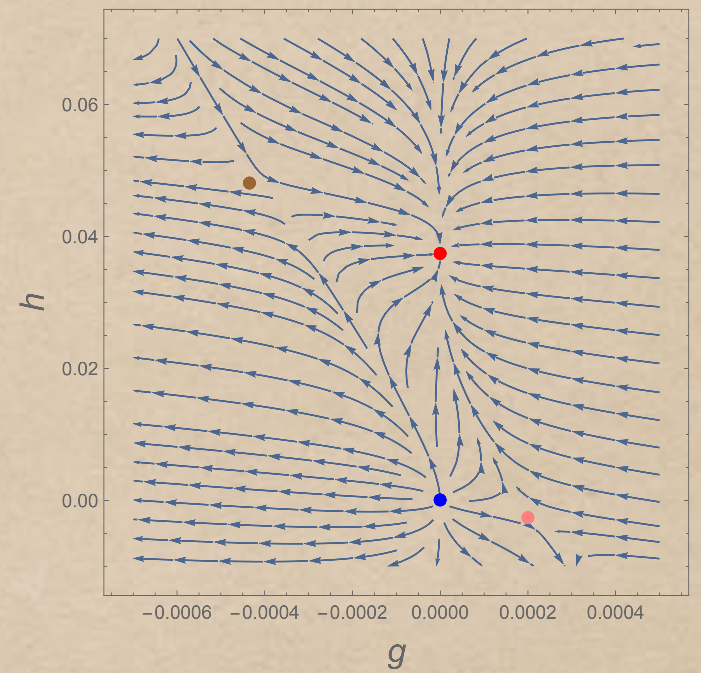
First kind

$n = 2$ (even)



Second kind

$n = 3$ (odd)



Red FP is IR attractive in (g,h)

The other two FPs have one more UV attractive direction.

They can also represent new Asympotically Safe theories but non unitary!

CFT

Using SDE based on: $\Box^2\phi = -2(2m+1)g\phi^{4m+1} + mh\phi^{2m-1}(\partial\phi)^2 + h\phi^{2m}\Box\phi$

Comparison of the anomalous dimensions

- γ_1

$$\Box_x^2\Box_y^2\langle\phi_x\phi_y\rangle \overset{\text{LO}}{=} -2^9\gamma_1\, c\,|x-y|^{-\frac{2}{m}-8}\prod_{i=0}^{3}(i+1/m)$$

$$\langle\Box_x^2\phi_x\Box_y^2\phi_y\rangle \overset{\text{LO}}{=} 4(2m+1)^2(4m+1)!g^2c^{4m+1}|x-y|^{-\frac{2}{m}-8} - 8(2m+1)!m^{-2}h^2c^{2m+1}|x-y|^{-\frac{2}{m}-8}$$

To compare with RG, rescale the couplings accordingly

- γ_i

$$\Box_x^2\langle\phi_x\phi_y^i\phi_z^{i+1}\rangle = \langle\left[-2(2m+1)g\phi^{4m+1} + mh\phi^{2m-1}(\partial\phi)^2 + h\phi^{2m}\Box\phi\right]_x\phi_y^i\phi_z^{i+1}\rangle$$

Evaluate r.h.s. at tree level and normalize the couplings $\longrightarrow$ $\gamma_{i+1}-\gamma_i = \binom{i}{m}\frac{(2m)!}{(m+1)!}h + 4\binom{i}{2m}\frac{(4m+1)!}{(2m)!}g$

Solve the recurrence relation with b.c. $\gamma_1 = O(g^2) \simeq 0$

Agreement between RG and CFT

Conclusions

- In perturbation theory it is possible to obtain leading non trivial results with renormalization group and with CFT techniques for the conformal universal data. Complete agreement where results overlap.
- This approach works both for unitary and non unitary theories. Tested on some non trivial scalar theories.
- Among non unitary theories we have identified and studied a non trivial universality class in 3 dimensions: the Blume-Capel or tri-critical Lee-Yang.
- To study higher derivative scalar theories we have employed RG which relies neither on unitarity nor conformal invariance
- Allows to identify scale invariant deformations of higher derivative free CFTs.
- In particular for theories of the "second type" pure potential deformations are not scale invariant (k=2, n=2m+1)
- We have confirmed most of our results using SDE and assuming conformal symmetry (provides evidence for conformal invariance)

Outlook

- Higher derivative models of “second type”: alternative CFT approaches (Conformal Bootstrap or structure of conformal blocks)
- Extension to higher derivative theories with odd interactions
- Higher order corrections
- Global symmetries: e.g. O(N) models
- Can we improve the methods on both RG and CFT sides?
- Theories for fields with non zero spin content
- More geometrical formulation of RG flow of QFTs
- Natural question: can one extend to a functional non perturbative RG framework some of these ideas?

Thank you!

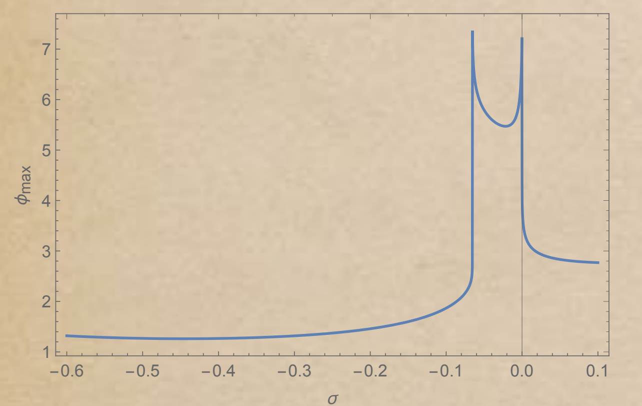
Non perturbative FRG

What about a non perturbative analysis of these models?
Conformal bootstrap may not be easy to apply, due to the lack of unitarity
One can proceed with FRG…

Let us consider the simpler case k=2 with <u>even n</u>
Use LPA with a cutoff: $R_k(p^2) = (k^4 - p^4)\theta(k^4 - p^4)$

Critical dimensions

$$8, \frac{16}{3}, \frac{24}{5}, \frac{32}{7}, \ldots$$



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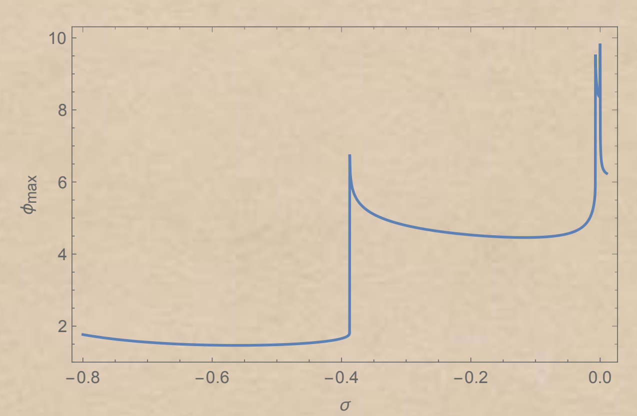

d=5

Leading non trivial eigenvalues for the ϕ^4 theory